AN IMPROVED ADAPTIVE APPROACH FOR ELITIST NONDOMINATED SORTING GENETIC ALGORITHM FOR MANY-OBJECTIVE OPTIMIZATION (A^2-NSGA-III)

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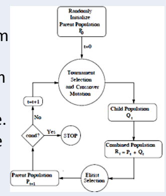
Introduction

Non-dominated Sorting Genetic Algorithm for Many Objective Optimization (NSGA-III) (KanGAL Report Number: 2012009 and 2012010):

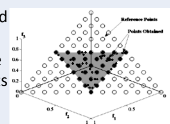
➢ Basic structure remains similar to NSGA-II with significant changes in Elitist Selection Mechanism and in Creation of offspring population.



➢**Elitist Selection Mechanism:** N *best* members are selected from combined parent and offspring population (R_t=P_t+Q_t) of size 2N.

- ➢R_t is classified into non-dominated levels and members from fronts 1 to L are included in S_t such that $|S_t| \geq N$.
- ➢Set of reference points is defined on normalized hyperplane.
- ➢**Normalization**: All members in S_t are normalized so as to be in same range as the reference points.
- ➢**Association:** Each member in S_t is associated with a reference point based on its perpendicular distance from the reference line.
- ➢**Niching:** Members from last front are selected such that population members are maximally distributed among the reference points.

➢ **Creation of offspring population:** Tournament selection operator is modified such that if both competing parents are feasible then one of them is chosen randomly, if one is feasible and another one is infeasible then feasible one is selected and if both are infeasible then least infeasible is selected.

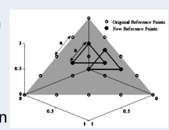


Limitations: One of the major limitations of using a predefined set of reference points is that in some cases there might not exist a Pareto-point corresponding to each and every reference point (as shown in figure) thus rendering those reference points non-useful and in the end one might not get a uniformly distributed set of Pareto- optimal solutions as expected.

Adaptive Relocation of Reference Points

➢ In order to overcome the above-stated limitation, an adaptive relocation mechanism was added to NSGA-III so that extra reference points are introduced in areas where Pareto-front exists while the non-useful reference points are simultaneously deleted.

A-NSGA-III (KanGAL Report No. 2012010):



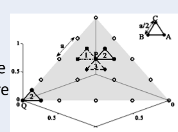
➢ **Addition:** Reference points having more than one population member associated to them are identified and N_{obj} extra reference points are added around it in form of (N_{obj}-1) dimensional simplex as shown in figure.

➢ **Deletion:** If there exist no reference point to which more than one population member is associated then all those reference points to whom no population member is associated are deleted.

➢ **Shortcomings:**

1. Not many extra reference points can be added by above procedure.
2. Introduction of reference points around the vertices of hyperplane not possible.
3. Since addition procedure is carried out right from first generation it may lead to premature introduction of extra reference points in unwanted regions.
4. It may be possible that deletion condition is never achieved and algorithm will keep on adding extra reference points thus increasing the computational cost.

Proposed Efficient Relocation Procedure A^2-NSGA-III: (EMO-2013): Following modifications are proposed so as to take care of the above stated shortcomings:



➢ Instead of adding simplex (ABC) around the identified reference point (P) it is added keeping P as vertex thereby adding (N_{obj}-1) extra points . N_{obj} such additions are possible (shown as configuration 1,2 and 3 in figure). Each time there is a requirement of addition one of the configurations is randomly added.

➢ If still more reference points need to be added around P the size of simplex ABC is halved and extra reference points are included. These two modifications take care of first two shortcomings.

➢ Addition procedure is initiated only when the number of reference points having more than one population member associated to them has settled down to a constant value. This removes the third limitation.

➢ The total number of reference points at a particular generation is limited by ten times the original number of reference points. In case the number shoots above this threshold, deletion procedure is carried out, thereby eliminating the last shortcoming.

Results : *Inverted* DTLZ Problems

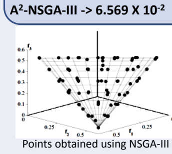
KanGAL Report: 2012009 | 2012010 | EMO-2013 Better all along

DTLZ1 Inverted

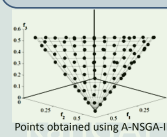
N_{obj} = 3, N_{gen} = 400, N_{ref} = 91
Hypervolume Values:
NSGA-III -> 6.229 X 10^{-2}
A-NSGA-III -> 6.540 X 10^{-2}
A^2-NSGA-III -> 6.569 X 10^{-2}

N_{obj} = 5, N_{gen} = 600, N_{ref} = 210
Hypervolume Values:
NSGA-III -> 1.982 X 10^{-3}
A-NSGA-III -> 2.125 X 10^{-3}
A^2-NSGA-III -> 2.711 X 10^{-3}

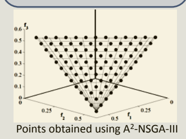
N_{obj} = 8, N_{gen} = 750, N_{ref} = 156
Hypervolume Values:
NSGA-III -> 3.386 X 10^{-6}
A-NSGA-III -> 3.422 X 10^{-6}
A^2-NSGA-III -> 4.835 X 10^{-6}



Points obtained using NSGA-III



Points obtained using A-NSGA-III



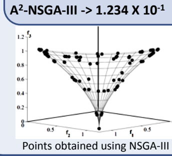
Points obtained using A^2-NSGA-III

DTLZ2 Inverted

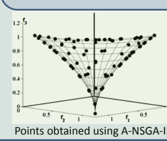
N_{obj} = 3, N_{gen} = 250, N_{ref} = 91
Hypervolume Values:
NSGA-III -> 1.095 X 10^{-1}
A-NSGA-III -> 1.195X 10^{-1}
A^2-NSGA-III -> 1.234 X 10^{-1}

N_{obj} = 5, N_{gen} = 350, N_{ref} = 210
Hypervolume Values:
NSGA-III -> 1.910 X 10^{-3}
A-NSGA-III -> 2.024 X 10^{-3}
A^2-NSGA-III -> 3.082 X 10^{-3}

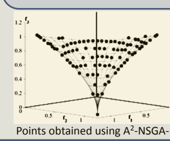
N_{obj} = 8, N_{gen} = 500, N_{ref} = 156
Hypervolume Values:
NSGA-III -> 1.481 X 10^{-6}
A-NSGA-III -> 1.481 X 10^{-6}
A^2-NSGA-III -> 2.322 X 10^{-6}



Points obtained using NSGA-III



Points obtained using A-NSGA-III



Points obtained using A^2-NSGA-III

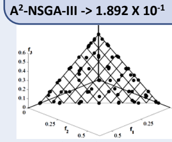
Results : Population Size > N_{ref}

DTLZ1

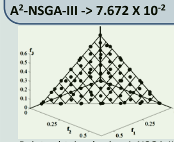
N_{obj} =3, N_{pop} = 92, N_{ref} = 28
Hypervolume Values:
NSGA-III -> 1.867 X 10^{-1}
A-NSGA-III -> 1.887X 10^{-1}
A^2-NSGA-III -> 1.892 X 10^{-1}

N_{obj} = 5, N_{pop} = 212, N_{ref} = 35
Hypervolume Values:
NSGA-III -> 7.636 X 10^{-2}
A-NSGA-III -> 7.636 X 10^{-2}
A^2-NSGA-III -> 7.672 X 10^{-2}

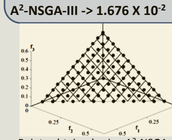
N_{obj} = 8, N_{pop} = 156, N_{ref} = 44
Hypervolume Values:
NSGA-III -> 1.675 X 10^{-2}
A-NSGA-III -> 1.675 X 10^{-2}
A^2-NSGA-III -> 1.676 X 10^{-2}



Points obtained using NSGA-III



Points obtained using A-NSGA-III



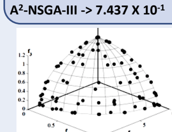
Points obtained using A^2-NSGA-III

DTLZ2

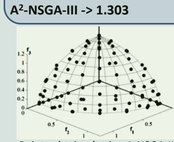
N_{obj} =3, N_{pop} = 92, N_{ref} = 28
Hypervolume Values:
NSGA-III -> 7.180 X 10^{-1}
A-NSGA-III -> 7.304X 10^{-1}
A^2-NSGA-III -> 7.437 X 10^{-1}

N_{obj} = 5, N_{pop} = 212, N_{ref} = 35
Hypervolume Values:
NSGA-III -> 1.266
A-NSGA-III -> 1.266
A^2-NSGA-III -> 1.303

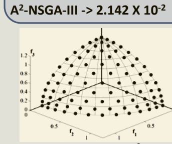
N_{obj} = 8, N_{pop} = 156, N_{ref} = 44
Hypervolume Values:
NSGA-III -> 1.679 X 10^{-2}
A-NSGA-III -> 1.679 X 10^{-2}
A^2-NSGA-III -> 2.142 X 10^{-2}



Points obtained using NSGA-III



Points obtained using A-NSGA-III

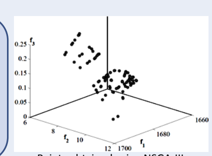


Points obtained using A^2-NSGA-III

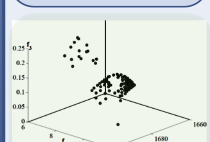
Results : Problems from Practice

Crash-worthiness in design of vehicles

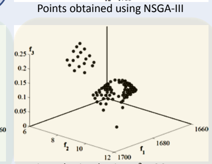
N_{obj} =3, N_{var} = 5 , N_{gen} = 500
N_{pop} = 92, N_{ref} = 91



Points obtained using NSGA-III



Points obtained using A-NSGA-III



Points obtained using A^2-NSGA-III

Machining Problem

N_{obj} =4, N_{var} = 3 , $N_{constraints}$ = 3
N_{gen} = 750, N_{pop} = 168, N_{ref} = 165

Hypervolume Values:
NSGA-III -> 2.268
A-NSGA-III -> 2.295
A^2-NSGA-III -> 2.305

Water Problem

N_{obj} =5, N_{var} = 3 , $N_{constraints}$ = 7
N_{gen} = 750, N_{pop} = 212, N_{ref} = 210

Hypervolume Values:
NSGA-III -> 0.530
A-NSGA-III -> 0.536
A^2-NSGA-III -> 0.542