


Approximation Model Guided Selection for Evolutionary Multiobjective Optimization

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Outline

- Background
- Approximation Model Guided Selection
- Experimental Results
- Discussions & Conclusions

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Multiobjective Optimization Problem

○ Definition

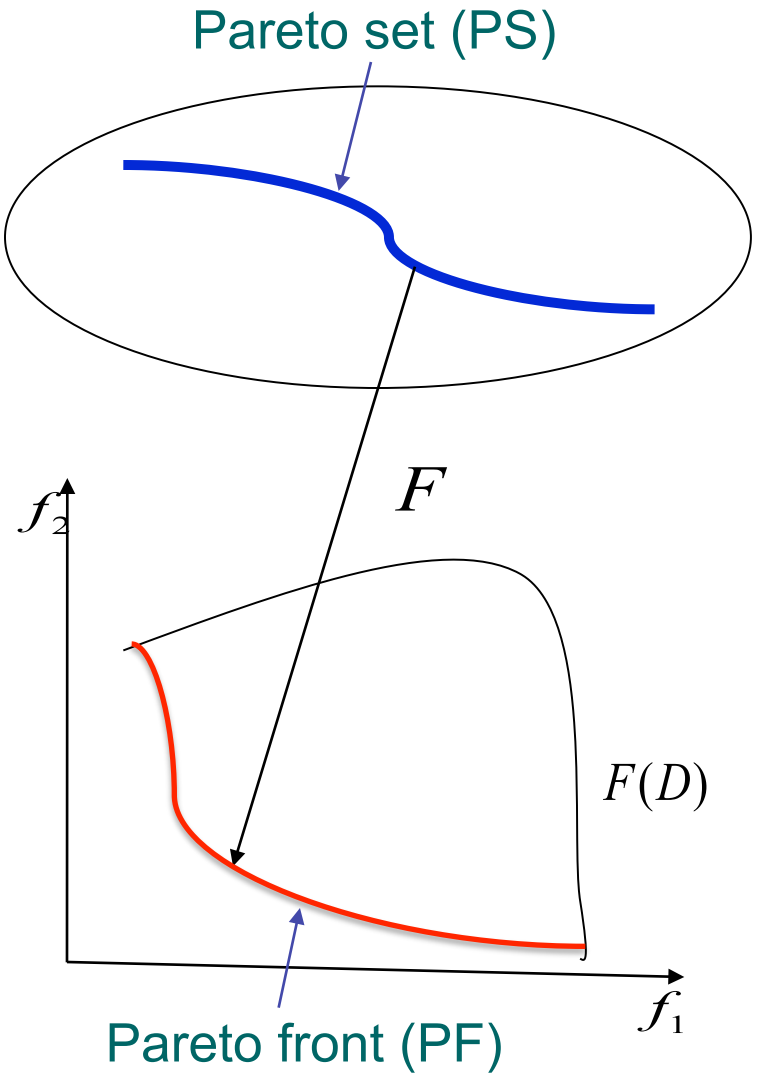
$$\min F(x) = (f_1(x), f_2(x), \ldots, f_m(x))$$
$$s.t \quad x \in D$$

where

D: decision (variable) space.

$f_i: D \to R$, objective function

$F: D \to R^m$, objective vector function



○ Pareto domination

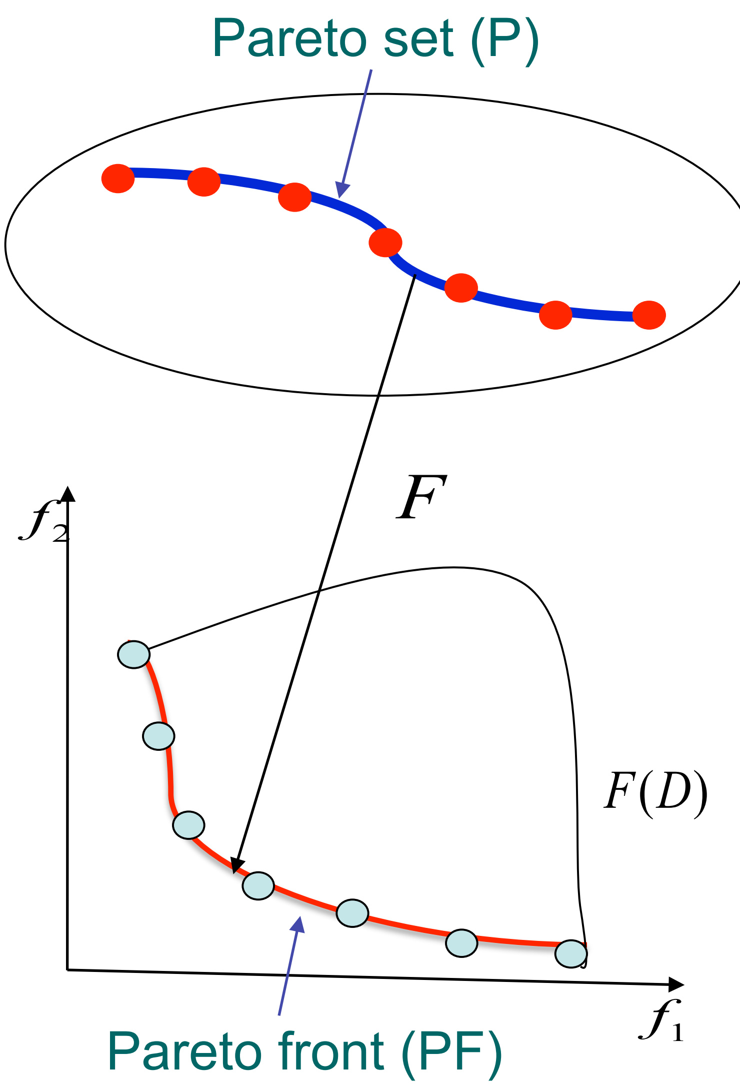
○ Optimum

- Pareto set (PS)
- Pareto front (PF)

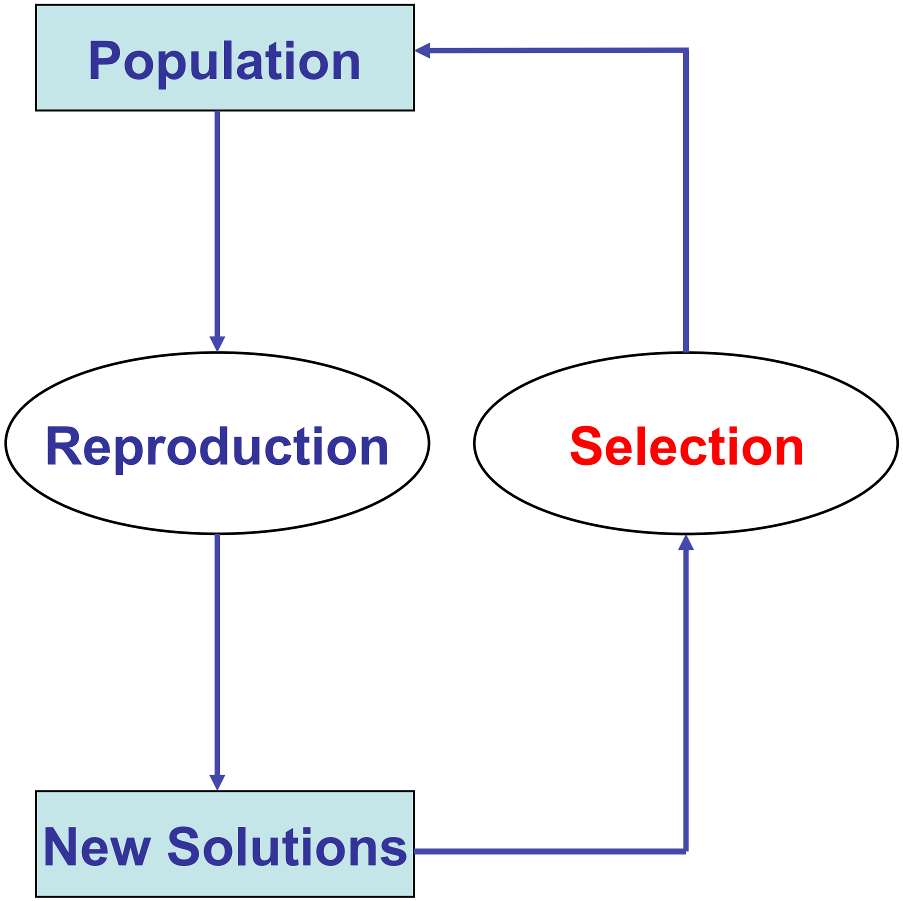
Target of MOEA

- Find an approximation set, which is
  - as **diverse** as possible
  - as **close** to the PF (PS) as possible

Lower dimensional problems!

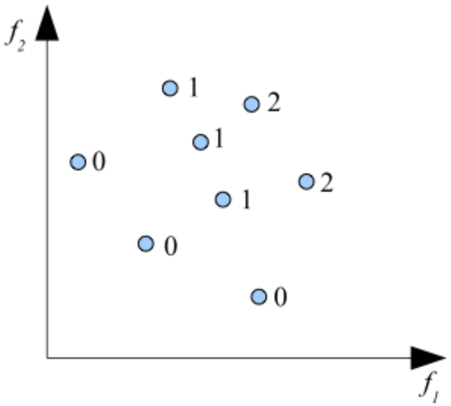


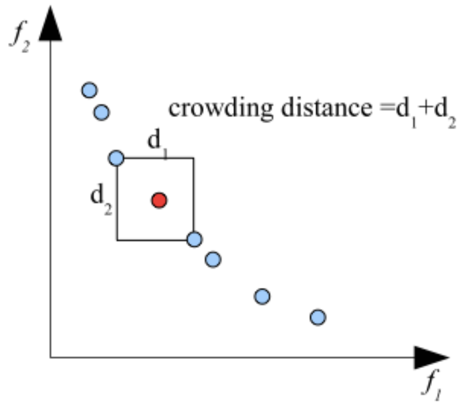
A General MOEA Framework



- **Reproduction**
  - Generate new trial solutions
- **Selection**
  - Select fittest ones into the next generation

Dominance based Selection



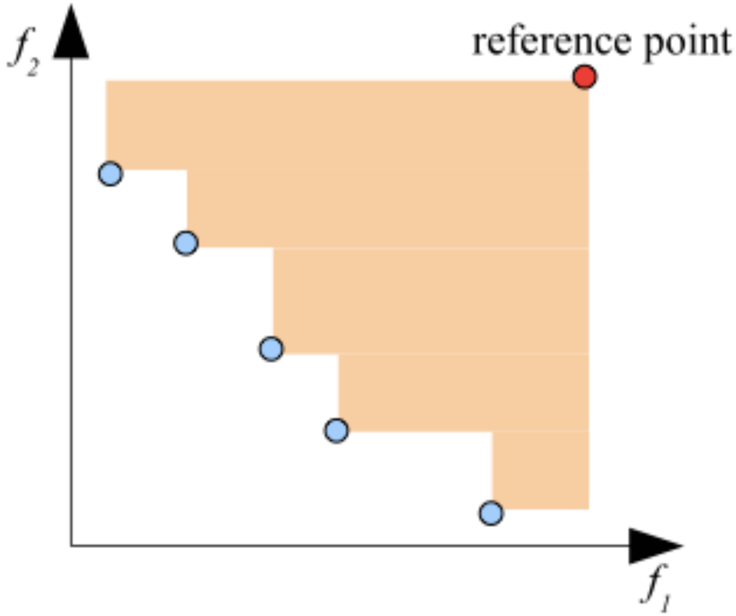


- **Step 1: rank population**
  - dominance rank
  - dominance count
  - dominance strength
- **Step 2: estimate density**
  - niche and fitness sharing
  - crowding distance
  - K-nearest neighbor
  - gridding

$$x \prec_i y, \text{ iff } \begin{cases} (x^{rnk} < y^{rnk}), \text{ or} \\ (x^{rnk} = y^{rnk} \text{ and } x^{den} < y^{den}) \end{cases}$$

Define a complete order over individuals!

Indicator based Selection

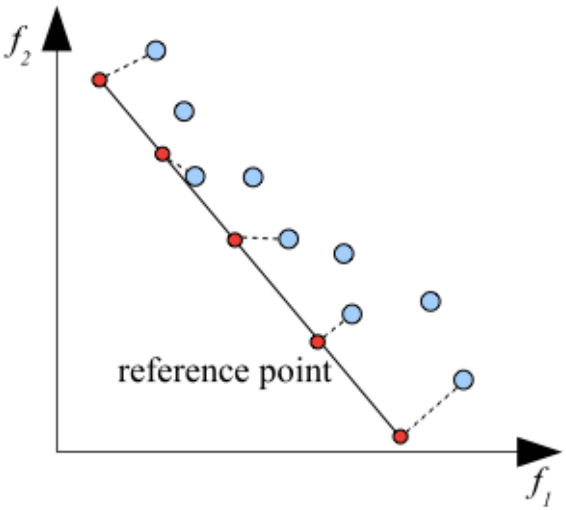


- Convert an MOP into an SOP
  - Obj. = performance metric

$$P \prec_p Q \text{ iff } I(P) < I(Q)$$

Define a complete order over populations!

Model Guided Selection



- Step 1: model the PF
- Step 2: choose reference points
- Step 3: select promising solutions

Much work needs to be done along this direction!

- **H. J. F. Moen, et al.,** *Many-objective Optimization Using Taxi-Cab Surface Evolutionary Algorithm, EMO, 2013.*
- **H. Jain, and K. Deb,** *An improved Adaptive Appraoch for Elitist Nondominated Sorting Genetic Algorithm for Many-Objective Optimization, EMO, 2013*.

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AMS Framework

Step 0: Set $P = \emptyset$.

Step 1: Build a utopian PF by using information extracted from Q to approximate the true PF.

Step 2: Define N single objective functions $G = \{g^i | i = 1, \cdots, N\}$ based on the utopian PF.

Step 3: Randomly choose $g \in G$, and find:

$$x^* = \arg\min_{x \in Q} g(x),$$

set $Q = Q \backslash \{x^*\}$, $G = G \backslash \{g\}$ and $P = P \cup \{x^*\}$.

Step 4: Repeat *Step 3* until $G = \emptyset$.

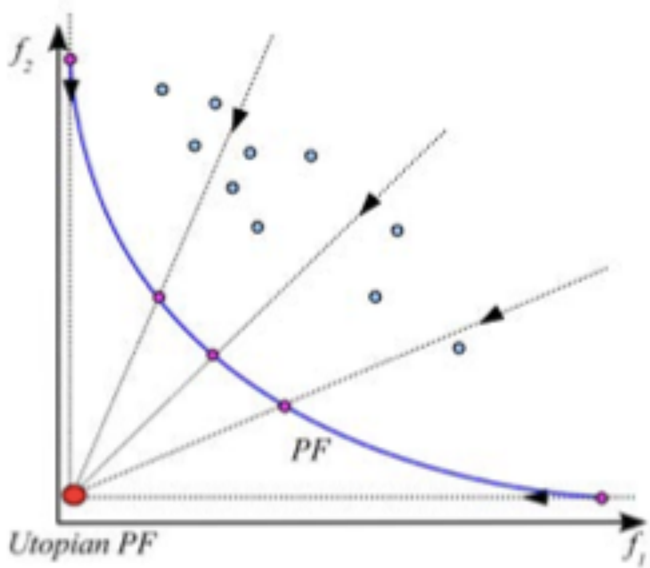
Model PF

Define sub-problem

Select

- **A. Zhou**, *Estimation of Distribution Algorithms for Continuous Multiobjective Optimization*, *Ph.D Thesis, University of Essex, 2009. (Chapter 5.3)*
- **A. Zhou, Q. Zhang, Y. Jin, and B. Sendhoff,** *Combination of EDA and DE for Continuous Biobjective Optimization, CEC 2008.*
- **Q. Zhang and H. Li**, *MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition, IEEE Trans. on Evolutionary Computation, 2007.*

Zero-order Approximation(AMS0)



- A single point to approximate the PF
- Model (ideal point)

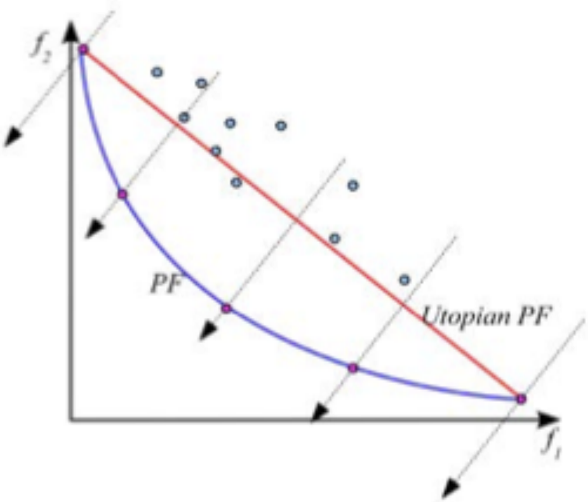
$$z_i^* = \min_{x \in Q} f_i(x), i = 1, \cdots, m.$$

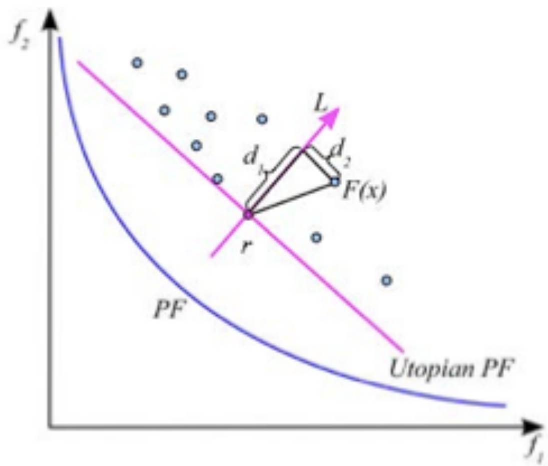
- Distance to utopian PF (sub-problem)

$$g^i(x) = g(x|\lambda^i, z^*) = \max_{1 \leq j \leq m} \lambda_j^i |f_j(x) - z_j^*|.$$

Simple, but does not consider the shape of PF!

First-order Approximation(AMS1)





- A simplex to approximate the PF
- Model (vertices of simplex)

$$v_j^i = \begin{cases} \min_{x \in NS(Q)} f_j(x) \text{ if } j \neq i \\ \max_{x \in NS(Q)} f_j(x) \text{ if } j = i \end{cases}$$

for $i, j = 1, \cdots, m.$

- Distance to simplex

$$g^i(x) = g(x|r^i) = d_1 + 2d_2$$

Simple, and consider the shape of PF in a sense!

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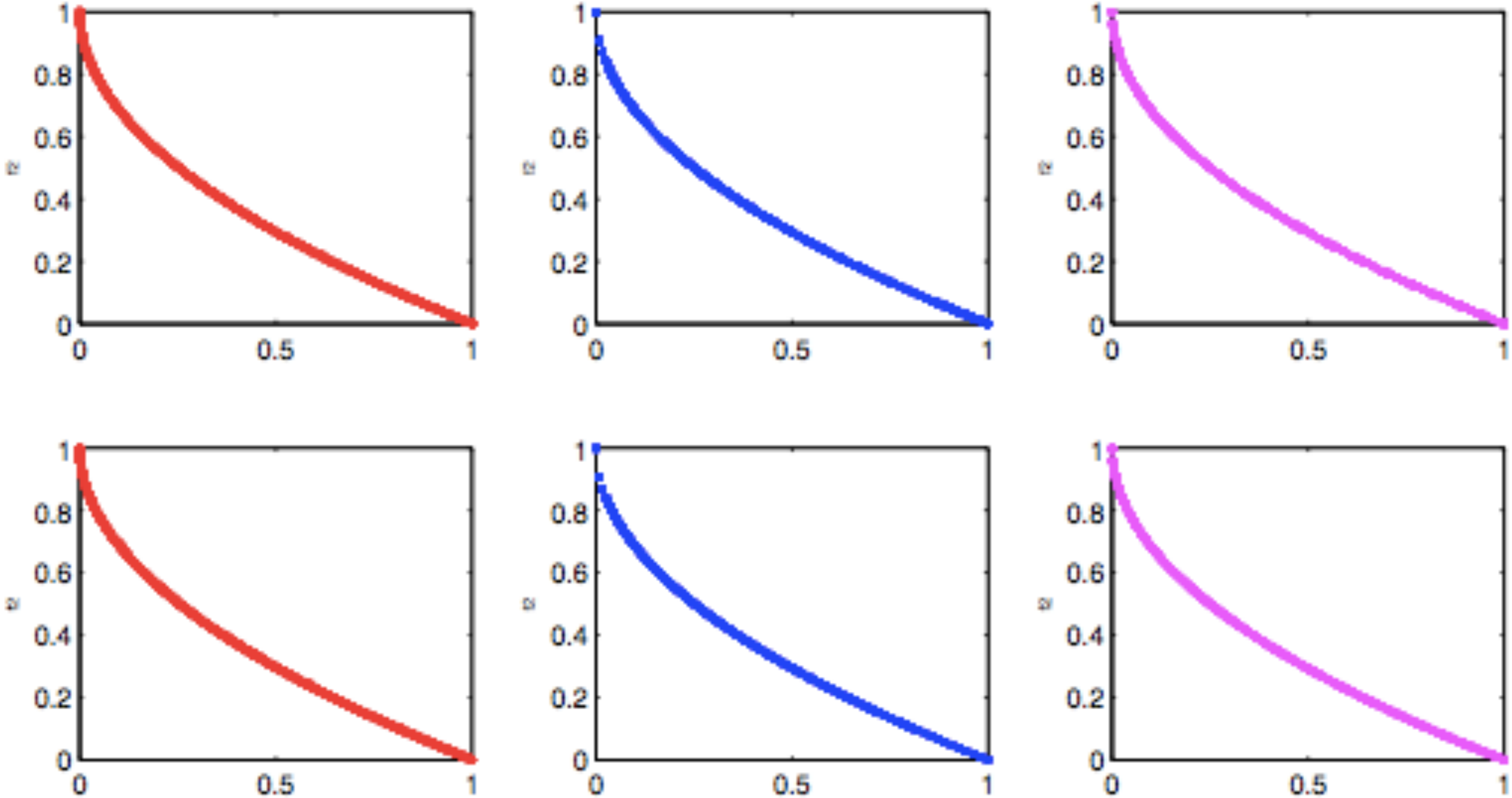
Settings

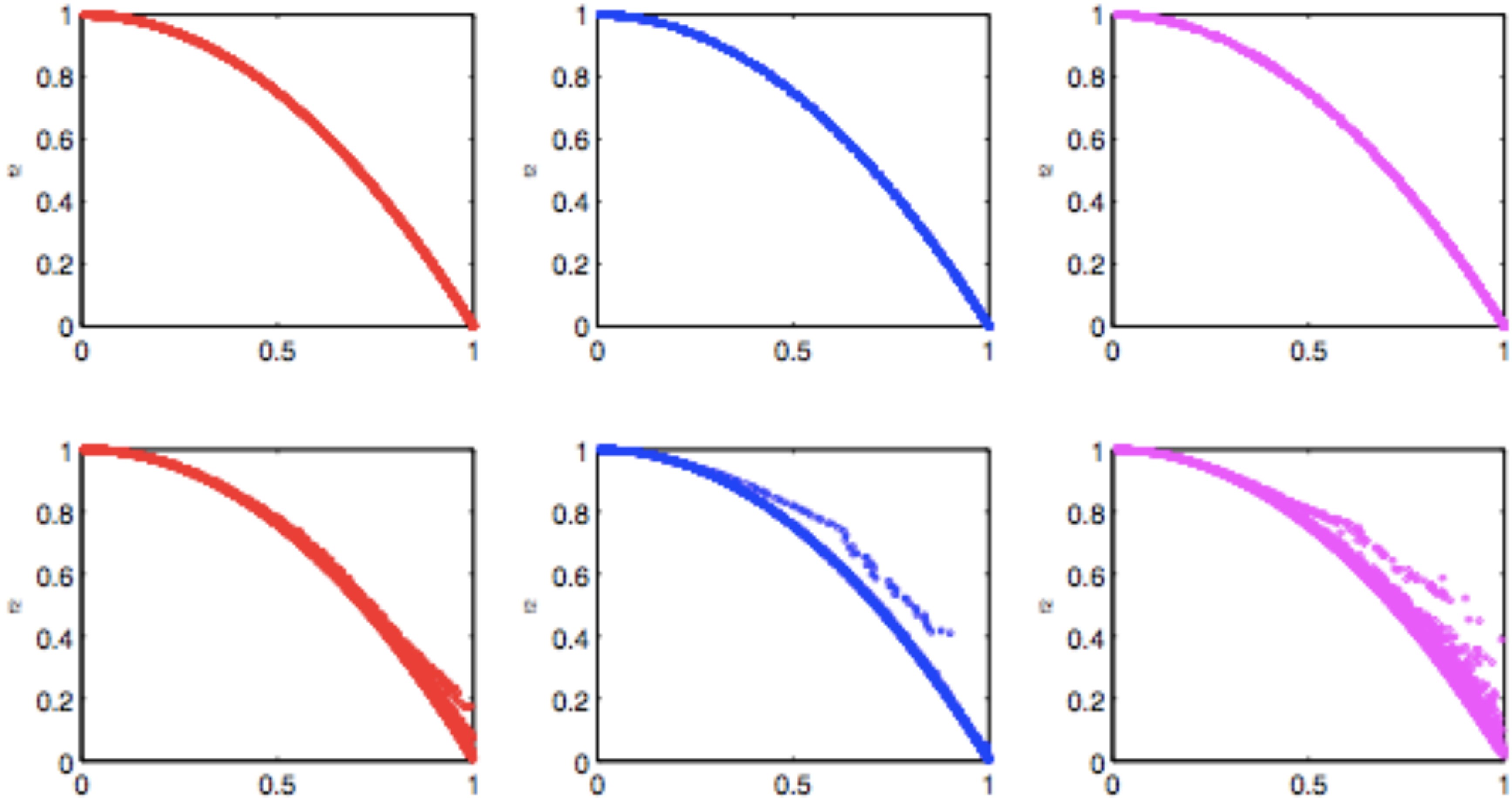
- Offspring reproduction operator
  - A probability model based reproduction operator (RM-MEDA)
- Comparison strategy
  - NDS: non-dominated sorting scheme (NSGA-II)
- Test instances
  - 8 instances with different properties
- Performance metric
  - IGD: inverted general distance

Convex PF



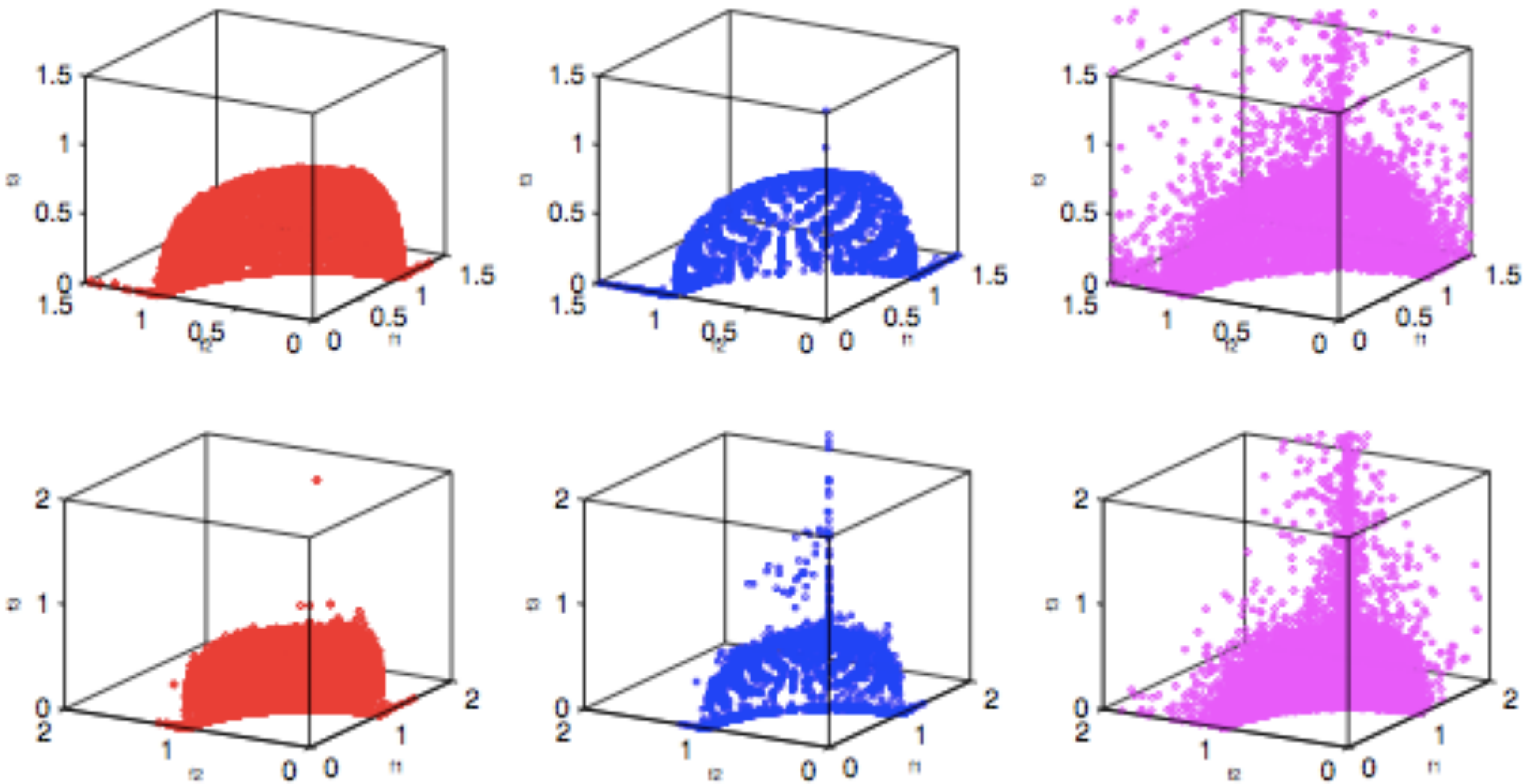
NDS **AMS0** **AMS1**

Concave PF



NDS **AMS0** **AMS1**

Tri-objective MOP



NDS **AMS0** **AMS1**

Statistical Results

Table 1. Statistical *IGD* values (mean±std.) on the test instances over 30 runs

	NDS	AMS0	AMS1
F_1	0.0043 ± 0.0001	0.0039 ± 0.0000	**0.0036** ± 0.0000
F_2	0.0041 ± 0.0001	**0.0038** ± 0.0000	**0.0038** ± 0.0000
F_3	0.0057 ± 0.0041	0.0154 ± 0.0102	**0.0032** ± 0.0011
F_4	0.0494 ± 0.0015	**0.0490** ± 0.0012	0.0830 ± 0.0549
F_5	0.0050 ± 0.0002	0.0042 ± 0.0001	**0.0038** ± 0.0001
F_6	0.0122 ± 0.0086	**0.0094** ± 0.0205	0.0333 ± 0.0560
F_7	0.1365 ± 0.1703	**0.1011** ± 0.0167	0.2044 ± 0.2616
F_8	**0.0657** ± 0.0041	0.0769 ± 0.0543	0.0830 ± 0.0405

NDS:1 AMS0:4 AMS1:4

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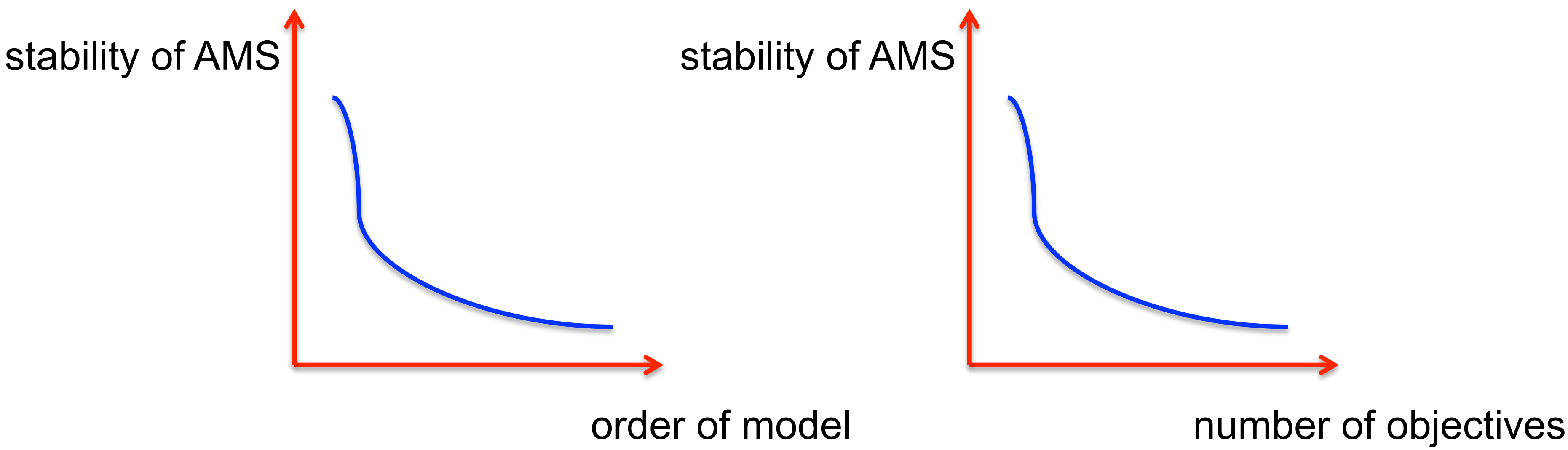
- AMS works better than NDS in most of the test instances
- AMS0 is more stable than AMS1
- AMS1 works better than AMS0 if a good simplex model can be found



- How to build a high-quality model?
  - model should be cheap
  - model should be stable
- What's the performance on complicated problems?
  - non-concave (non-convex)
  - with disconnected PF
- What's the performance on many-objective problems?
  - interesting sub-problems (reference points, targets points)

Thanks!

The source code is available from amzhou@cs.ecnu.edu.cn