Generalized Decomposition

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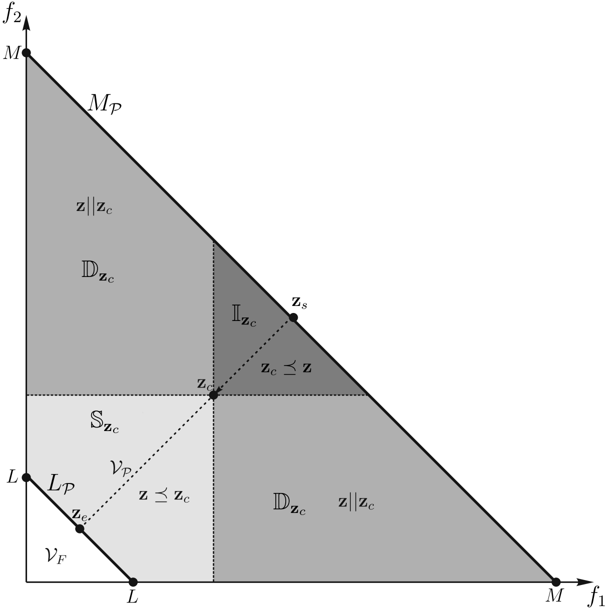
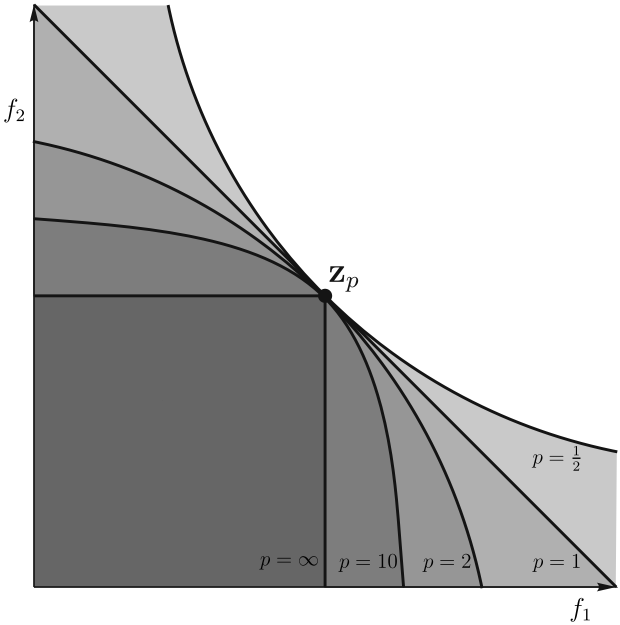
Contents

Why decomposition for many-objective problems?

- Search can be directed at will.
- Variety of norms and scalarizing functions (we'll see why this is beneficial).
- No incomparable solutions with respect to the ℓ_p-norm used.
- It is unclear how to distribute solutions along the Pareto front
- Not every scalarizing function can guarantee that all Pareto optimal solutions are reachable.



f_2
M
$M_{\mathcal{P}}$
$\mathbf{z}||\mathbf{z}_c$
$\mathbb{D}_{\mathbf{z}_c}$
$\mathbb{I}_{\mathbf{z}_c}$
$\mathbf{z}_s$
$\mathbf{z}_c \preceq \mathbf{z}$
$\mathbf{z}_c$
$\mathbb{S}_{\mathbf{z}_c}$
$\mathcal{V}_{\mathcal{P}}$
L
$L_{\mathcal{P}}$
$\mathbf{z} \preceq \mathbf{z}_c$
$\mathbb{D}_{\mathbf{z}_c}$
$\mathbf{z}||\mathbf{z}_c$
$\mathbf{z}_e$
$\mathcal{V}_F$
L
M
f_1


f_2
$\mathbf{z}_p$
$p = \frac{1}{2}$
$p = \infty$
$p = 10$
$p = 2$
$p = 1$
f_1

Evenly Distributed

Evenly distributed weighting vectors

$$\left\{\frac{0}{H}, \frac{1}{H}, \ldots, \frac{H}{H}\right\}, \tag{1}$$

Uniformly Distributed

Uniformly distributed weighting vectors

$$\mathbf{w} = \{w_1, \ldots, w_k\},$$
$$w_i = 1 - \sum_{m=1}^{i-1} w_m - (\mathcal{U}(0,1))^{i-k}, \text{ for all } i = 1, \ldots, k. \tag{2}$$

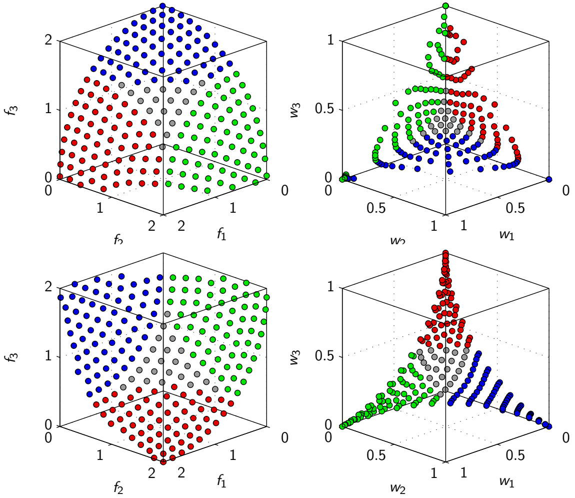
Optimal Weighting Vector Choice

Definition

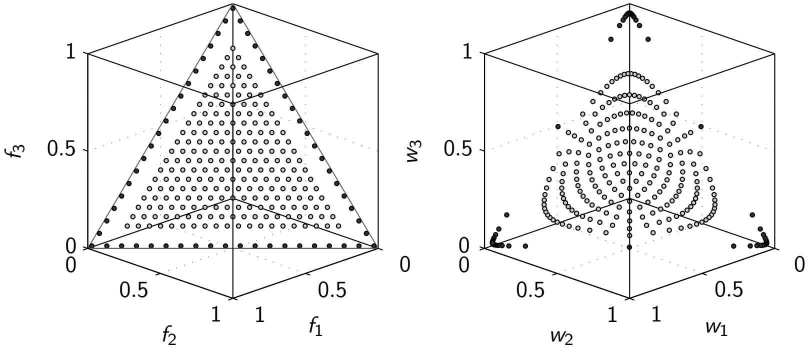
Generalised Decomposition

$$\begin{aligned} &\min_{\mathbf{w}} \|\mathbf{w} \circ \mathbf{F}(\mathbf{x})\|_p \\ &\text{s.t. } \mathbf{1}^T\mathbf{w} = 1 \\ &\mathbf{w} \succeq 0\,, \mathbf{F}(\mathbf{x}) \succeq 0. \end{aligned} \tag{3}$$

Pareto Front Geometries



Practical Considerations



Practical Considerations (cont'd)

Definition

$$\min_{\mathbf{x}} g_p(\mathbf{x}, \mathbf{w}^s, \mathbf{z}^\star) = \| \mathbf{w}^s \circ (|\mathbf{F}(\mathbf{x}) - (\mathbf{z}^\star - \epsilon)| + \rho \sum_{i=1}^{k} |f_i(\mathbf{x}) - (z_i^\star - \epsilon)|) \|_p \quad \forall\, s = \{1, \ldots, N\}, \quad \text{subject to } \mathbf{x} \in \mathbf{S}, \tag{4}$$

Practical Considerations (cont'd)

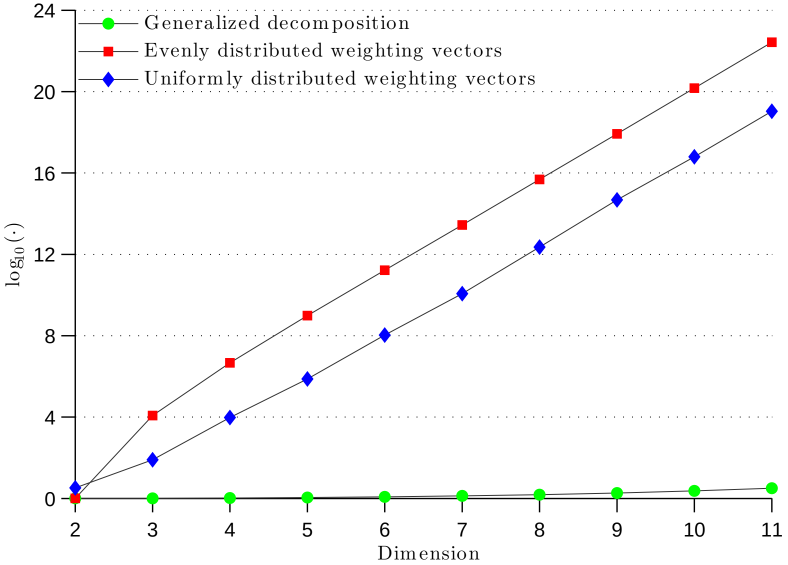
Definition

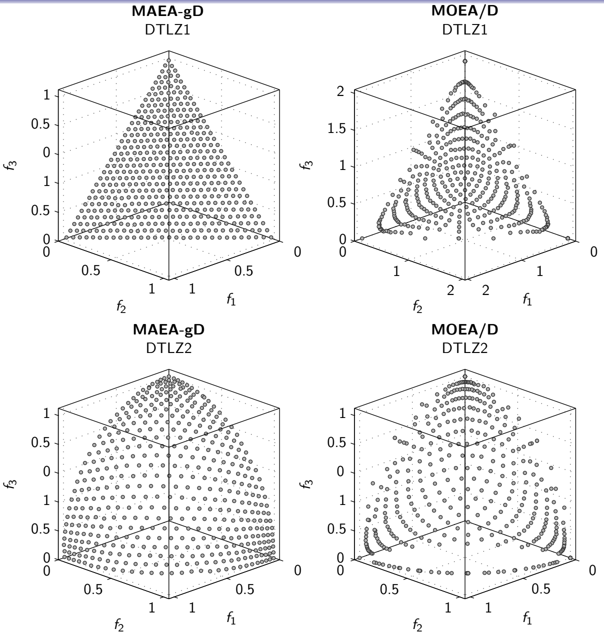
$$\min_{\mathbf{w}} \|\mathbf{w} \circ (\mathbf{F}(\mathbf{x}) + \rho \cdot C(k))\|_p,$$
$$\text{subject to } \sum_{i=1}^{k} w_i = 1, \tag{5}$$
$$\text{and } w_i \geq 0, \forall\, i \in \{1, \ldots, k\}, \mathbf{F}(\mathbf{x}) \geq 0.$$

Definition

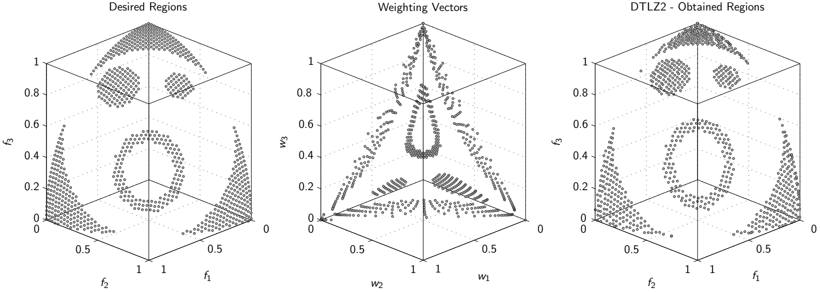
Coulomb Potential Energy

$$E(\mathbf{x}; s) = \sum_{1 \le i \le j \le N} \|\mathbf{x}_i - \mathbf{x}_j\|^{-s},\ s > 0 \tag{6}$$


Generalized decomposition
Evenly distributed weighting vectors
Uniformly distributed weighting vectors
24
20
16
12
8
4
0
$\log_{10}(\cdot)$
2
3
4
5
6
7
8
9
10
11
Dimension


MAEA-gD
DTLZ1
MOEA/D
DTLZ1
MAEA-gD
DTLZ2
MOEA/D
DTLZ2
f_1
f_2
f_3

Generalized Decomposition - Preference Articulation



Conclusions

Given a measure and a scalarizing function and information about the Pareto front geometry, generalized decomposition can:

- Identify the optimal weighting vectors that minimize the given measure.
- Direct solutions in specific regions of the Pareto front.

Why a Different Scalarization?