$\boxed{2011}$ PHY 330: second problem sheet.

a) i) $\hbar\omega = E_g + \frac{\hbar^2 k_e^2}{2m_e} + \frac{\hbar^2 k_h^2}{2m_h}$; $k_e \approx k_h \equiv k$

$$k = \sqrt{(\hbar\omega - E_g)\cdot\frac{2m_e m_h}{(m_e+m_h)}\cdot\frac{1}{\hbar^2}}$$

ii) $E_g = 0{,}8\,eV$; $m_e = 0{,}014\,m_0$; $\hbar\omega = 0{,}5\,eV$

$m_h = 0{,}18\,m_0$;

$$\hbar\omega = E_g + \frac{\hbar^2 k_e^2}{2m_e} + \frac{\hbar^2 k_h^2}{2m_h} \qquad k_e = k_h \equiv k$$

$$0{,}32\,eV = \frac{\hbar^2 k^2}{2m_0}\left(\frac{1}{0{,}014} + \frac{1}{0{,}18}\right) = \frac{\hbar^2 \cdot k^2}{2m_0}\cdot(71{,}42 + 5{,}55) =$$

$$= 76{,}97\cdot\frac{\hbar^2 k^2}{2m_0} \Rightarrow \frac{\hbar^2 k^2}{2m_0} = 0{,}00415\,eV$$

<u>Electron kinetic energy</u> $\frac{\hbar^2 k^2}{2\cdot m_0\cdot 0{,}014} = \frac{0{,}00415\,eV}{0{,}014} = 0{,}296\,eV$

<u>hole kinetic energy</u> $= \frac{\hbar^2 k^2}{2m_0\cdot 0{,}18} = \underline{0{,}023\,eV}$

b) Donor is an impurity atom, ~~sub~~ which has a valency higher than that of atom in crystal lattice. when donor substitutes one of the atoms in crystal lattice it provides extra electron to conduction band in a semiconductor

<u>GaAs:</u> donor S, Se, Te if on As site

Si on Ga site

Acceptor C on As site

Zn on Ga site.

c)

$$n_e = N_c e^{-\frac{E_d}{kT}} \; ; \; N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

$n_e = 1{,}84 \cdot 10^{10}\ m^{-3}$; $T = 5K$; $m_e = 0{,}56 m_0$

$N_c =$

$kT = 1{,}38 \cdot 10^{-23} \cdot 5$

$$N_c = 2\left(\frac{6{,}28 \cdot 0{,}56 \cdot 9{,}1 \cdot 10^{-31} \cdot 6{,}9 \cdot 10^{-23}}{(6{,}62)^2 \cdot 10^{-68}}\right)^{3/2} =$$

$$= 2 \cdot \left(5 \cdot 10^{14}\right)^{3/2} = 22{,}6 \cdot 10^{21}$$

$$\ln\left(\frac{1{,}84 \cdot 10^{10}}{22{,}6 \cdot 10^{21}}\right) = -\frac{E_d}{6{,}9 \cdot 10^{-23}\ J} =$$

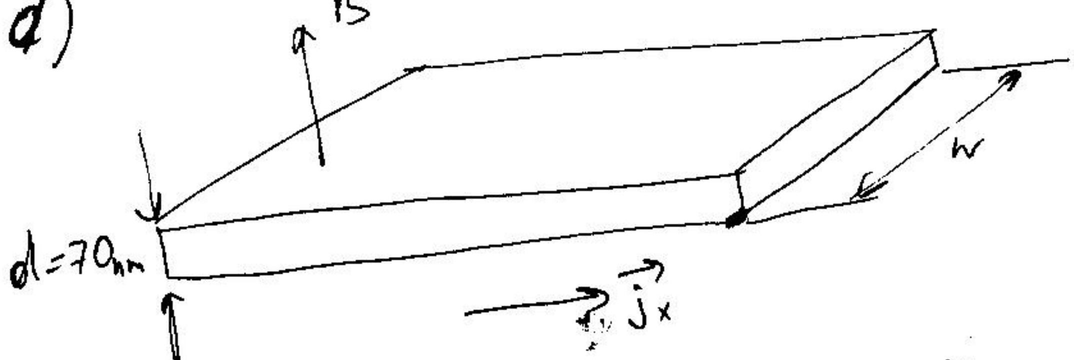
$$= \ln\left(0{,}085 \cdot 10^{-11}\right) = \ln(0{,}085) - 11 \ln 10 =$$

$$= -2{,}45 - 25.32 = -27{,}77;$$

$$E_d = 27{,}77 \cdot 6{,}9 \cdot 10^{-23} = 191 \cdot 10^{-23}\ J =$$

$$= \frac{191 \cdot 10^{-23}}{1{,}6 \cdot 10^{-19}} = 119 \cdot 10^{-4}\ eV = \underline{\underline{11{,}9\ meV}};$$

d)



Hall voltage $V_H = 0{,}72\,\mu V$; Current $I = 0{,}7\,mA$;

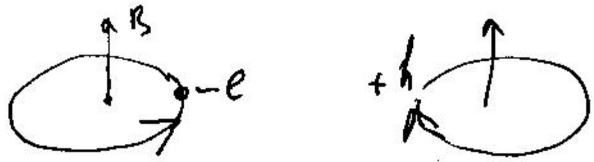
current density $j_x = \frac{I}{w \cdot d} = \frac{0{,}7\,mA}{70\,nm \cdot w}$;

Hall field $E_H = \frac{0{,}72\,\mu V}{w} = R_H \cdot j_x \cdot B = R_H \cdot \frac{0{,}7\,mA}{w \cdot 70\,nm} \cdot 1\,T \Rightarrow$

$$\Rightarrow R_H = \frac{0{,}72 \cdot 10^{-6}\,V \cdot 70 \cdot 10^{-9}\,m}{0{,}7 \cdot 10^{-3}\,A \cdot 1\,T} = 72 \cdot 10^{-12}\,\frac{m^3}{C} = 7{,}2 \cdot 10^{-11}\,\frac{m^3}{C}$$

$$|R_H| = \frac{1}{ne} \Rightarrow n = \frac{1}{R_H \cdot e} = 0{,}86 \cdot 10^{28}\,m^{-3}$$

e) Cyclotron resonance is observed at frequency $\omega_c = \frac{eB}{m_e^*}$ => if we know the external field B, we can obtain electron (or hole) effective mass



electron (hole) move anticlockwise (clockwise) in external B-field. Therefore σ^+ or σ^- circularly polarised AC. E-field is required. => we can obtain sign of the dominant charge carriers in a sample.

Four resonances are observed in Si because

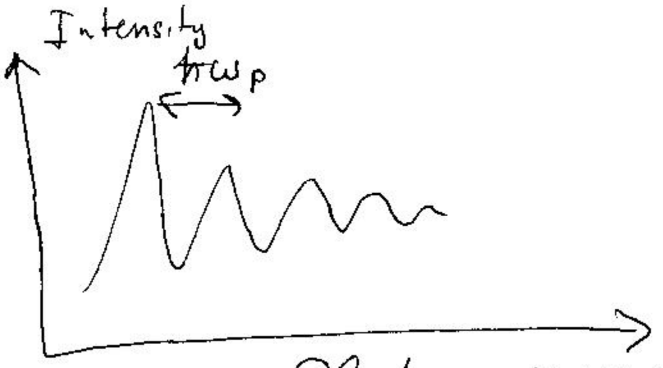
1) electron dispersion is anysotropic, electron travelling in different directions have different effective mass
2) hole valence band is degenerate $_{\text{at } k=0}$ consisting of 2 different energy bands described by different masses.

~~2~~ f) Plasmon is a quant of longitudinal plasma oscillations.

To observe one measures energy loss spectra for electron reflected from metallic films

$$E_{electron}^{incident} = E_{electron}^{reflected} + n \cdot \underbrace{\hbar\omega_p}_{\text{energy of plasmon}}$$

$n = 1, 2, \ldots$ ω_p – plasma frequency.



g) $E_n = \dfrac{n^2 \hbar^2 (2\pi)^2}{8 m_e d^2}$; $m_e = 0{,}066\, m_0$, $d = 140 \cdot 10^{-10}$ m, $m_0 =$

$$E_2 - E_1 = (4-1) \cdot \frac{\hbar^2 \cdot (2\pi)^2}{8 \cdot m_e d^2} = \frac{3 \hbar^2 (2\pi)^2}{8 \cdot 0{,}06 m_0 d^2}$$

$$= \left| \begin{array}{l} m_0 = 9{,}1 \cdot 10^{-31}\, kg \\ \hbar = 6{,}6 \cdot 10^{-34} J \cdot s \end{array} \right| = \frac{3 \cdot 6{,}6^2 \cdot 10^{-68}}{8 \cdot 0{,}06 \cdot 9{,}1 \cdot 10^{-31} \cdot (140 \cdot 10^{-10})^2} =$$

$$= \frac{3 \cdot 6{,}6^2 \cdot 10^{-68}}{8 \cdot 0{,}06 \cdot 9{,}1 \cdot (140)^2 \cdot 10^{-51}} = 0{,}0015 \cdot 10^{-17} J =$$

$$= \frac{0{,}0015 \cdot 10^{-17}}{1{,}6 \cdot 10^{-19}} = 0{,}00093 \cdot 10^2 eV = 0{,}093 eV = \underline{93\ meV}$$