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Periodic boundary conditions (box, side L)

$$\psi(x+L,y,z)=\psi(x,y,z)$$

$$\psi_k(r)=e^{ik.r}=e^{i(k_x x+k_y y+k_z z)}$$ is solution

provided that k_x = 0, ±2π/L, ±4π/L .... 2πn/L, where n is a positive or negative integer

Proof: $$\exp ik_x(x+L)=\exp i\frac{2\pi n}{L}(x+L)$$

$$=\exp i\frac{2\pi nx}{L}\exp i2\pi n$$ → $\cos 2\pi n+i\sin 2\pi n=1+0=1$

$$=\exp\frac{i2\pi nx}{L}=\exp ik_x x$$

Substitute $\psi_k(r) = e^{ik.r}$ into Schrödinger equation gives

$$E_k = \frac{\hbar^2}{2m}(k_x^2 + k_y^2 + k_z^2) = \frac{\hbar^2}{2m}k^2$$