

Contributions

This paper proposes an improved version of the PESA-II algorithm (called IPESA-II) to enhance the performance of the algorithm in terms of

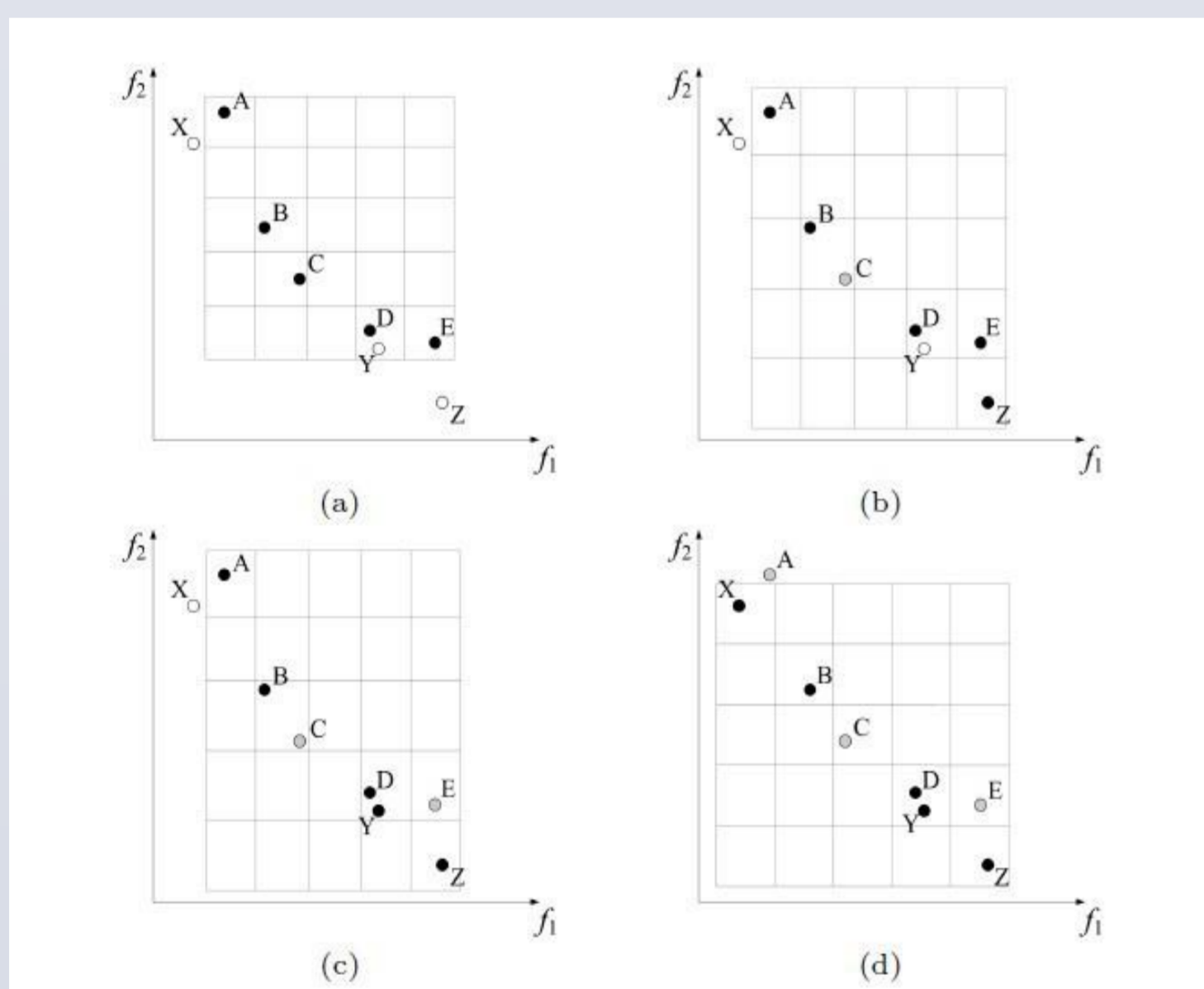
- minimizing the distance from the resulting solutions to the Pareto front (i.e., Convergence)
- maintaining the uniform distribution of the solutions (i.e., uniformity)
- maximizing the distribution range of the solutions along the Pareto front (i.e., extensivity)

Problems of PESA-II

Uniformity Problem

The environmental selection process of the original PESA-II adopts an incremental update mode. Once a candidate has entered the archive, the grid environment will need to be checked. Any excess of the grid boundary or the upper limit of the archive size will lead to the adjustment (or even reconstruction) of the grid environment. This not only causes extra time consumption, but also affects the uniformity of the final archive set since different sequences that the candidates enter the archive result in different distributions.

Figure 1: An example of environmental selection in PESA-II, where individuals A–E are the current members in the archive set (the size of the archive set is five), and individuals X–Z are the candidates to be archived. (b)–(d) show the archiving process of the candidates in the order of Z, Y, and X. Black points correspond to the current individuals in the archive, hollow points stand for the candidates, and gray points denote the individuals removed from the archive.



The final archive set is formed by X, B, D, Y, and Z. Obviously, the obtained archive is not an ideal distribution result. A better archive is that individual C is preserved and either D or Y is removed, which, in fact, is the result of the entry of the candidates into the archive in the order of X, Y, and Z. In addition, if the enter order is Y, Z, and X, both the above results may occur with equal probability.

Extensivity Problem

In PESA-II, extreme solutions do not get special treatment—they could be eliminated with the same probability as other solutions, which leads the solution set of PESA-II to have a poorer distribution range than that of other algorithms.

Convergence Problem

In environmental selection of PESA-II, when the archive set is overfull, a nondominated solution will be randomly eliminated in the most crowded hyperbox. However, the Pareto dominance relation is a qualitative metric of distinguishing individuals, which fails to give a quantitative difference of objective values among individuals. That is, two individuals are incomparable even if the former is largely superior to the latter in most of the objectives but only slightly inferior to the latter in one or a few objectives.

IPESA-II

IPESA-II introduces three simple but effective improvements in the algorithm's environmental selection:

- Maintaining the archive after all individuals in the internal population have entered it, instead of doing step by step.
- Extending the distribution range of the solution set by keeping the boundary individuals.
- Improving the convergence of the solution set by removing the worst-performed individual (i.e., the individual that is farthest away from the best corner of the hyperbox) in the most crowded hyperbox.

Results

We compare IPESA-II with PESA-II and five other well-known EMO algorithms:

- Nondominated Sorting Genetic Algorithm II (NSGA-II)
- Strength Pareto Evolutionary Algorithm 2 (SPEA2)
- Indicator-Based Evolutionary Algorithm (IBEA)
- ϵ -dominance Multiobjective Evolutionary Algorithm (ϵ -MOEA)
- Territory Defining Evolutionary Algorithm (TDEA)

Performance Metrics and Test Problems

- Generational Distance (GD), Spacing (SP), Maximum Spread (MS), and Hypervolume (HV)
- ZDT and DTLZ

Table 1: Comparison results between PESA-II and IPESA-II regarding GD, SP, and MS, where the top and bottom values in each cell correspond to PESA-II and IPESA-II, respectively, and the better mean is highlighted in boldface.

Problem	GD	SP	MS
ZDT1	2.048e-4(1.4e-4) [†] 4.930e-5(2.3e-5)	6.771e-3(5.7e-4) [†] 4.468e-3(2.2e-4)	9.762e-1(1.0e-2) [†] 9.997e-1(1.7e-4)
ZDT2	2.040e-4(8.1e-5) [†] 4.733e-5(2.7e-5)	6.793e-3(6.2e-4) [†] 4.268e-3(3.3e-4)	9.779e-1(1.5e-2) [†] 9.994e-1(3.0e-4)
ZDT3	1.216e-4(4.5e-5) [†] 5.120e-5(1.9e-5)	8.181e-3(1.1e-3) [†] 8.415e-3(7.7e-4)	9.817e-1(1.0e-2) [†] 9.995e-1(5.7e-4)
ZDT4	4.628e-2(8.7e-2) [†] 2.805e-4(1.5e-4)	3.669e-1(7.5e-1) [†] 6.202e-3(7.0e-4)	9.726e-1(3.6e-2) [†] 9.964e-1(1.2e-3)
ZDT6	1.014e-2(9.5e-3) [†] 3.472e-4(7.7e-4)	7.236e-2(8.0e-2) [†] 5.112e-3(7.3e-3)	9.930e-1(3.8e-3) [†] 9.978e-1(2.9e-4)
DTLZ1	6.746e-2(1.0e-1) [†] 2.293e-2(3.5e-2)	1.463e-1(2.0e-1) [†] 5.997e-2(6.4e-2)	9.862e-1(2.3e-2) [†] 9.963e-1(6.4e-3)
DTLZ2	1.500e-3(2.0e-4) [†] 8.186e-4(2.2e-4)	3.674e-2(3.4e-3) [†] 2.905e-2(4.6e-3)	9.901e-1(5.4e-3) [†] 9.998e-1(4.0e-4)
DTLZ3	1.039e+0(8.8e-1) [†] 5.153e-1(6.2e-1)	3.065e+0(2.0e+0) [†] 1.172e+0(2.6e+0)	9.999e-1(3.8e-4) [†] 1.000e+0(0.0e+0)
DTLZ4	1.209e-3(5.7e-4) [†] 7.622e-4(2.7e-4)	3.460e-2(1.6e-2) [†] 3.489e-2(1.6e-2)	8.313e-1(3.7e-1) [†] 9.031e-1(2.0e-1)
DTLZ5	5.703e-4(4.4e-5) [†] 4.577e-4(3.5e-5)	7.385e-3(9.3e-4) [†] 6.232e-3(6.2e-4)	9.894e-1(1.1e-2) [†] 9.980e-1(1.8e-3)
DTLZ6	7.980e-2(1.0e-2) [†] 4.961e-2(1.1e-2)	8.172e-2(4.5e-2) [†] 5.577e-2(3.1e-2)	9.889e-1(3.0e-2) [†] 1.000e+0(1.5e-6)
DTLZ7	3.315e-3(1.1e-3) [†] 2.077e-3(6.8e-4)	4.206e-2(1.4e-2) [†] 4.643e-2(7.5e-3)	8.688e-1(1.4e-1) [†] 9.979e-1(2.4e-3)

^{††} indicates that the p -value of 58 degrees of freedom is significant at a 0.05 level of significance by a two-tailed t -test.

- For the convergence metric GD, IPESA-II outperforms PESA-II in all the 12 problems, and also with statistical significance
- For the uniformity metric SP, IPESA-II outperforms PESA-II in 9 out of the 12 problems, and with statistical significance for 8 problems
- For the extensivity metric MS, IPESA-II outperforms PESA-II in all the 12 problems, and with statistical significance for 11 problems

Figure 2: The final solution set of PESA-II and IPESA-II on ZDT6

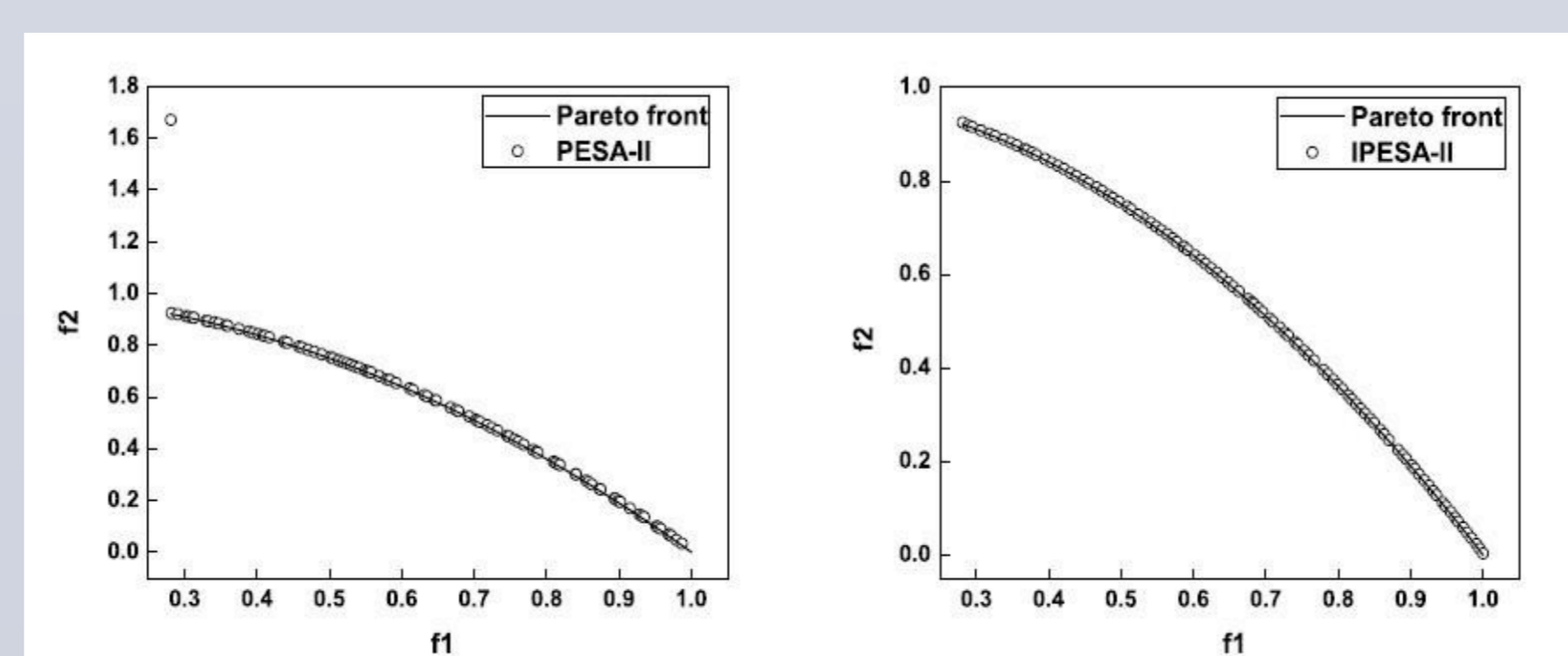


Figure 3: The final solution set of PESA-II and IPESA-II on DTLZ7

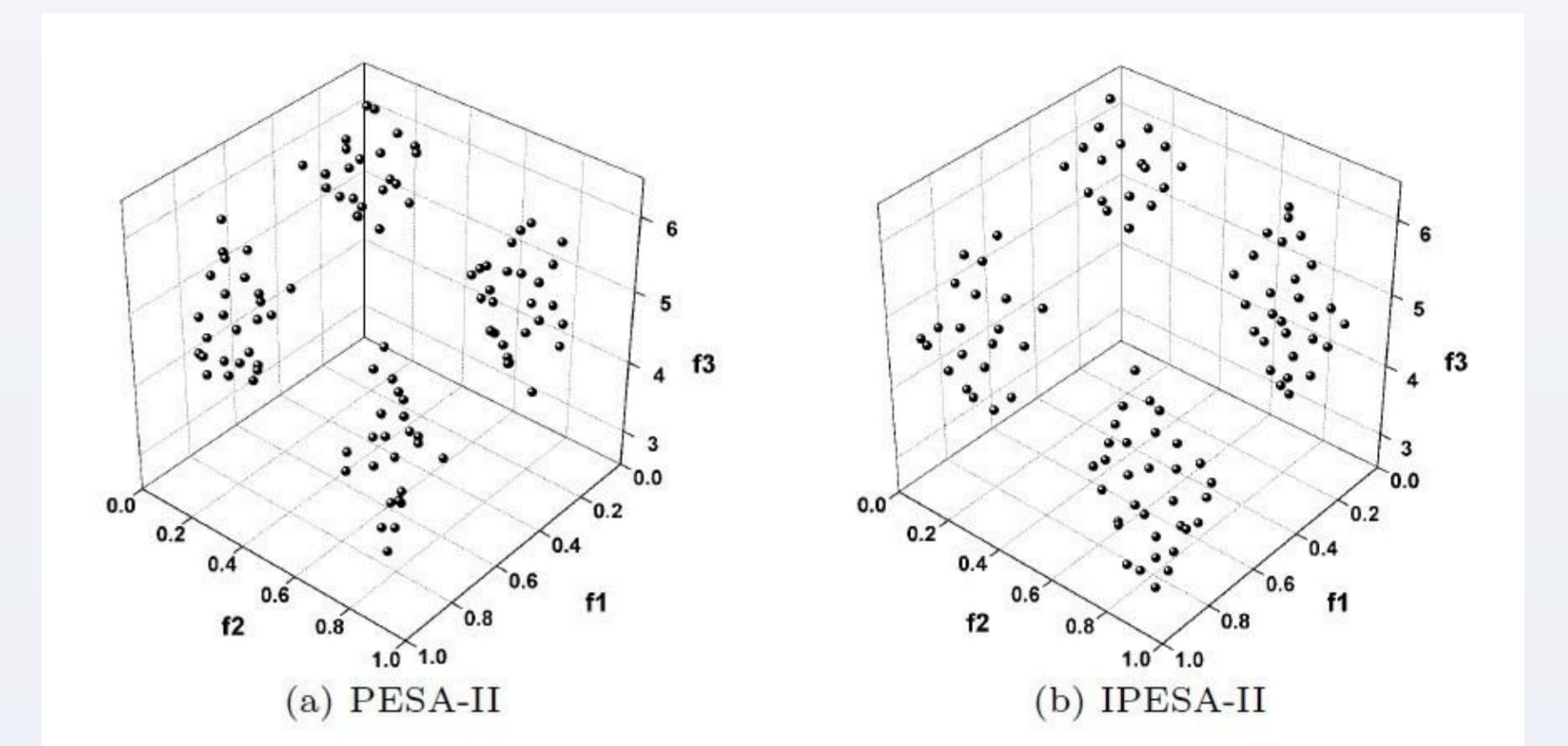


Table 2: HV comparison of the six EMO algorithms. The best mean for each problem is highlighted in boldface.

Problem	NSGA-II	SPEA2	IBEA	ϵ -MOEA	TDEA	IPESA-II
ZDT1	3.659e+0 [†] (4.1e-4)	3.659e+0 [†] (4.7e-4)	3.659e+0 [†] (8.0e-4)	3.648e+0 [†] (1.7e-3)	3.657e+0 [†] (1.8e-3)	3.661e+0 (2.5e-4)
ZDT2	3.325e+0 [†] (5.8e-4)	3.325e+0 [†] (8.9e-4)	3.324e+0 [†] (2.8e-4)	3.323e+0 [†] (1.6e-3)	3.319e+0 [†] (3.1e-3)	3.327e+0 (5.6e-4)
ZDT3	4.812e+0 [†] (4.7e-4)	4.812e+0 [†] (5.1e-4)	4.806e+0 [†] (2.1e-4)	4.809e+0 [†] (1.1e-3)	4.804e+0 [†] (3.6e-3)	4.813e+0 (6.6e-4)
ZDT4	3.651e+0 [†] (7.8e-3)	3.650e+0 [†] (8.6e-3)	2.482e+0 [†] (2.1e-1)	3.635e+0 [†] (2.1e-2)	3.631e+0 [†] (4.1e-2)	3.652e+0 (9.9e-3)
ZDT6	3.022e+0 [†] (2.7e-3)	3.023e+0 [†] (2.1e-3)	3.037e+0[†] (5.7e-4)	3.028e+0 [†] (2.2e-3)	3.024e+0 [†] (2.7e-3)	3.035e+0 (8.4e-4)
DTLZ1	9.673e-1 [†] (5.3e-4)	9.721e-1[†] (1.0e-3)	8.977e-1 [†] (1.0e-3)	9.561e-1 [†] (1.2e-2)	9.707e-1 (5.1e-4)	9.695e-1 (2.8e-3)
DTLZ2	7.351e+0 [†] (1.9e-2)	7.391e+0 [†] (6.9e-3)	5.702e+0 [†] (2.9e+0)	7.389e+0 [†] (9.0e-3)	7.401e+0 (1.2e-2)	7.398e+0 (2.6e+0)
DTLZ3	6.919e-1 (2.3e+0)	6.369e-1 (1.7e+0)	6.228e+0 [†] (2.1e+0)	6.326e+0[†] (2.0e+0)	9.825e-01 (2.2e+0)	1.668e+00 (2.6e+0)
DTLZ4	6.888e+0 (5.9e-1)	6.912e+0 (5.5e-1)	6.411e+0 [†] (1.2e+0)	7.013e+0 (4.0e-1)	6.942e+0 (4.5e-1)	7.128e+0 (6.3e-1)
DTLZ5	6.099e+0 [†] (6.8e-4)	6.100e+0 [†] (7.2e-4)	6.088e+0 [†] (1.1e-3)	6.100e+0 [†] (2.1e-3)	6.101e+0[†] (1.2e-3)	6.097e+0 (6.4e-3)
DTLZ6	4.098e+0 [†] (2.6e-1)	4.044e+0 [†] (2.6e-1)	5.974e+0[†] (9.7e-2)	5.292e+0 [†] (1.2e-1)	5.255e+0 [†] (1.1e-1)	5.101e+0 (1.3e-1)
DTLZ7	1.326e+1 [†] (5.7e-2)	1.333e+1 [†] (2.0e-1)	1.026e+1 [†] (2.7e+0)	1.312e+1 [†] (2.0e-2)	1.328e+1 [†] (4.8e-2)	1.340e+1 (5.0e-2)

^{††} indicates that the p -value of 58 degrees of freedom is significant at a 0.05 level of significance by a two-tailed t -test.

- IPESA-II outperforms NSGA-II in 11 out of the 12 problems, and with statistical significance for 8 problems
- IPESA-II outperforms SPEA2 in 10 out of the 12 problems, and with statistical significance for 7 problems
- IPESA-II outperforms IBEA in 9 out of the 12 problems, and with statistical significance for 9 problems
- IPESA-II outperforms ϵ -MOEA in 9 out of the 12 problems, and with statistical significance for 8 problems
- IPESA-II outperforms TDEA in 8 out of the 12 problems, and with statistical significance for 6 problems

Conclusions

IPESA-II has a clear advantage over PESA-II in finding a near-optimal, uniformly-distributed, and well-extended solution set.

IPESA-II is very competitive with the other five algorithms in balancing convergence and diversity.

Future work

- Investigation of the effect of the grid division parameter to the algorithm's performance
- investigation of IPESA-II on more test problems, such as many-objective problems

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