Relation between Neighborhood Size and MOEA/D Performance on Many-Objective Problems

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Content of This Presentation

1. Motivation

We briefly explain our motivation.

2. MOEA/D and Its Modified Version

We explain MOEA/D and its slightly modified version with two neighborhood structures.

3. Experimental Results

We report some interesting observations on the behavior of MOEA/D with various settings of neighborhood size.

Motivation
Performance of MOEA/D

In a series of our studies on MOEA/D, its performance was almost always high.

[1] Ishibuchi et al.: "Adaptation of Scalarizing Functions in MOEA/D". EMO 2009.

[2] Ishibuchi et al.: "Evolutionary Many-Objective Optimization by NSGA-II and MOEA/D with Large Populations". SMC 2009.

[3] Ishibuchi et al.: "Simultaneous Use of Different Scalarizing Functions in MOEA/D". GECCO 2010.

[4] Ishibuchi et al. "Effects of the Existence of Highly Correlated Objectives on the Behavior of MOEA/D". EMO 2011.

[5] Ishibuchi et al.: "A Study on the Specification of a Scalarizing Function in MOEA/D for Many-Objective Knapsack Problems". LION 2013.

Motivation
Performance of MOEA/D

However, in other authors' studies, **the performance of MOEA/D was sometimes surprisingly bad** when it was used for performance comparison with their EMO algorithms.

Question:

Why was the performance of MOEA/D poor in their computational experiments (whereas its performance was high in our own studies)?

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Possibilities:

(0) Use of different test problems.

(1) Use of inappropriate scalarizing functions:
In MOEA/D, each solution is evaluated by a scalarizing function with a different weight vector.

(2) Use of inappropriate neighborhood specifications:

- **Parents are selected from neighbors (Local Mating).**
- **Generated offspring are compared with neighbors (Local Replacement).**

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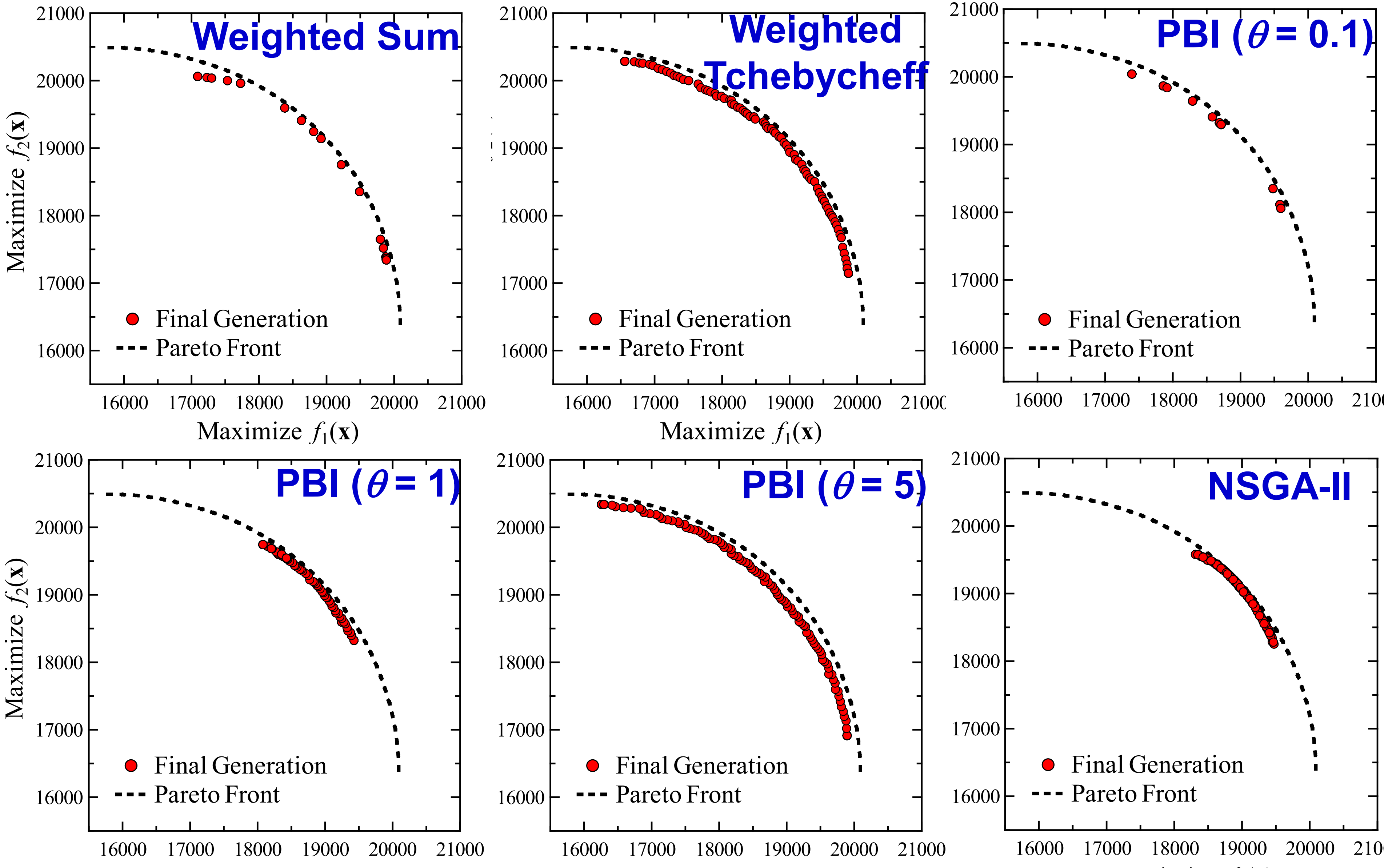
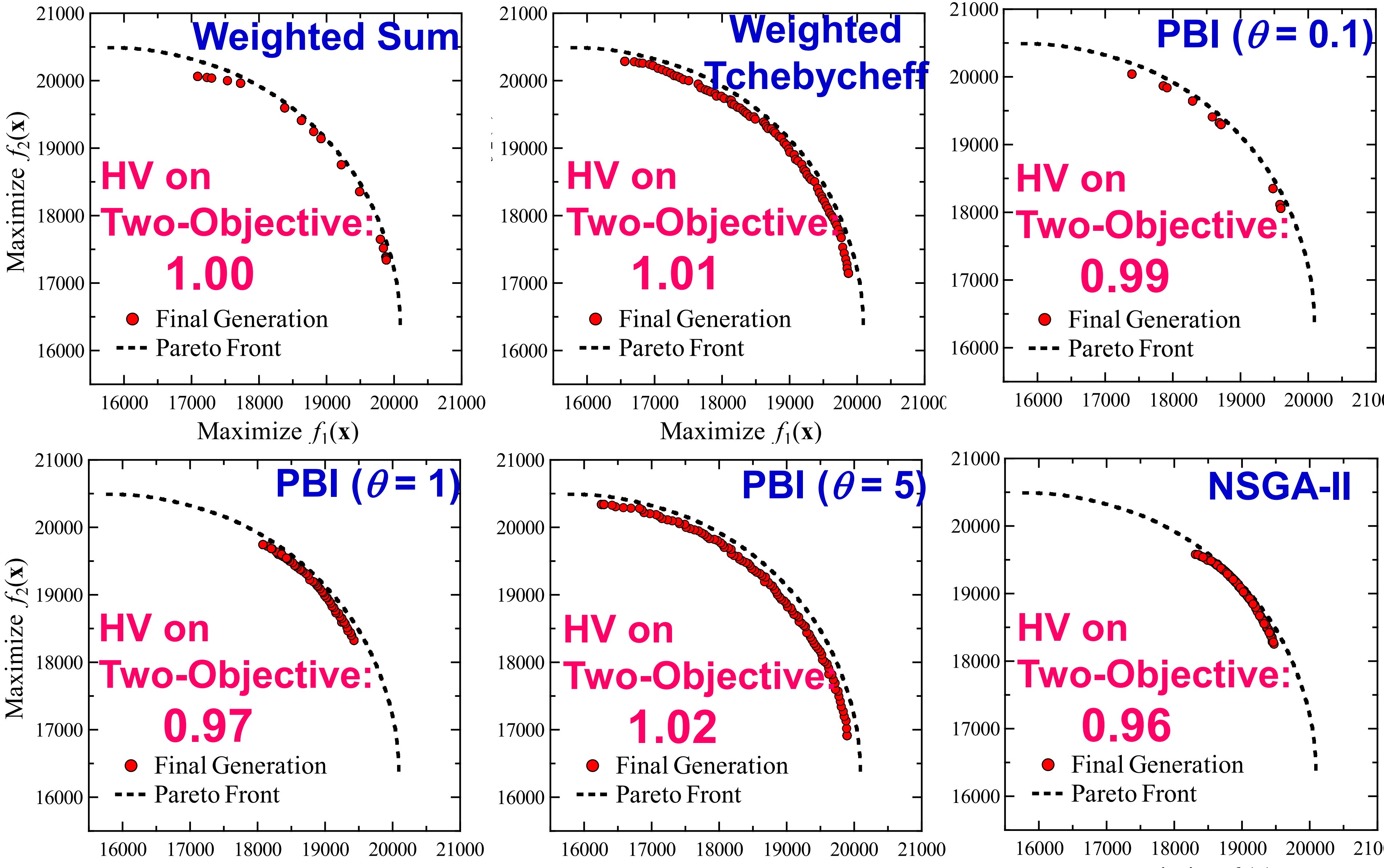
Results on Two-Objective Knapsack Problem (LION 2013)

Performance of MOEA/D depends on Scalarizing Function



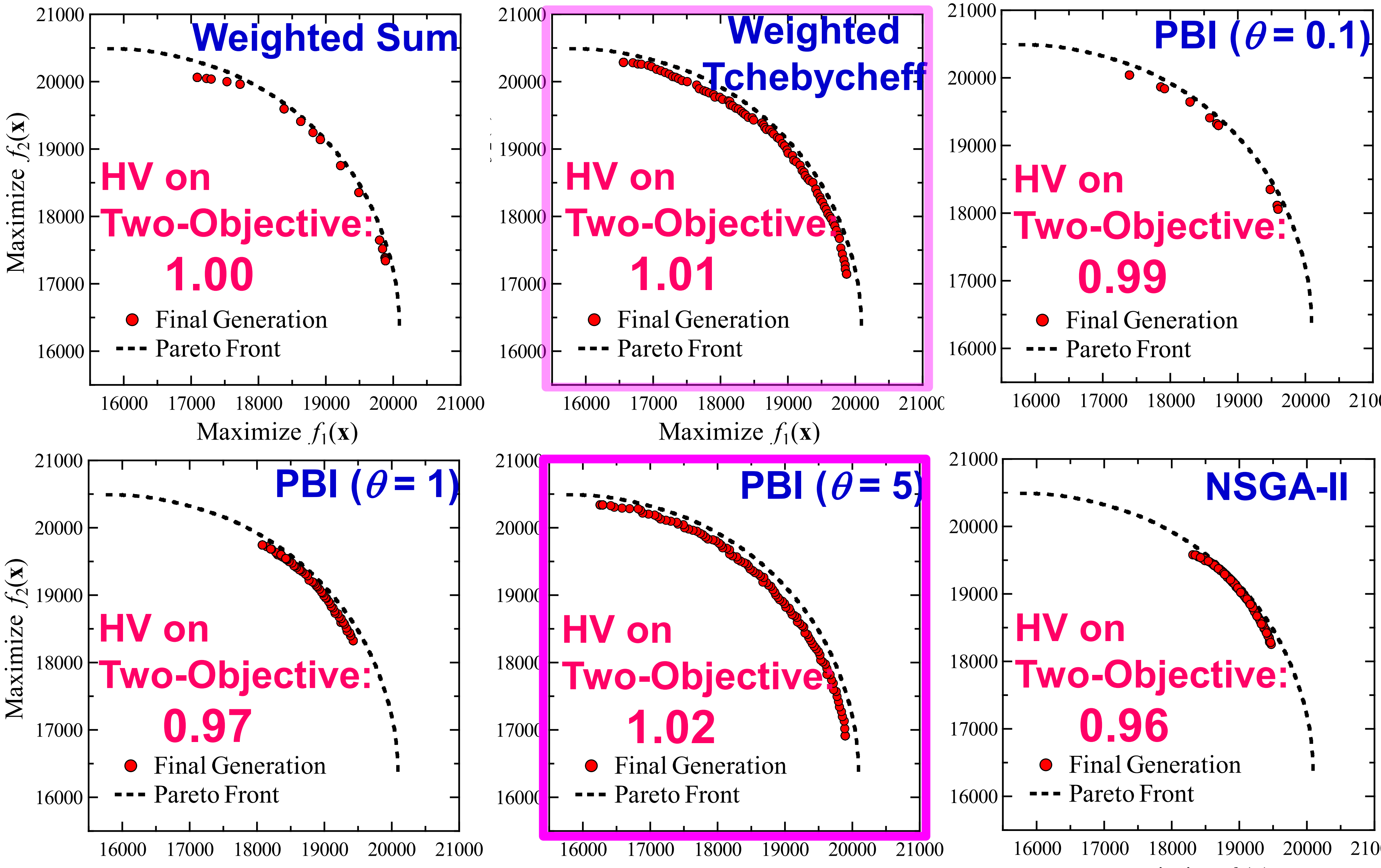
Average Hypervolume on Two-Objective Problem (100 runs)

Performance of MOEA/D depends on Scalarizing Function

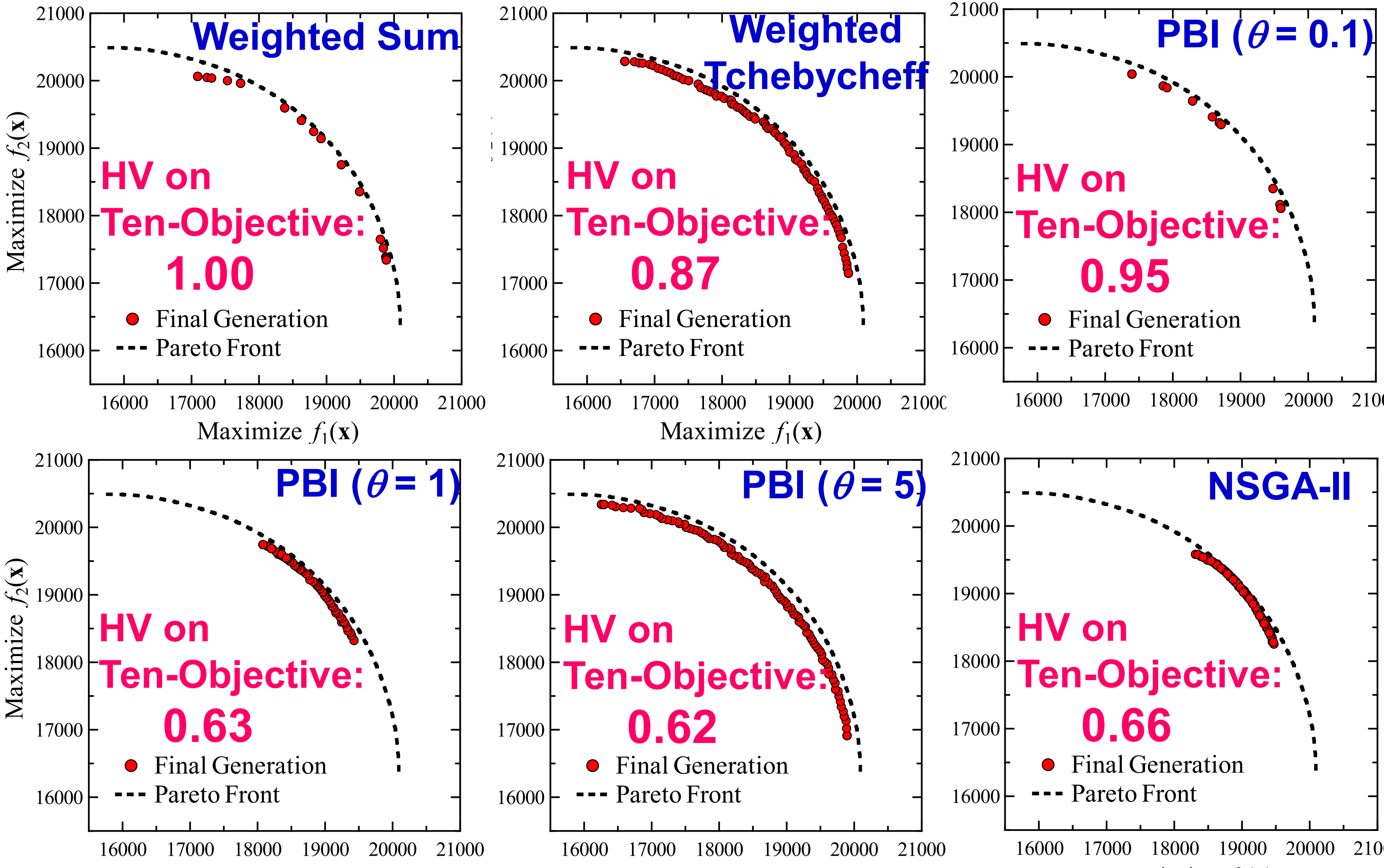


Best Specification for Two-Objective Problem

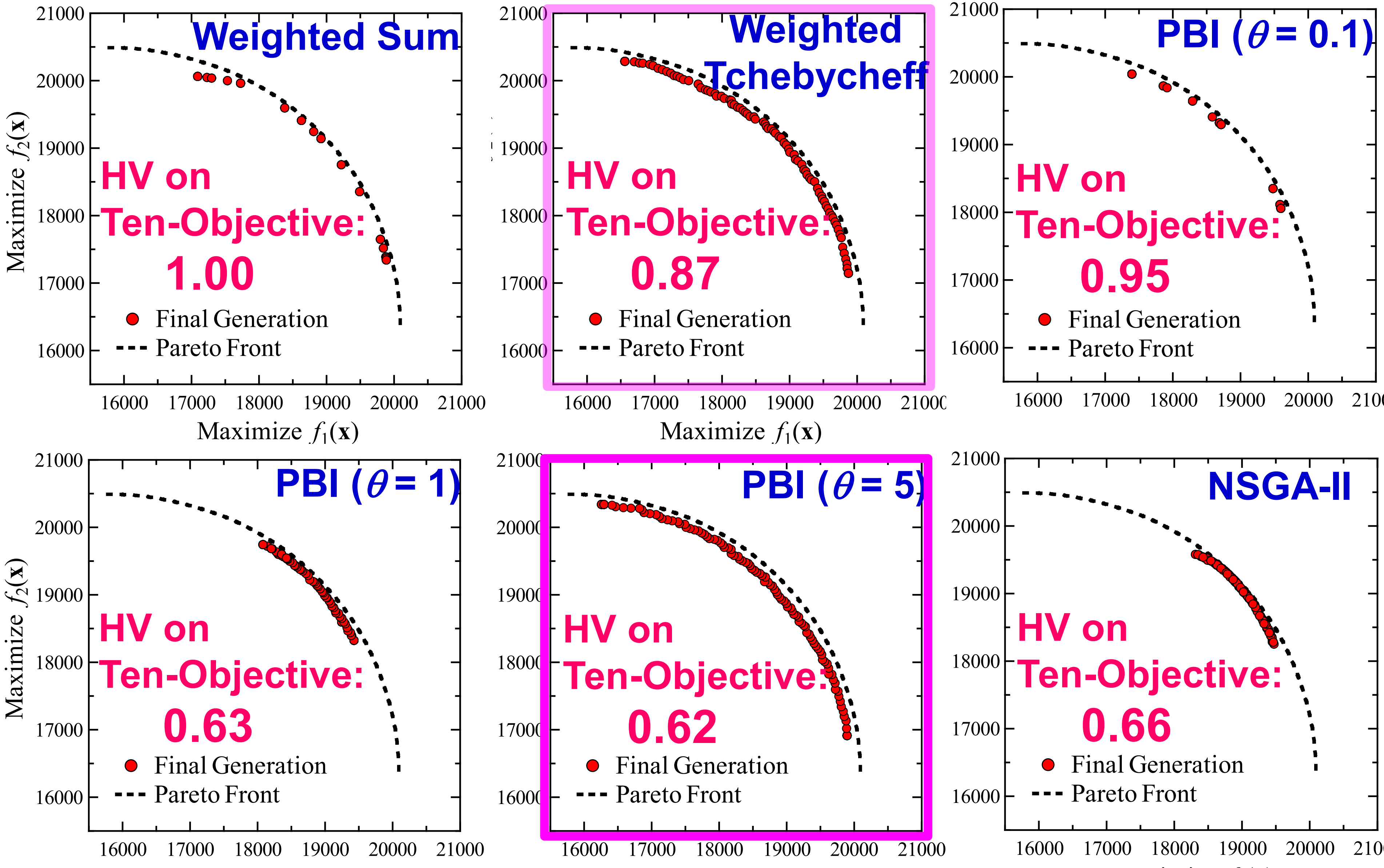
PBI Function with $\theta = 5$



Average Hypervolume on Ten-Objective Problem



The best specification for the two-objective problem is the worst for the ten-objective problem



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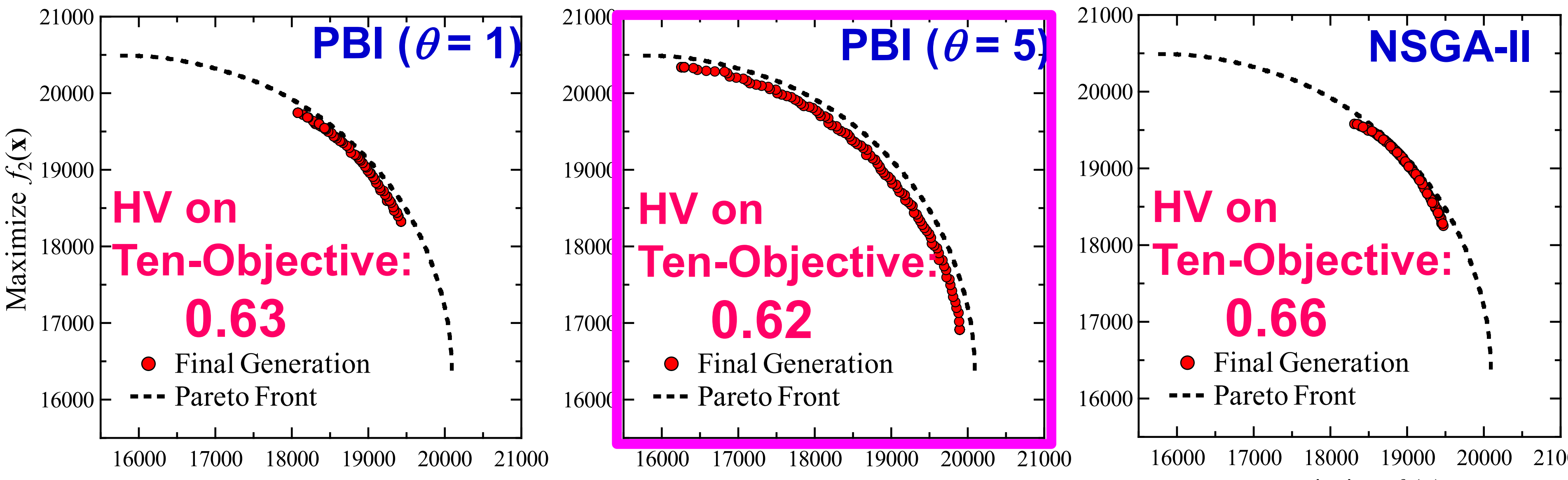
Observation (when PBI (θ = 5) was used):

MOEA/D is outperformed by NSGA-II in their applications to the 10-objective problem.



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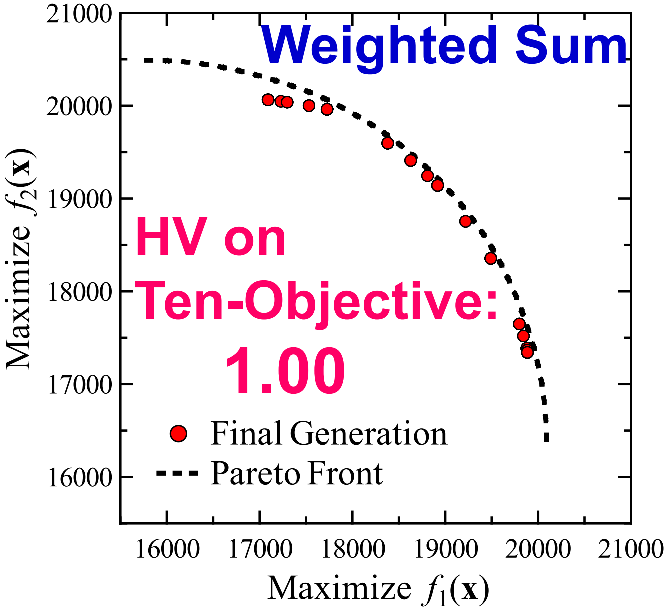
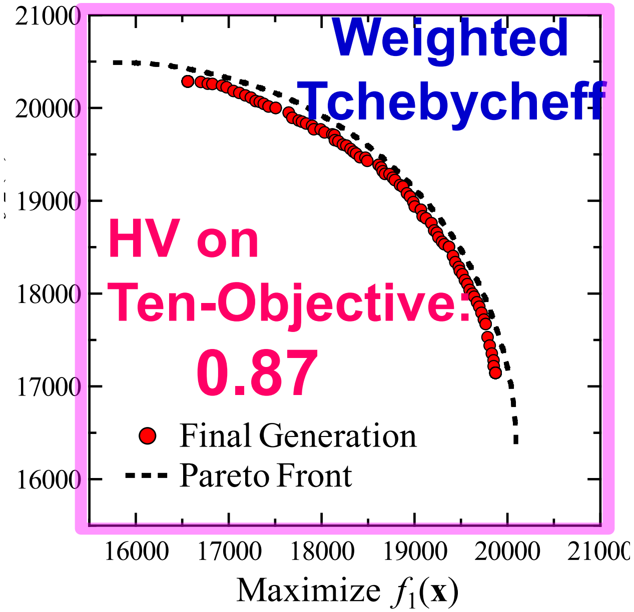
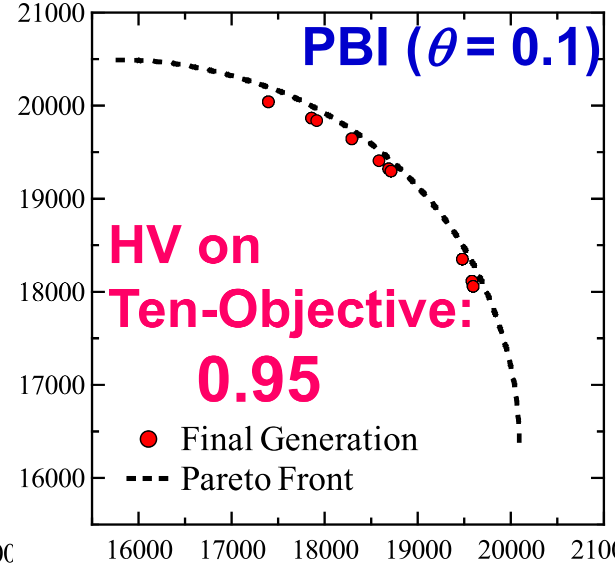
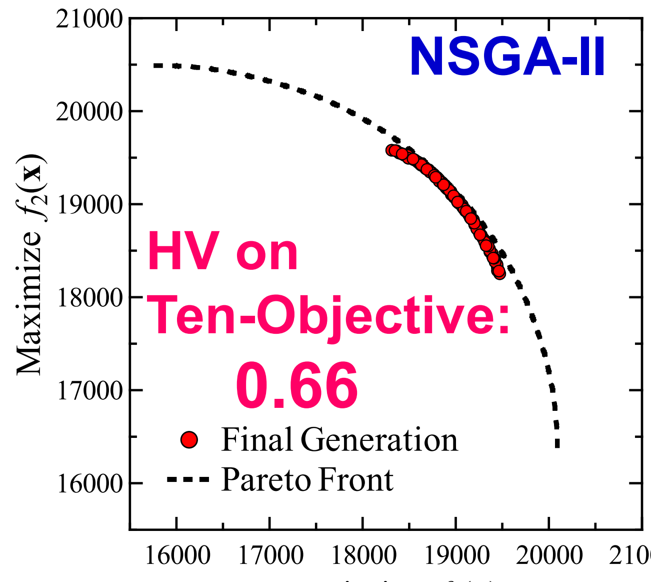






Observation from the weighted sum:

MOEA/D is much better than NSGA-II in their applications to the 10-objective problem.



Motivation
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Possibilities:

(1) Use of inappropriate scalarizing functions:
In MOEA/D. each solution is evaluated by a scalarizing function with a different weight vector. ==> LION 2013

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In EMO 2013, we examine the effect of neighborhood size.

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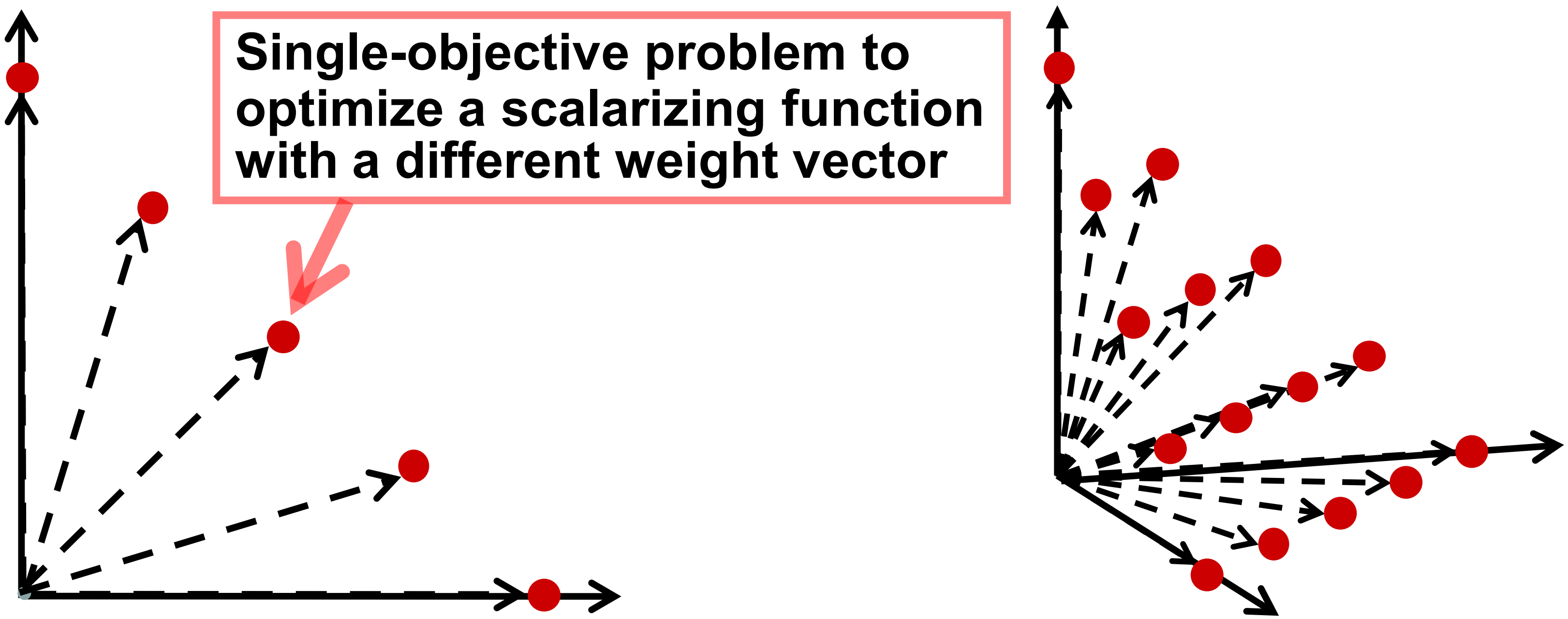
Basic Idea of MOEA/D

Q. Zhang and H. Li (IEEE TEVC 2007)

Decomposition: A multi-objective problem is handled as a set of scalarizing function optimization problems with different weight vectors. **Uniformly distributed weight vectors were used in the original study (2007).**



Weight vectors (2-objective case)

Weight vectors (3-objective case)

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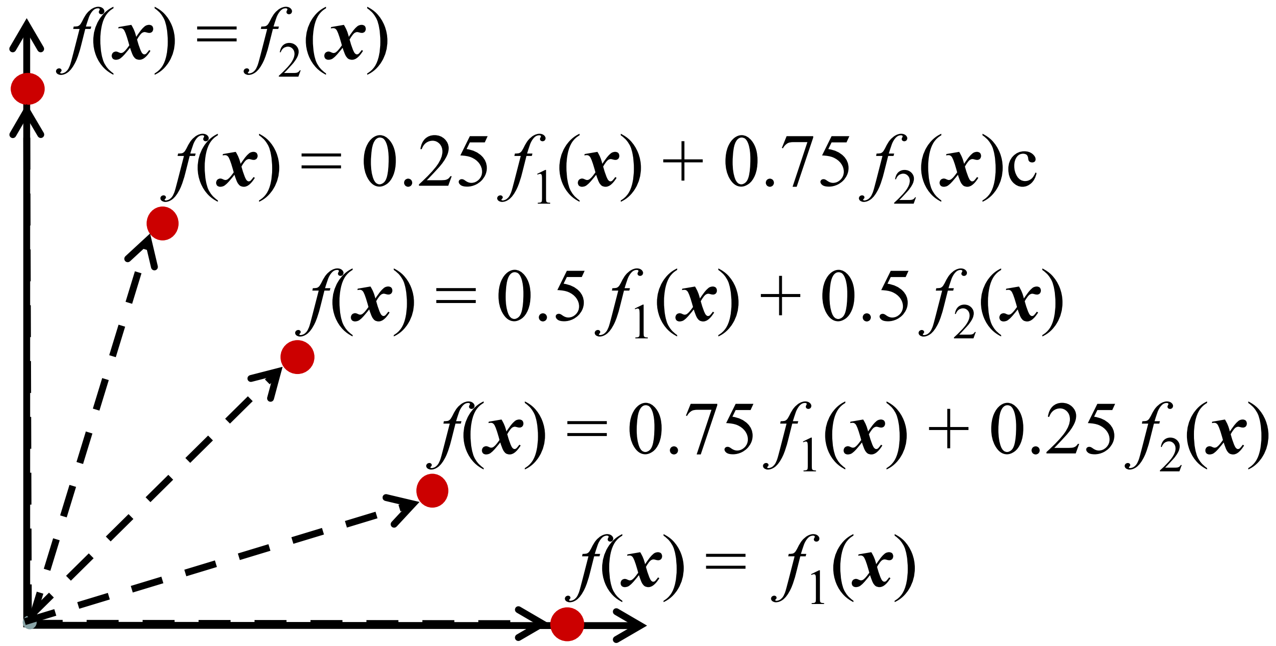
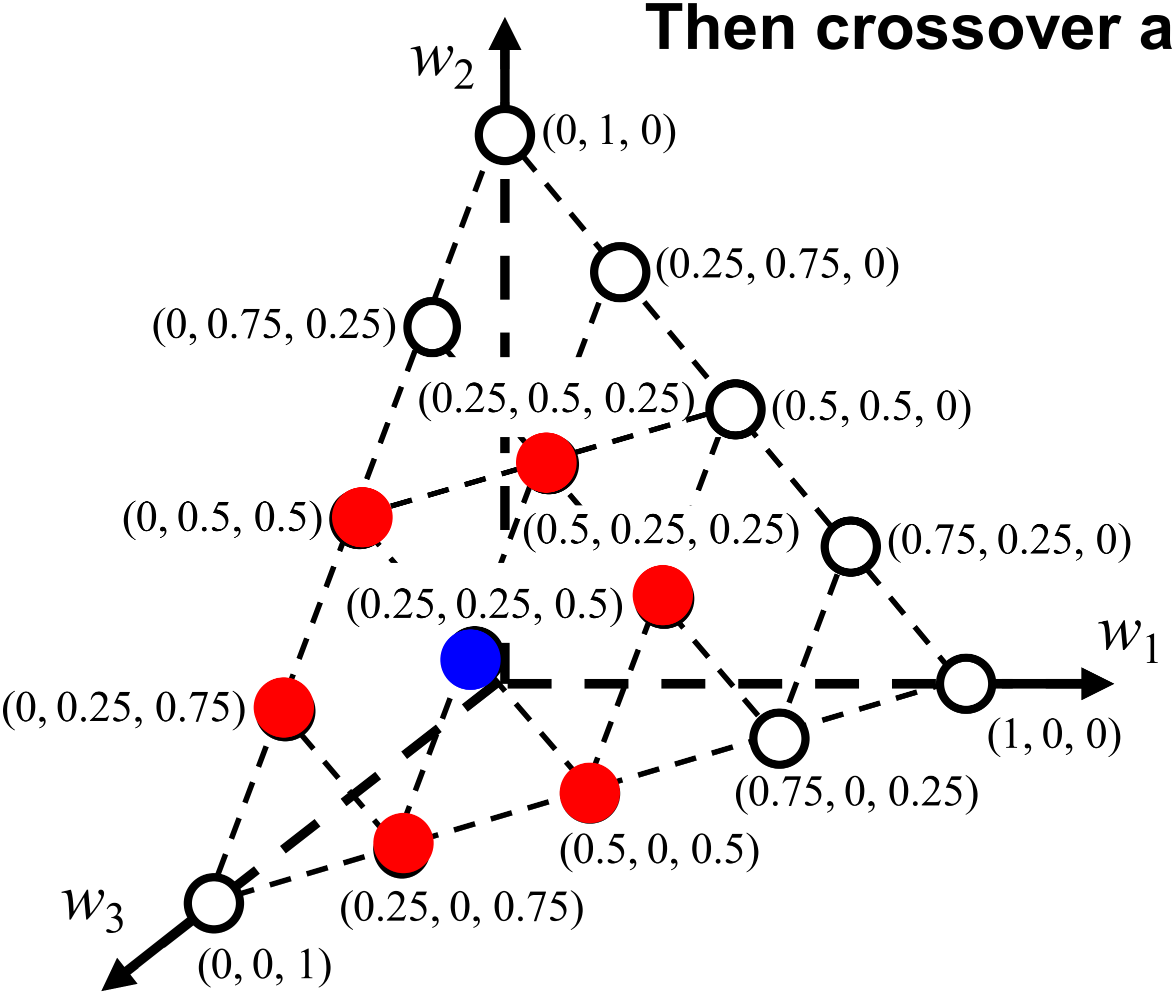


Weighted Sum

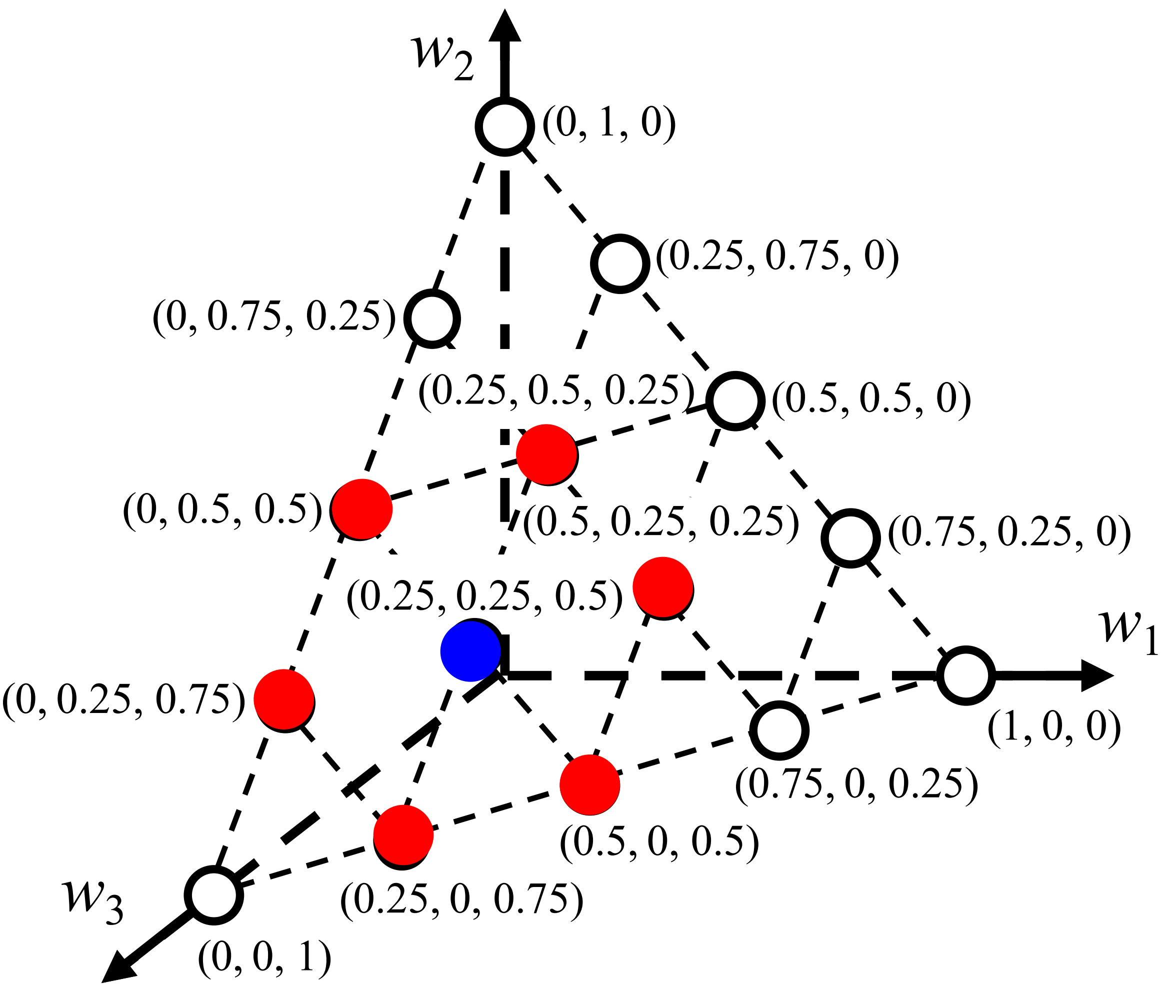
Characteristic Features of MOEA/D
Local Mating in Parent Selection

To generate a new offspring for the current solution (blue dot), a pair of parents are selected from its neighbors (red dot). Then crossover and mutation are used.



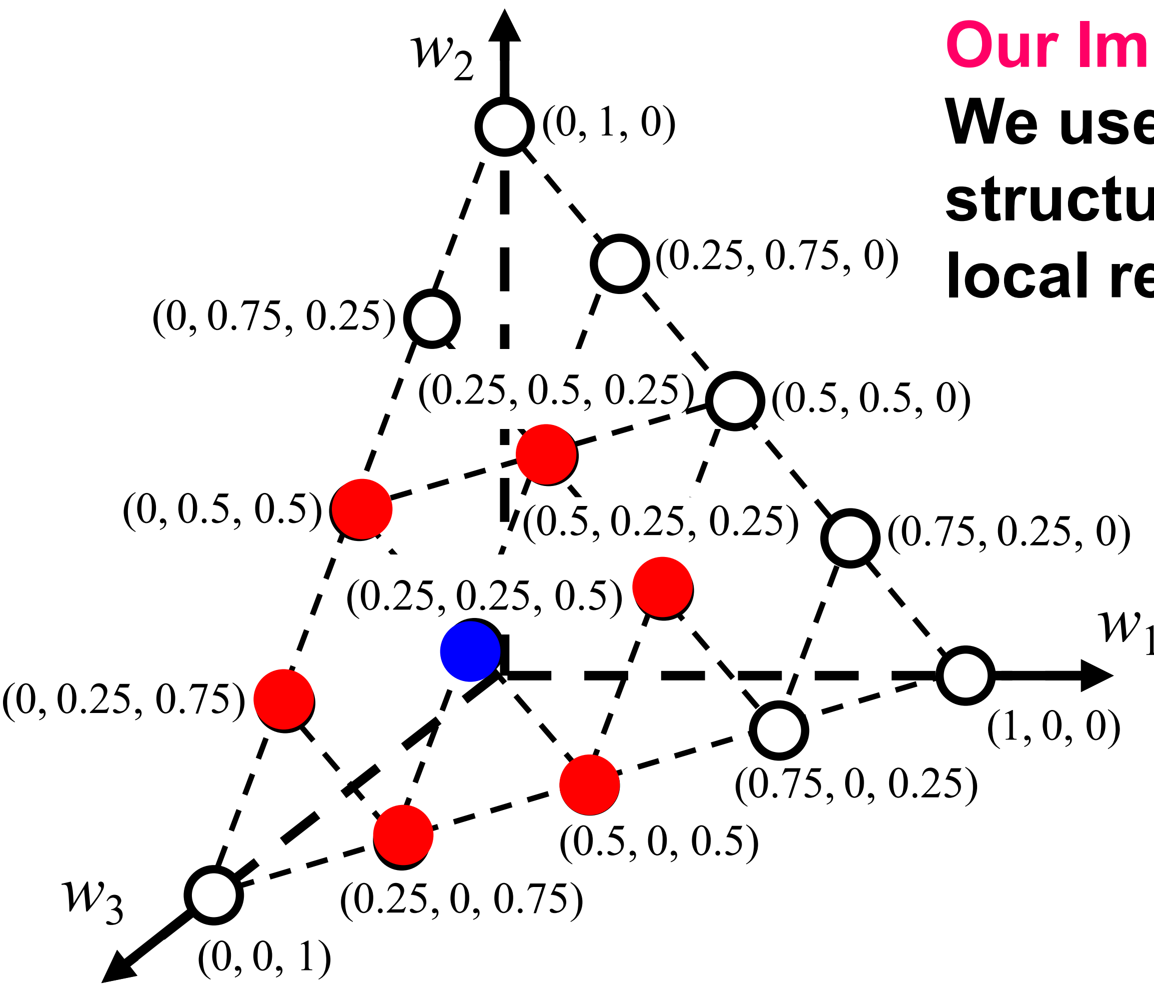
Characteristic Features of MOEA/D
Local Replacement in Solution Update

A newly generated offspring is compared with the current solution (blue) and its neighbors (red). (If better then replace).



Our Model: Slightly Modified Version Local Mating and Local Replacement

In MOEA/D, the same neighborhood structure was used for both the local mating and the local replacement.



Our Implementation:
We use different neighborhood structures for local mating and local replacement.

Mating ●

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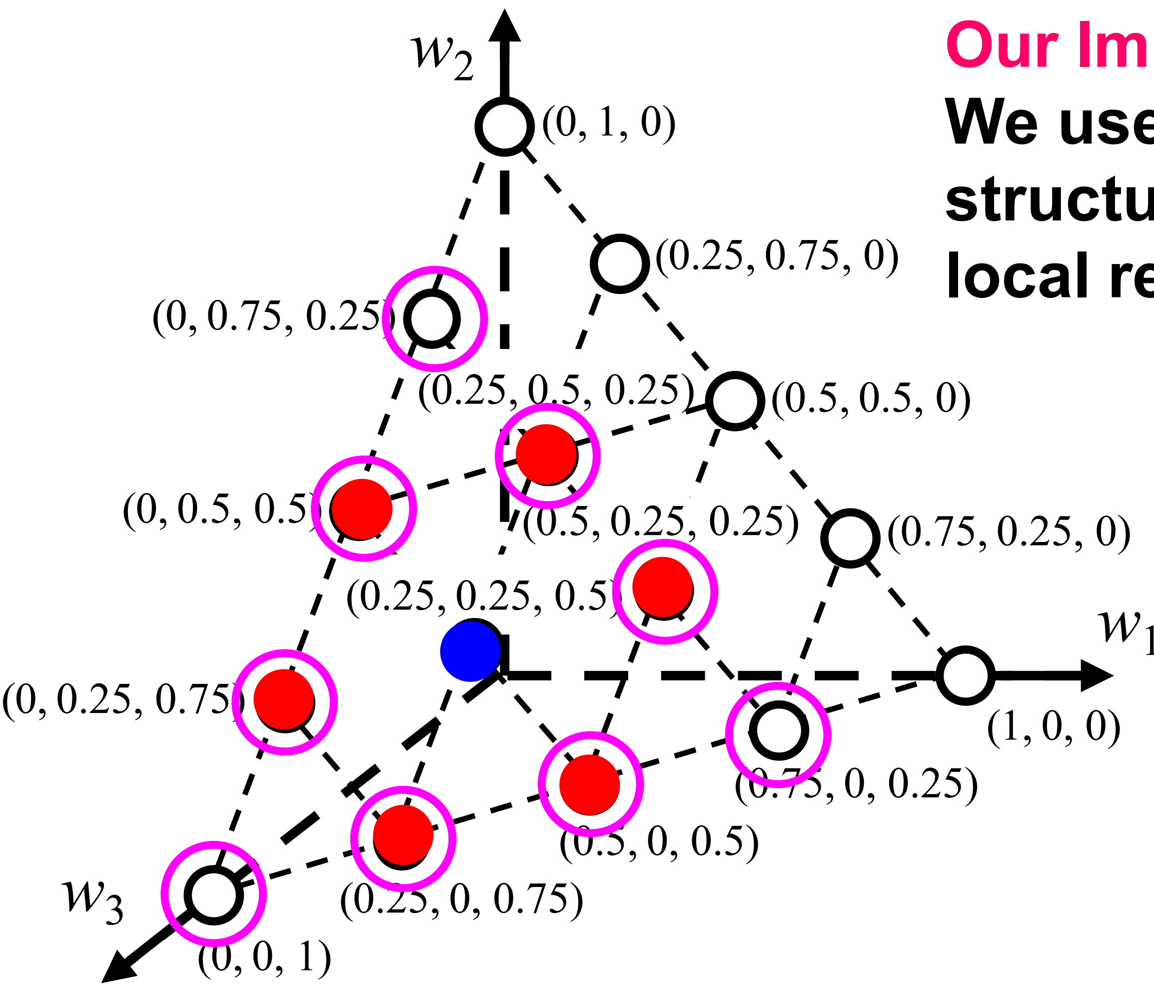

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Mating
Replacement

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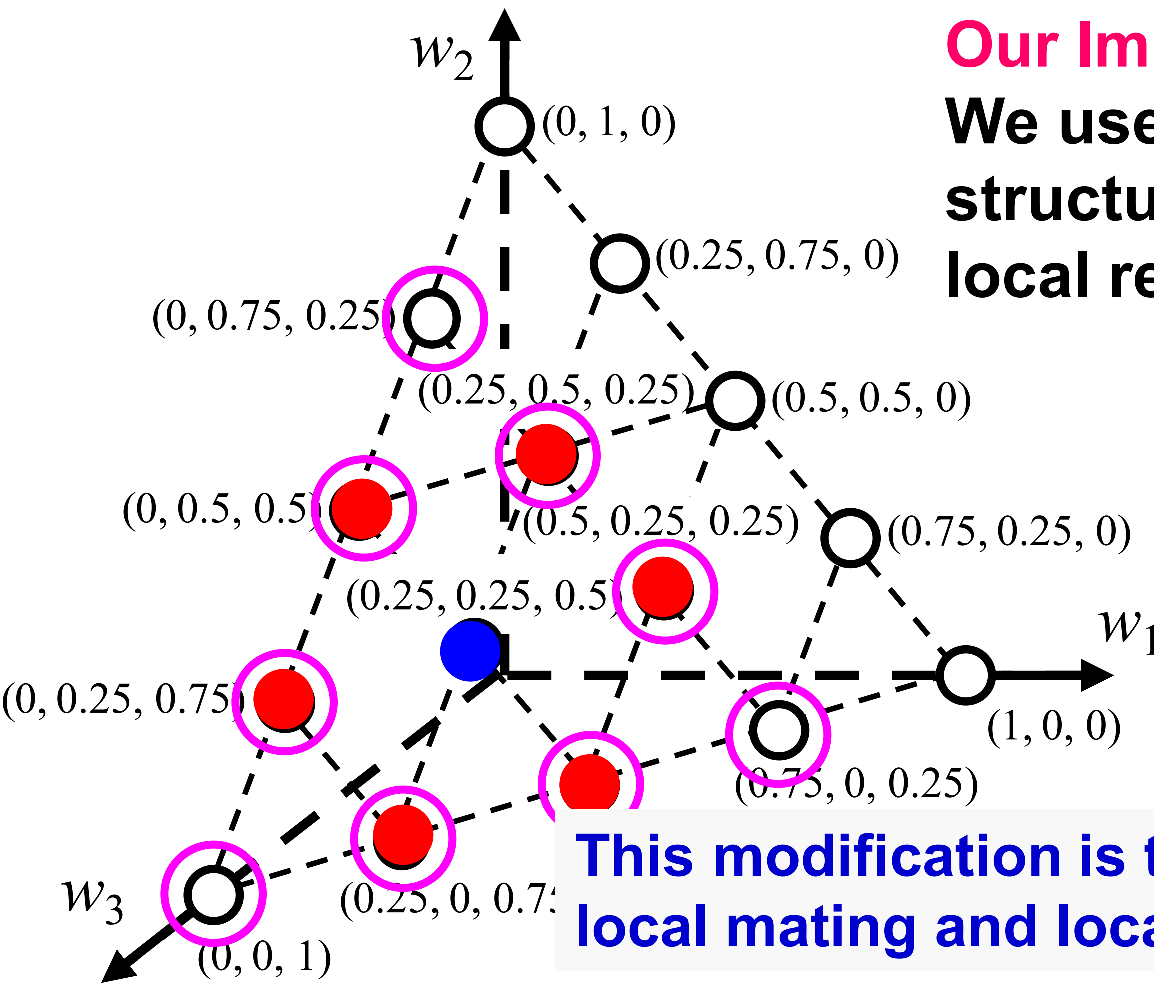


Our Implementation:
We use different neighborhood structures for local mating and local replacement.

Mating ●
Replacement ○

This modification is to examine the effects of local mating and local replacement separately.

Scalarizing Functions

We used the following scalarizing functions:

1. **Weighted Sum:** $$g^{WS} = \sum_{i=1}^{m} \lambda_i \cdot f_i(\mathbf{x})$$

Six- and Eight-Objective Knapsack Problems

2. **Weighted Tchebycheff:** $$g^{TE} = \max_{i=1,2,\ldots,m} \left\{ \lambda_i \cdot | z_i^* - f_k(\mathbf{x}) | \right\}$$

$$z_i^* = 1.1 \cdot \max \left\{ f_i(\mathbf{x}) \mid \mathbf{x} \in \Omega(t) \right\}$$

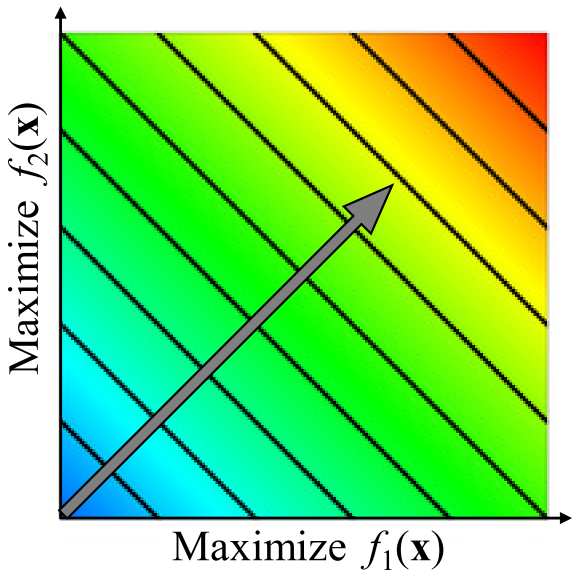
Two- and Four-Objective Knapsack Problems

Scalarizing Functions
Their Contour Lines

Contour line of each scalarizing function for the weighted vector λ = (0.5, 0.5)

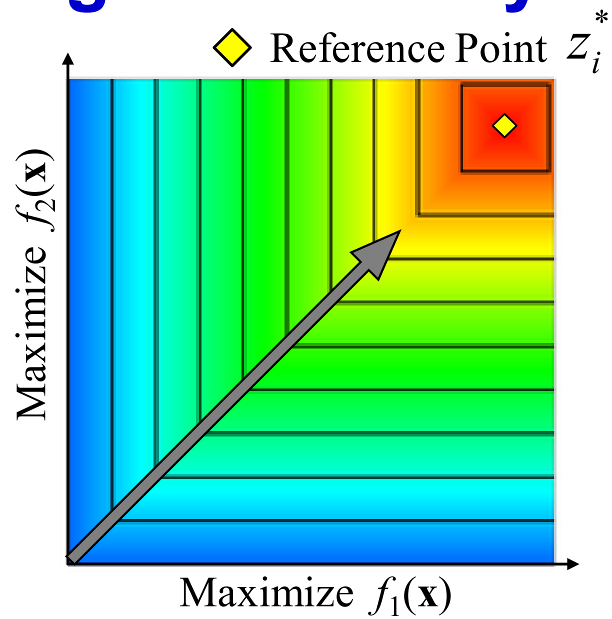
Bad ⟶ Good

Weighted Sum



$$g^{WS} = \sum_{i=1}^{m} \lambda_i \cdot f_i(\mathbf{x})$$

Weighted Tchebycheff



$$g^{TE} = \max_{i=1,2,\ldots,m} \left\{ \lambda_i \cdot | z_i^* - f_k(\mathbf{x}) | \right\}$$

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(1) Mating neighborhood: α% of the population size
(2) Replacement neighborhood: β% of the population size

$$\alpha = \{1, 2, 4, 6, 8, 10, 20, 40, 60, 80, 100\}$$
$$\beta = \{1, 2, 4, 6, 8, 10, 20, 40, 60, 80, 100\}$$

Computational Experiments

Original 2-500 Knapsack Problem

Two-objective 500-item knapsack problem in Zitzler & Thiele (IEEE TECV 1999) with randomly generated two objectives.

Maximize $f_i(\mathbf{x}) = \sum_{j=1}^{n} p_{ij} x_j$, i = 1, 2

Subject to $\sum_{j=1}^{n} w_{ij} x_j \leq c_i$, i = 1, 2

p_{ij} : Profit of item j according to knapsack i

w_{ij} : Weight of item j according to knapsack i

c_{ij} : Capacity of knapsack i

Value of p_{ij} and w_{ij} were randomly specified in [10, 100], and w_{ij} is 50% of the sum of w_{ij}.

Computational Experiments
Settings of Computational Experiments

EMO Algorithms: MOEA/D

Coding: Binary string of length 500

Crossover: Uniform crossover with the probability 0.8

Mutation: Bit-flip mutation with the probability 1/500

Constraint Handling: Greedy repair in Zitzler & Thiele (1999)

Termination: 400,000 solution evaluations

Neighborhood Size in MOEA/D:

Mating Neighborhood: α % of the population size

α = 1, 2, 4, 6, 8, 10, 20, 40, 60, 80, 100

Replacement Neighborhood: β % of the population size

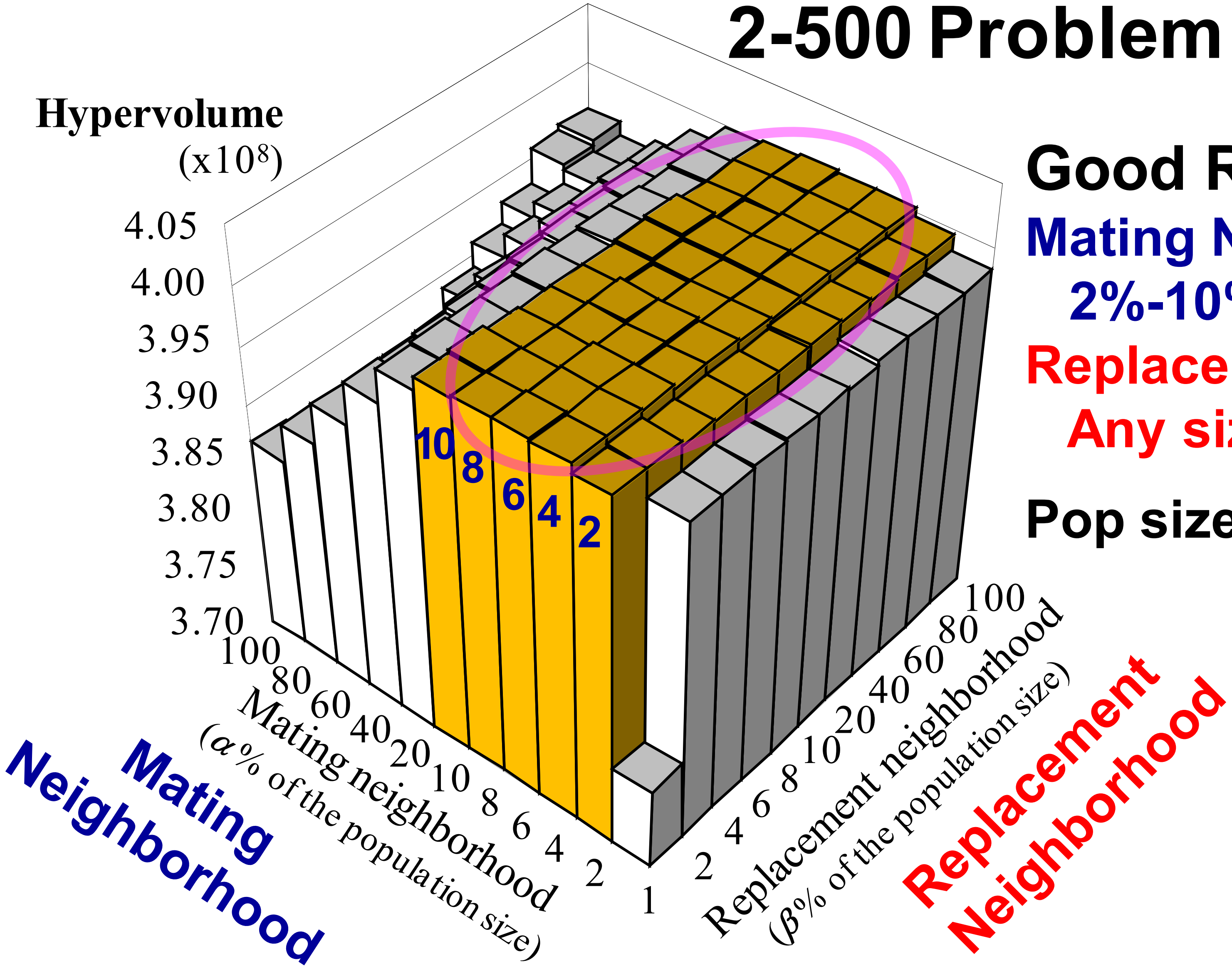
β = 1, 2, 4, 6, 8, 10, 20, 40, 60, 80, 100

Scalarizing Function in MOEA/D: Weighted Tchebycheff

Population Size: 200

Two-Objective 500-Item Problem
Average Hypervolume over 100 runs

2-500 Problem



Good Results:

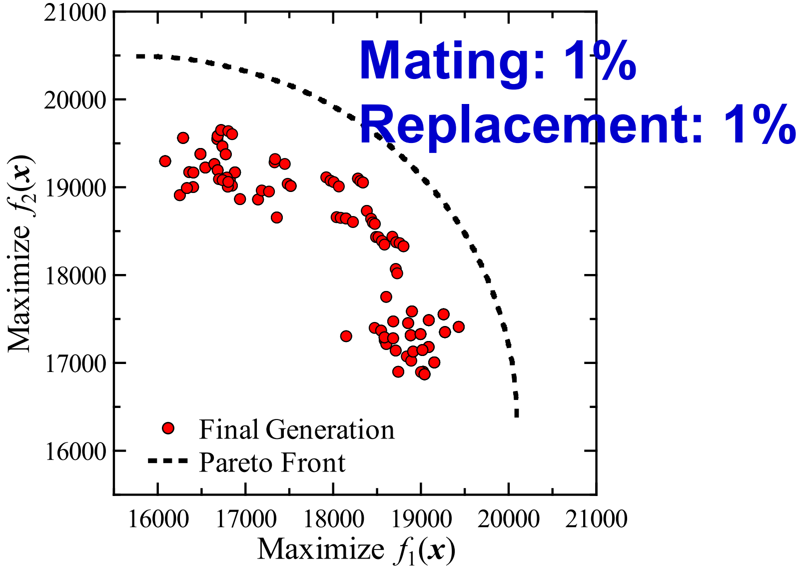
Mating Neighborhood:
2%-10% of Pop size

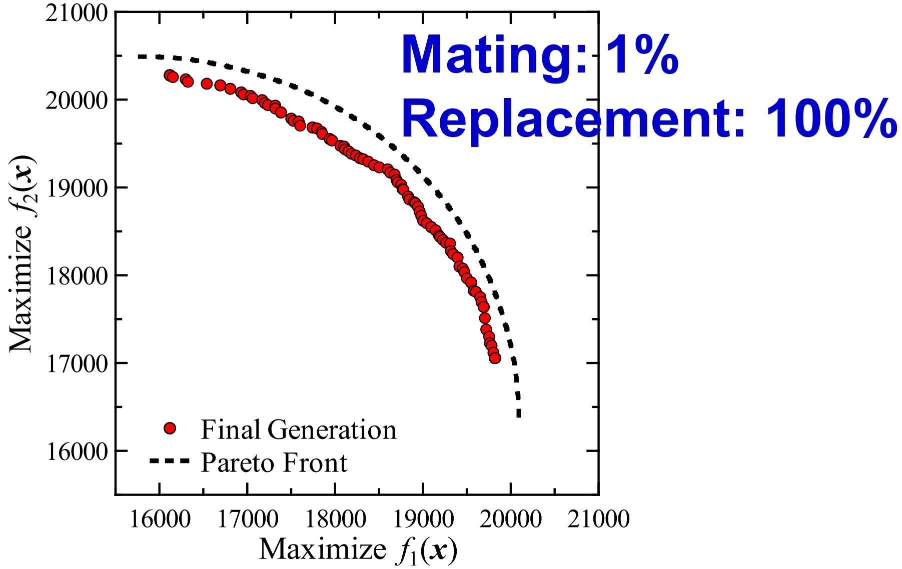
Replacement:
Any size is OK.

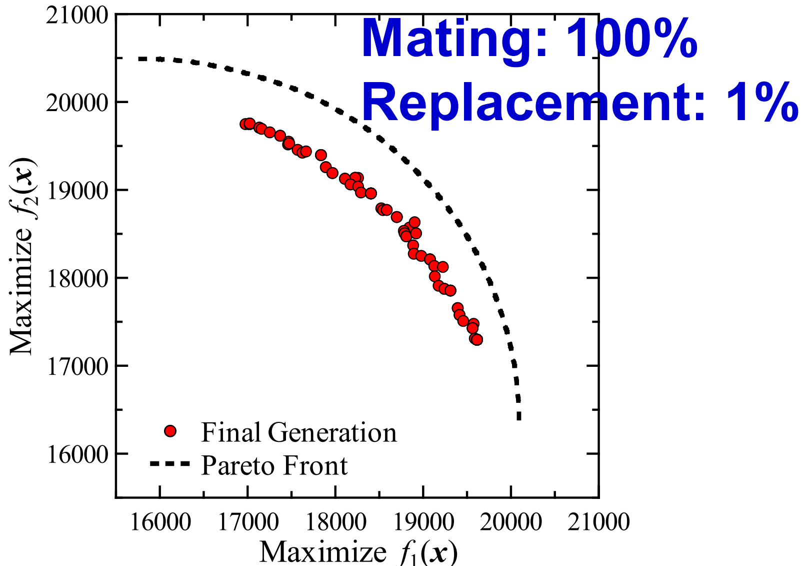
Pop size: 200

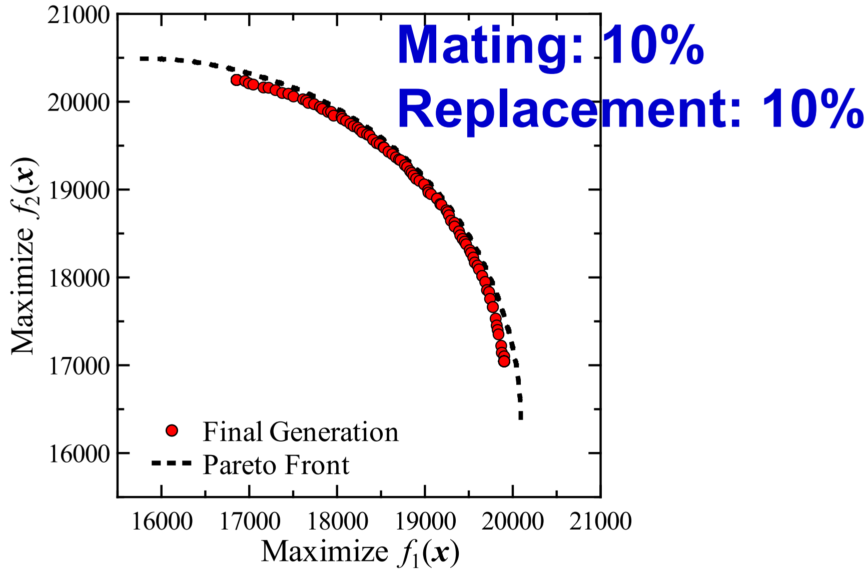
Two-Objective 500-Item Problem

Results of a Single Run









Knapsack Problems with 4-10 Objectives (4-500, 6-500, 8-500, 10-500)

Many-Objective Knapsack Problems (n-500 Problems)

Randomly generated objectives were added to the original 2-500 problems (n = 4, 6, 8)

Maximize $f_i(\mathbf{x}) = \sum_{j=1}^{n} p_{ij} x_j$, i = 1, 2, ..., n

Subject to $\sum_{j=1}^{n} w_{ij} x_j \leq c_i$, i = 1, 2

p_{ij} : Profit of item j according to knapsack i

w_{ij} : Weight of item j according to knapsack i

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Replacement Neighborhood: β % of the populations size

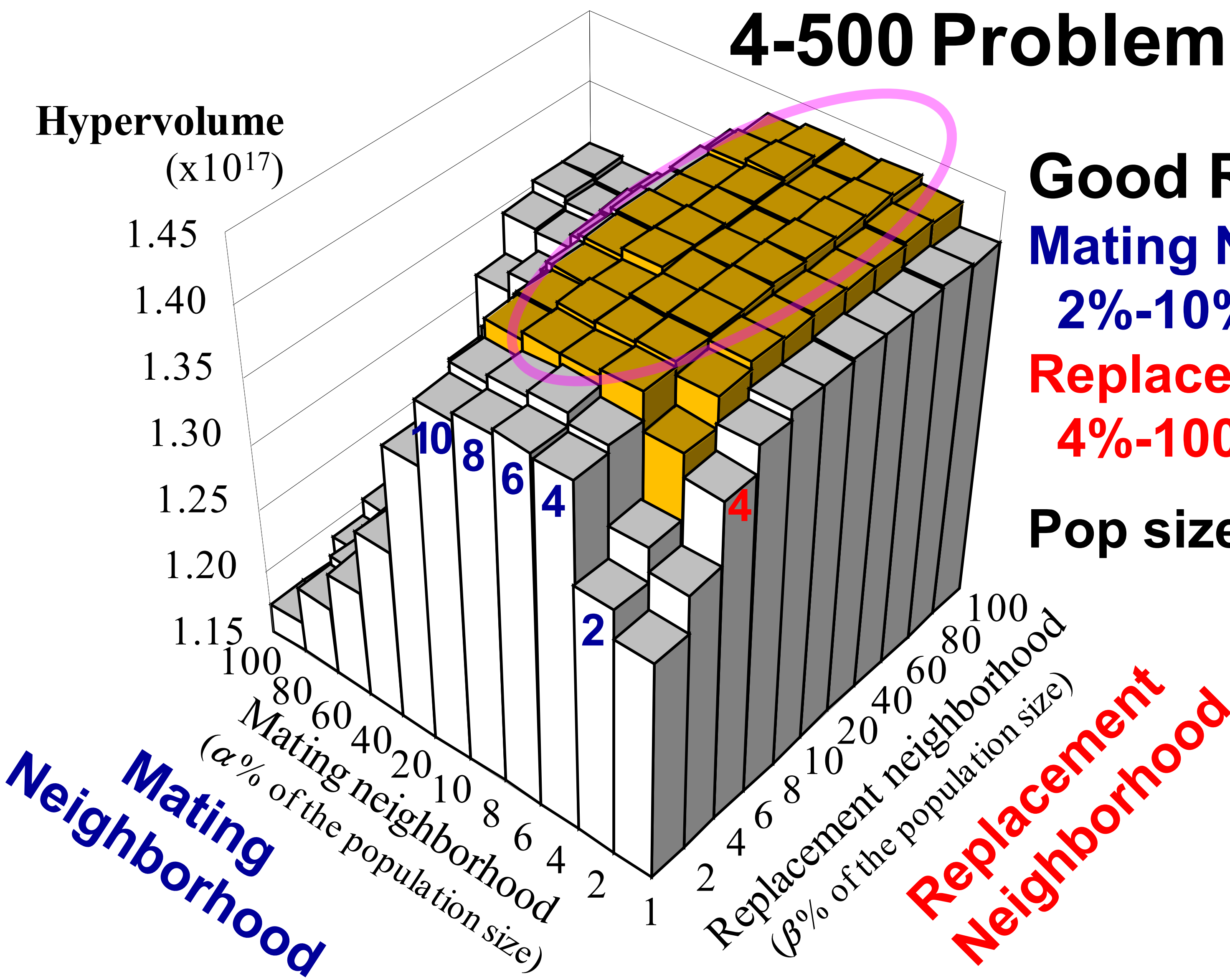
β = 1, 2, 4, 6, 8, 10, 20, 40, 60, 80, 100

Scalarizing Function in MOEA/D:

Tchebycheff (4-500), Weighted Sum (6-500, 8-500)

Population Size: 220 (4-500), 252 (6-500), 120 (8-500)

Four-Objective 500-Item Problem
Average Hypervolume over 100 runs



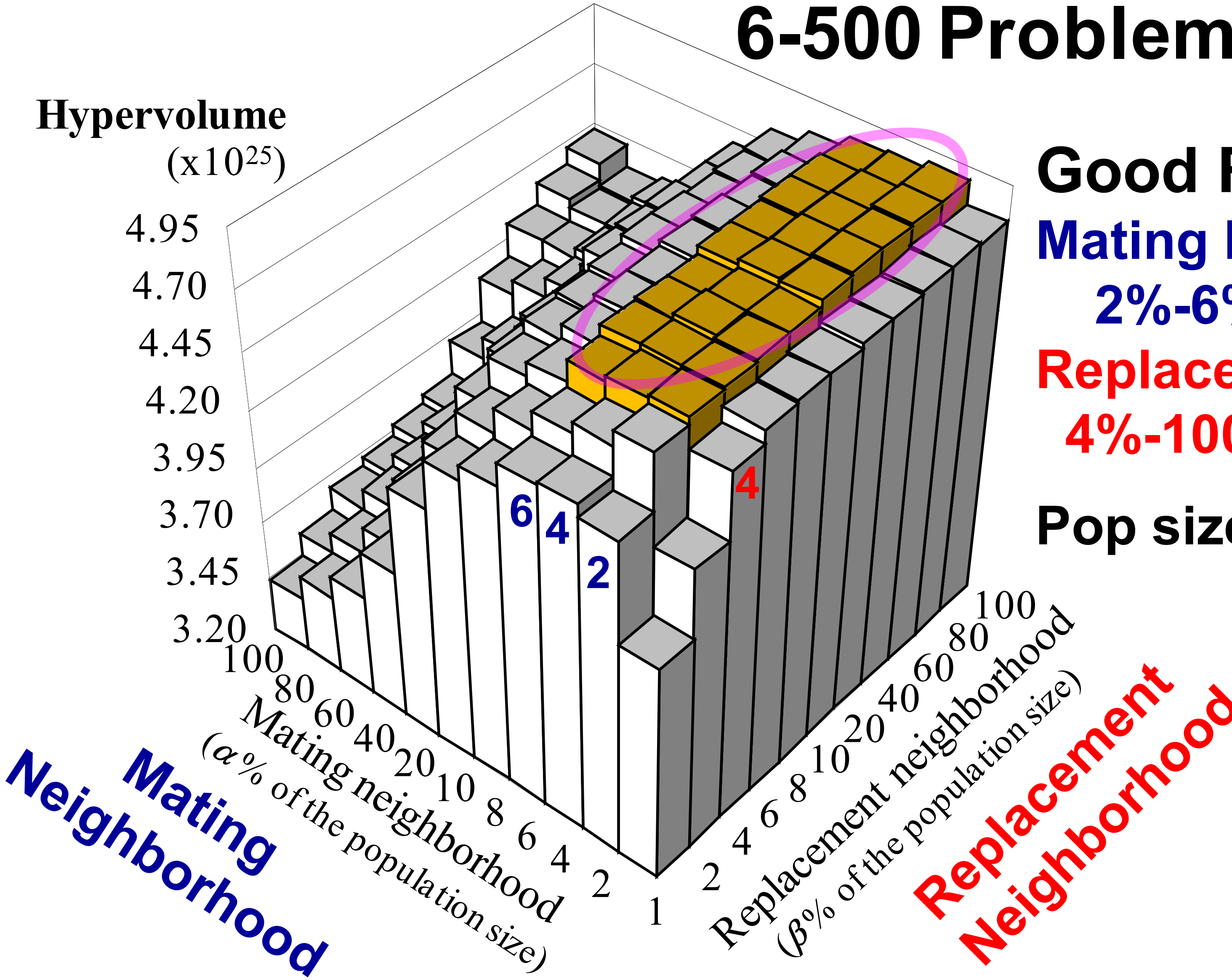
Good Results:

Mating Neighborhood:
2%-10% of Pop size

Replacement:
4%-100% of Pop size

Pop size: 220

Six-Objective 500-Item Problem
Average Hypervolume over 100 runs



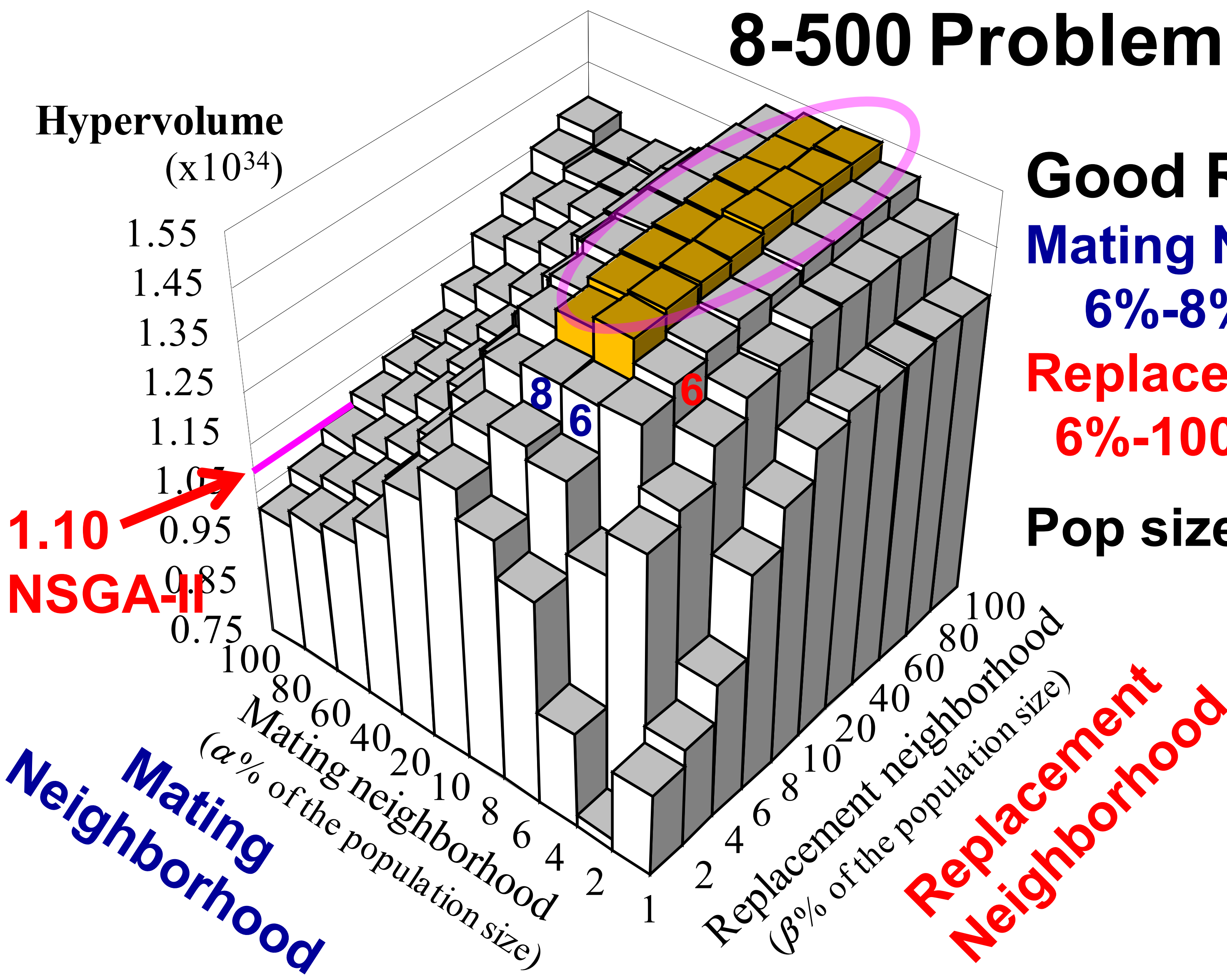
Good Results:

Mating Neighborhood:
2%-6% of Pop size

Replacement:
4%-100% of Pop size

Pop size: 252

Eight-Objective 500-Item Problem
Population Size: 120



Good Results:

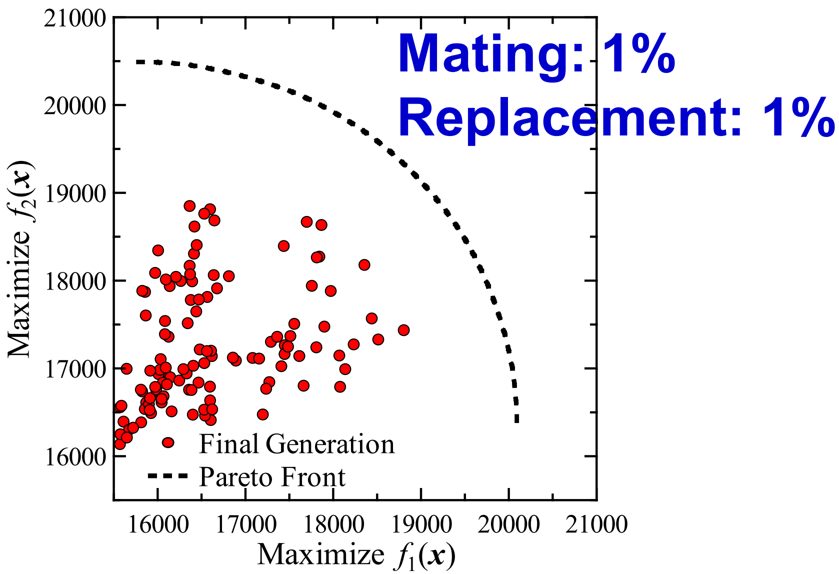
Mating Neighborhood: 6%-8% of Pop size

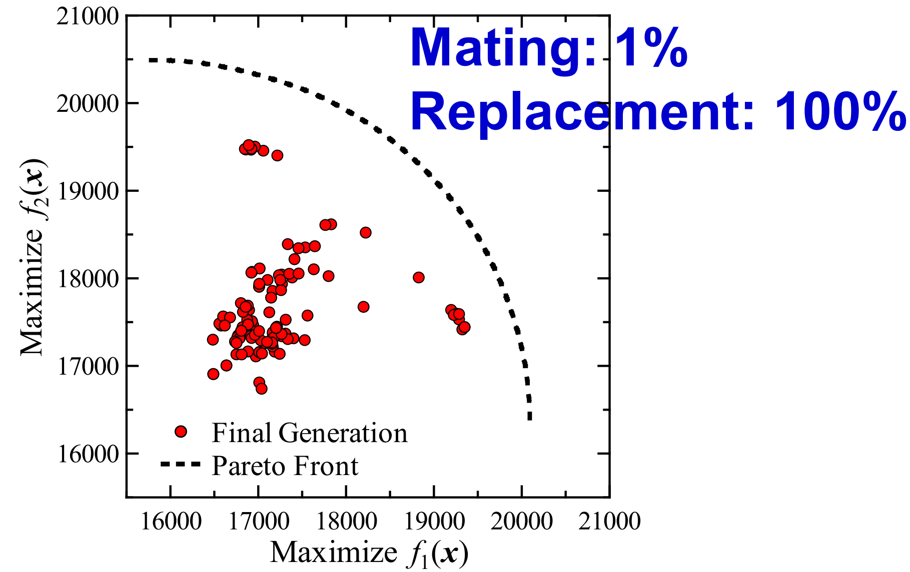
Replacement: 6%-100% of Pop size

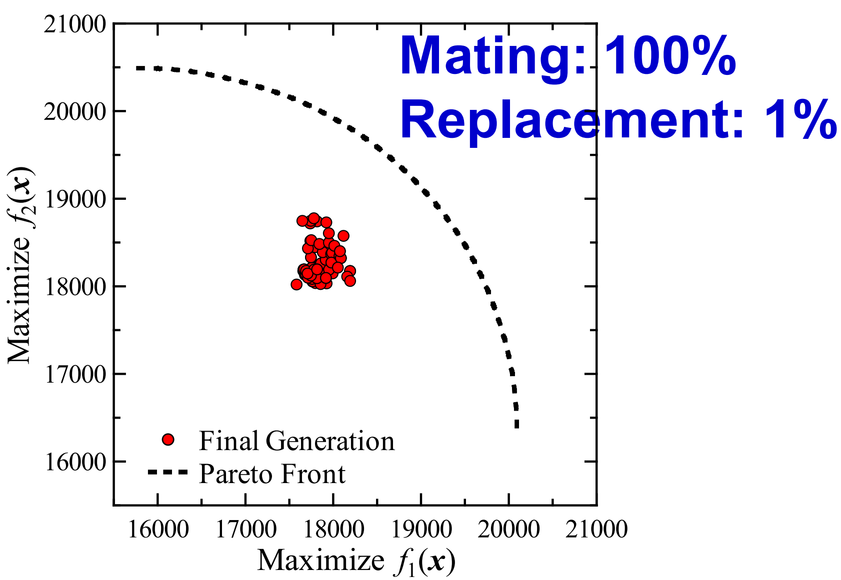
Pop size: 120

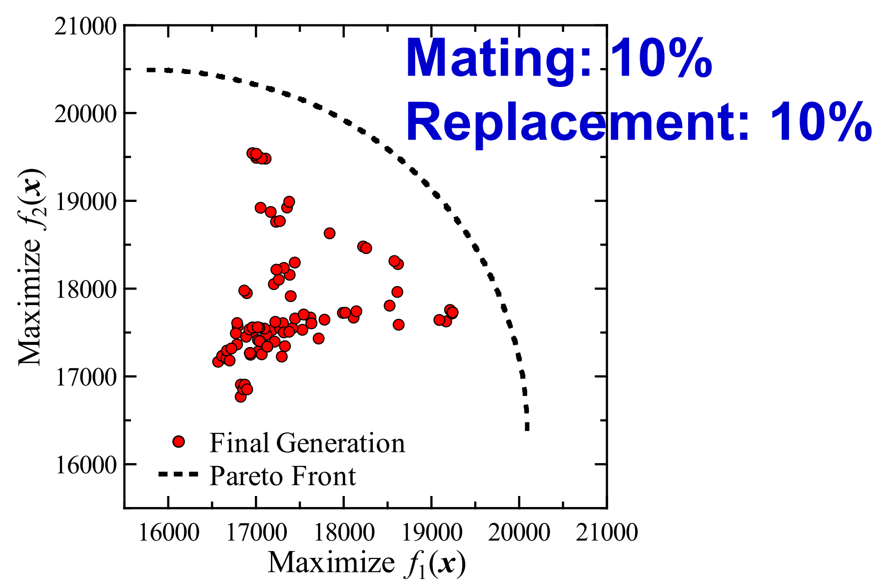
Eight-Objective 500-Item Problem

Projection onto the f1-f2 Subspace





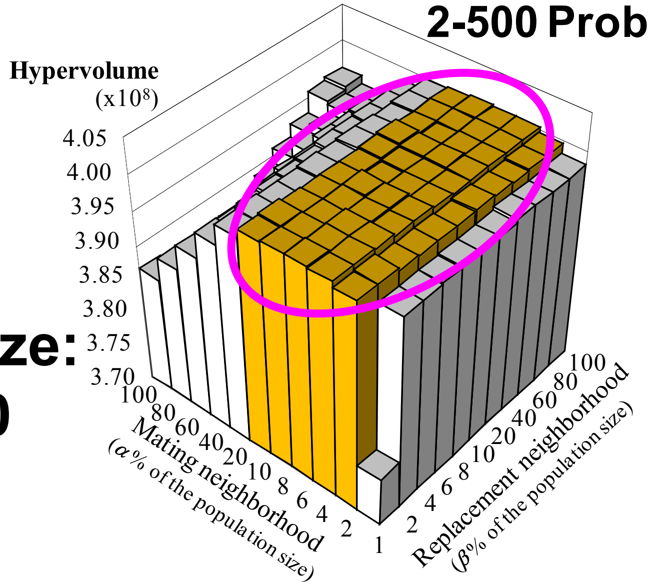




Knapsack Problems with 2-8 Objectives

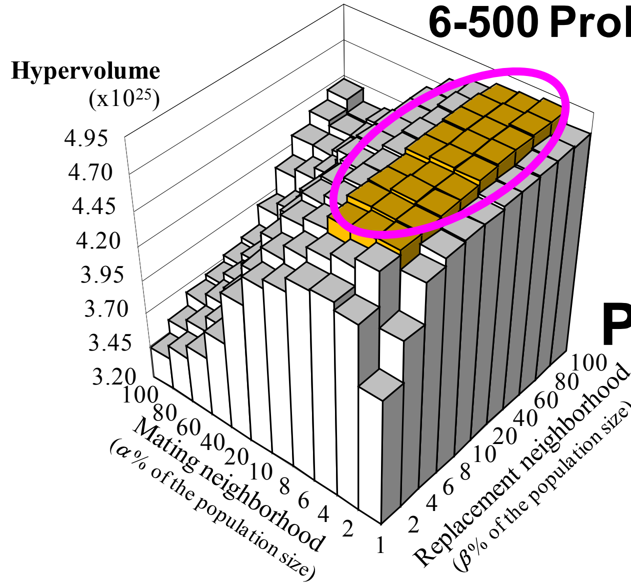
Average Hypervolume

2-500 Problem



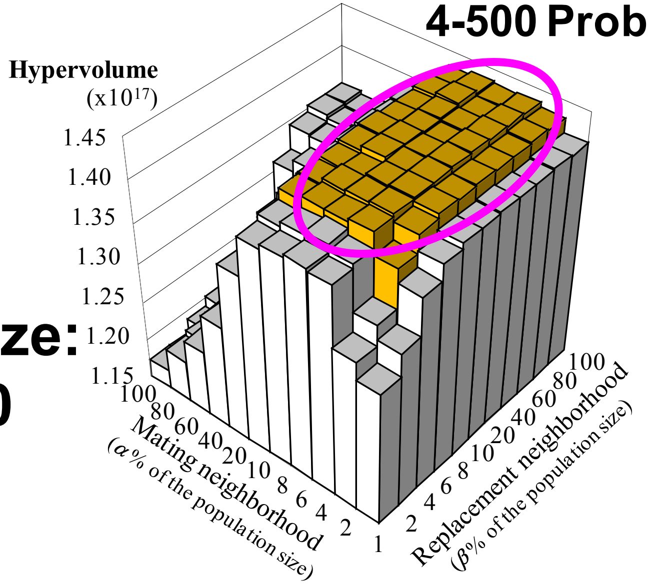
Pop size: 200

6-500 Problem



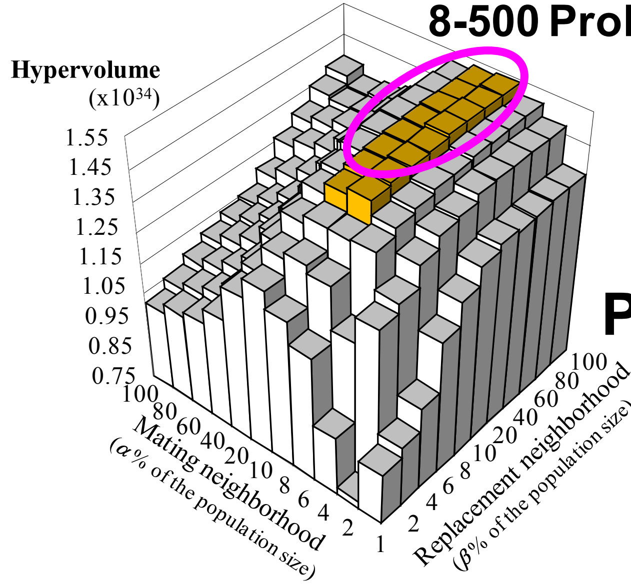
Pop size: 256

4-500 Problem



Pop size: 220

8-500 Problem

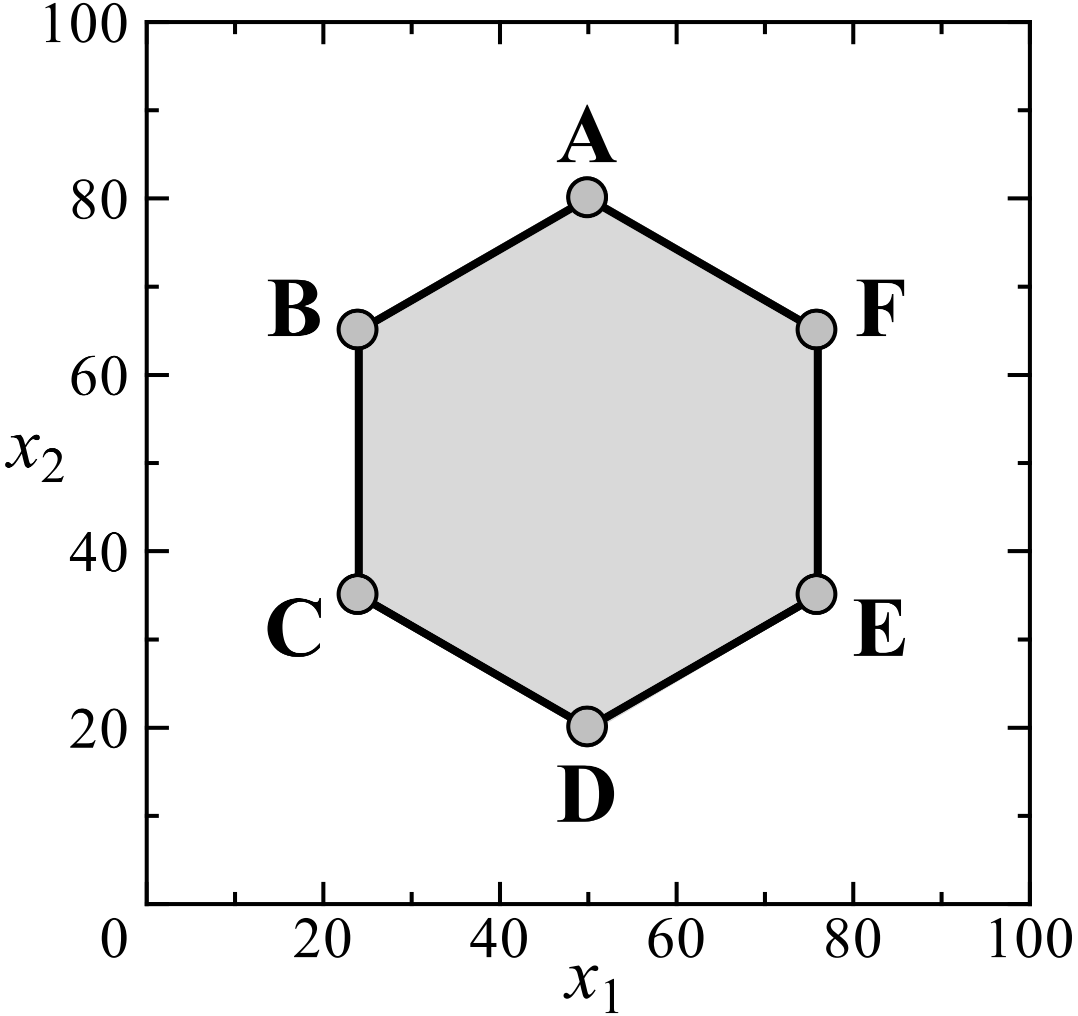


Pop size: 120

Other Test Problem

Six-Objective Distance Minimization Problem

Minimize $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), f_4(\mathbf{x}), f_5(\mathbf{x}), f_6(\mathbf{x}))$



$f_1(\mathbf{x}) = d(\mathbf{x}, \mathrm{A})$

$f_2(\mathbf{x}) = d(\mathbf{x}, \mathrm{B})$

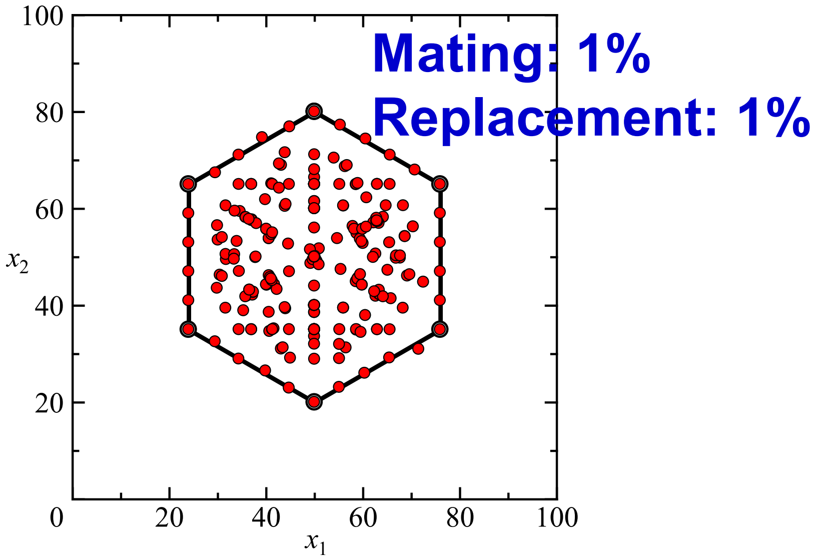
$f_3(\mathbf{x}) = d(\mathbf{x}, \mathrm{C})$

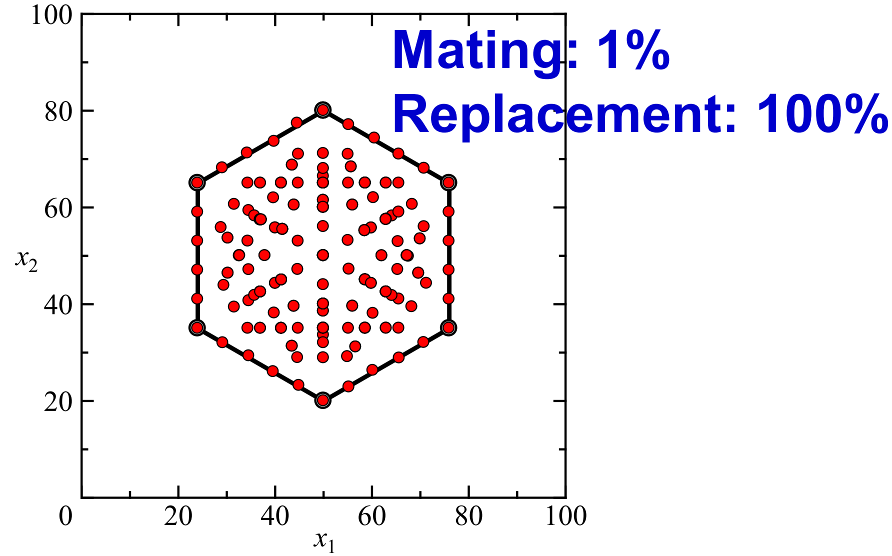
$f_4(\mathbf{x}) = d(\mathbf{x}, \mathrm{D})$

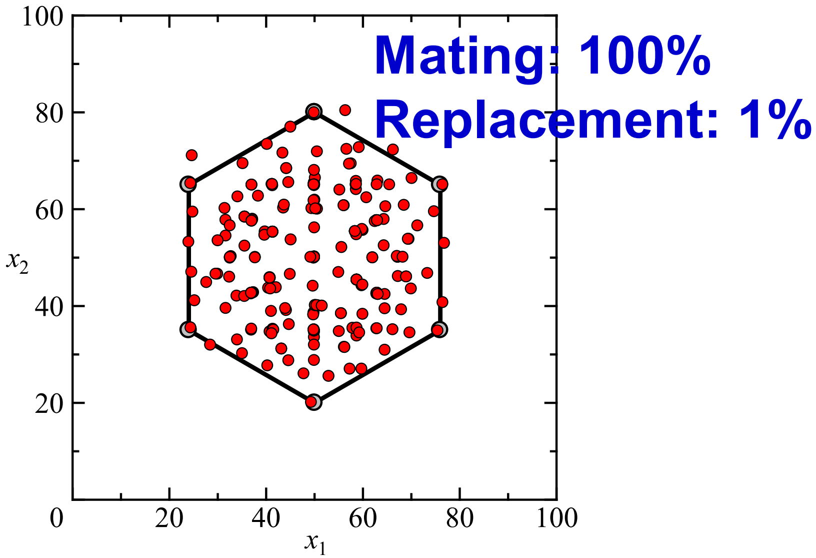
$f_5(\mathbf{x}) = d(\mathbf{x}, \mathrm{E})$

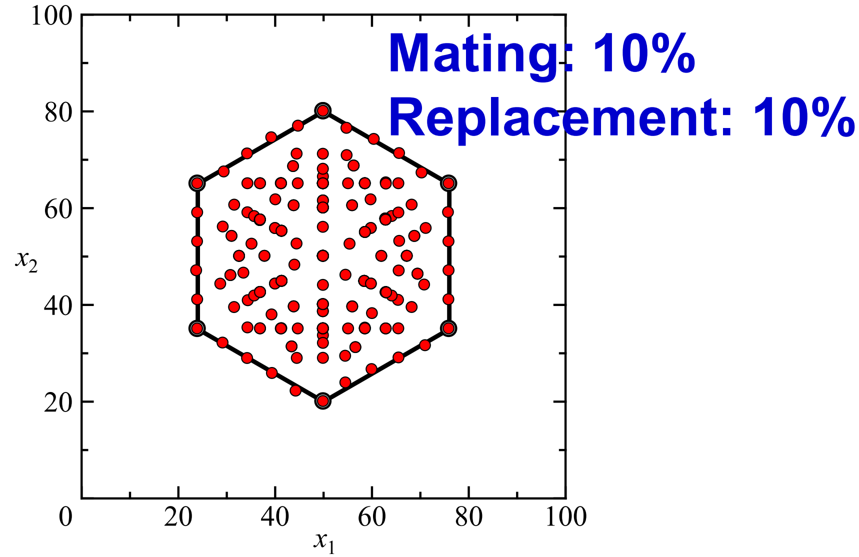
$f_6(\mathbf{x}) = d(\mathbf{x}, \mathrm{F})$

Other Test Problem
Six-Objective Distance Minimization Problem



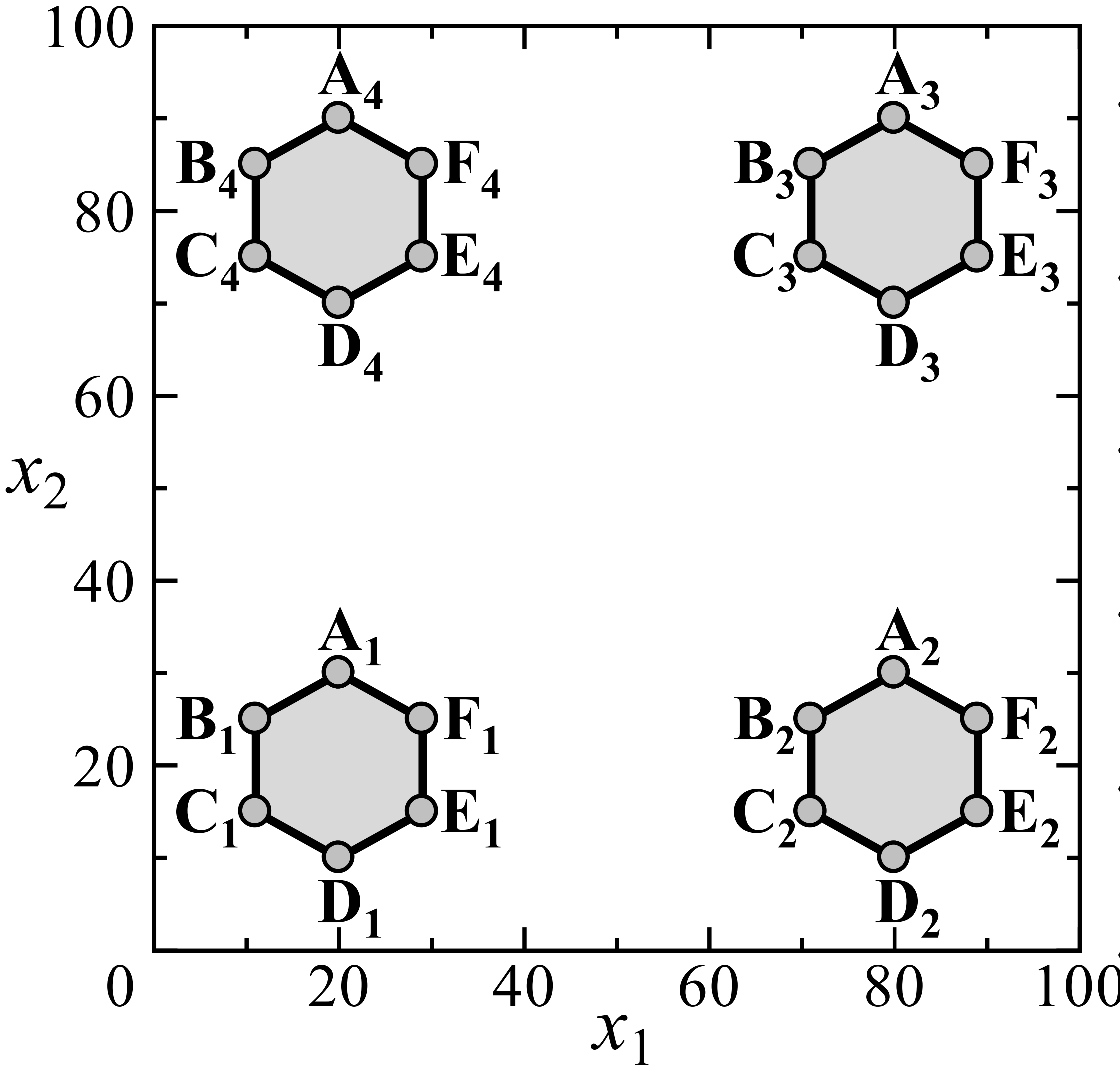






Six-Objective Problem with Four Equivalent Pareto Regions

Minimize $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), f_4(\mathbf{x}), f_5(\mathbf{x}), f_6(\mathbf{x}))$



f_1: $\min\{d(\mathbf{x}, A_1), d(\mathbf{x}, A_2), d(\mathbf{x}, A_3), d(\mathbf{x}, A_4)\}$

f_2: $\min\{d(\mathbf{x}, B_1), d(\mathbf{x}, B_2), d(\mathbf{x}, B_3), d(\mathbf{x}, B_4)\}$

f_3: $\min\{d(\mathbf{x}, C_1), d(\mathbf{x}, C_2), d(\mathbf{x}, C_3), d(\mathbf{x}, C_4)\}$

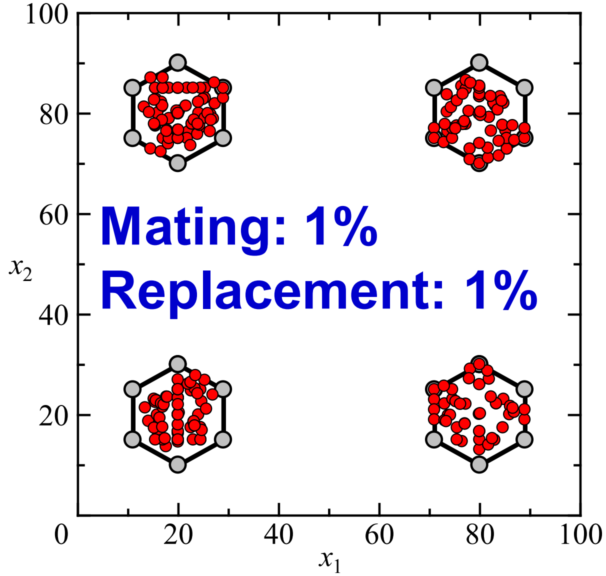
f_4: $\min\{d(\mathbf{x}, D_1), d(\mathbf{x}, D_2), d(\mathbf{x}, D_3), d(\mathbf{x}, D_4)\}$

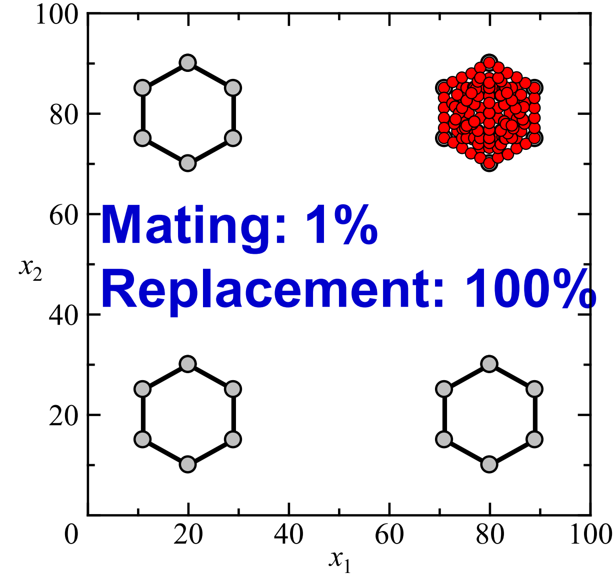
f_5: $\min\{d(\mathbf{x}, E_1), d(\mathbf{x}, E_2), d(\mathbf{x}, E_3), d(\mathbf{x}, E_4)\}$

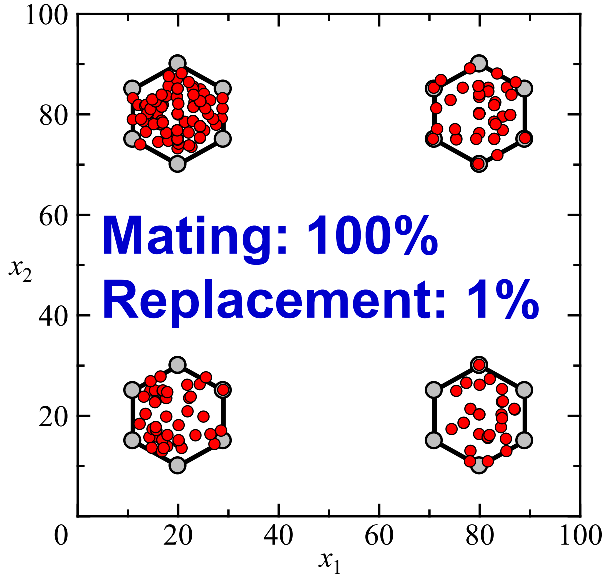
f_6: $\min\{d(\mathbf{x}, F_1), d(\mathbf{x}, F_2), d(\mathbf{x}, F_3), d(\mathbf{x}, F_4)\}$

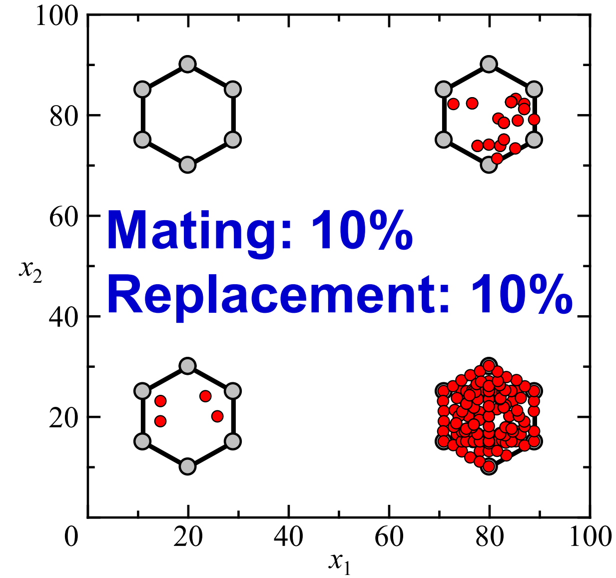
Other Test Problem

Six-Objective Distance Minimization Problem









Conclusions

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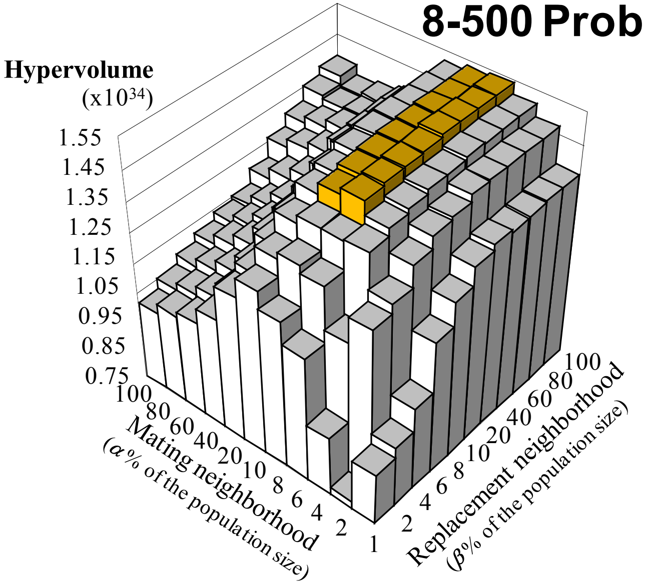
8-500 Problem



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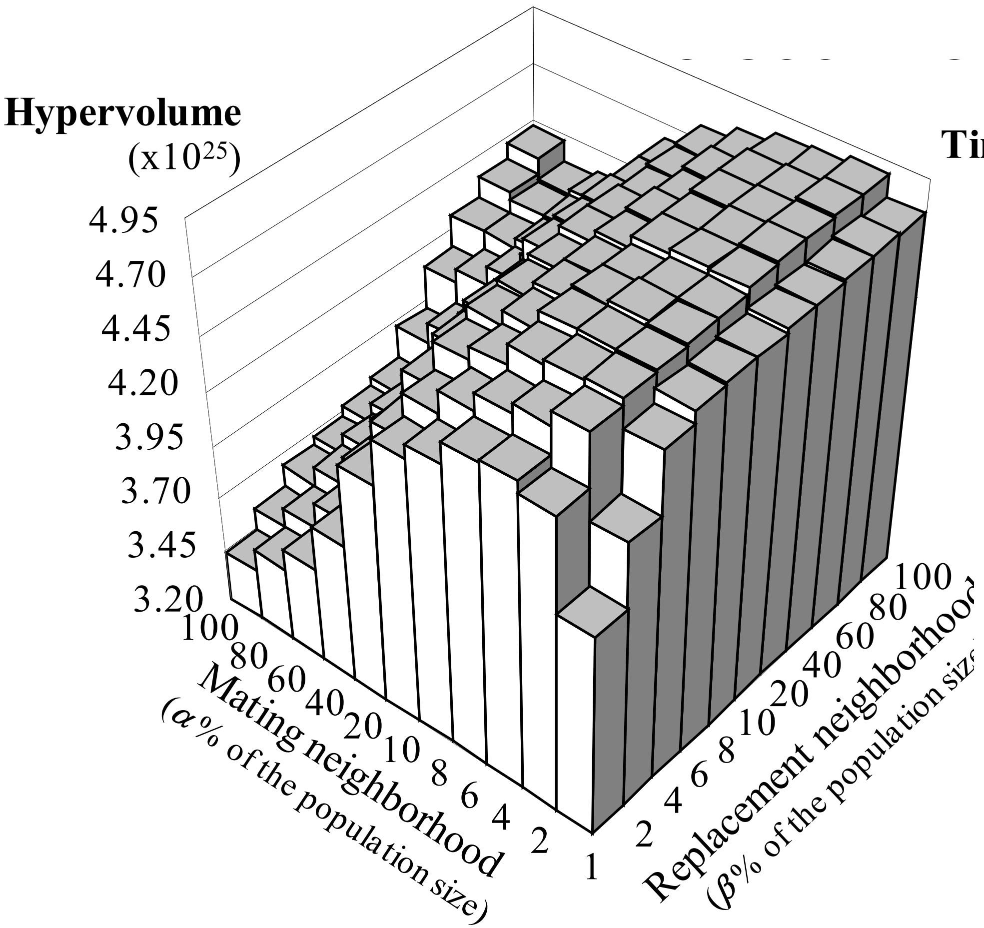
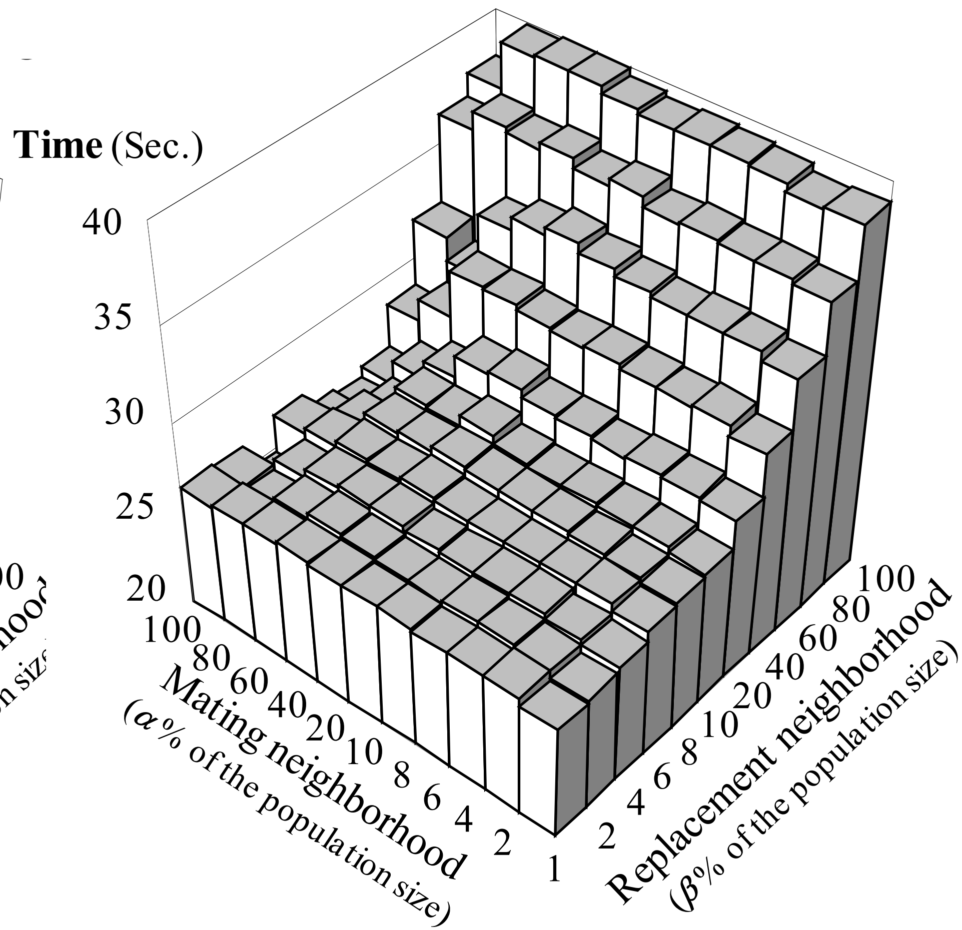
Conclusions

Thank you very much !

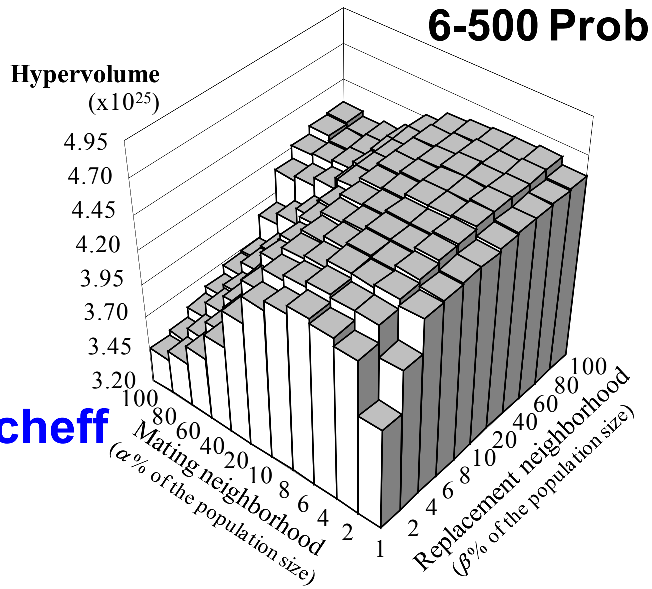
Average Computation Time

Six-Objective Knapsack Problem

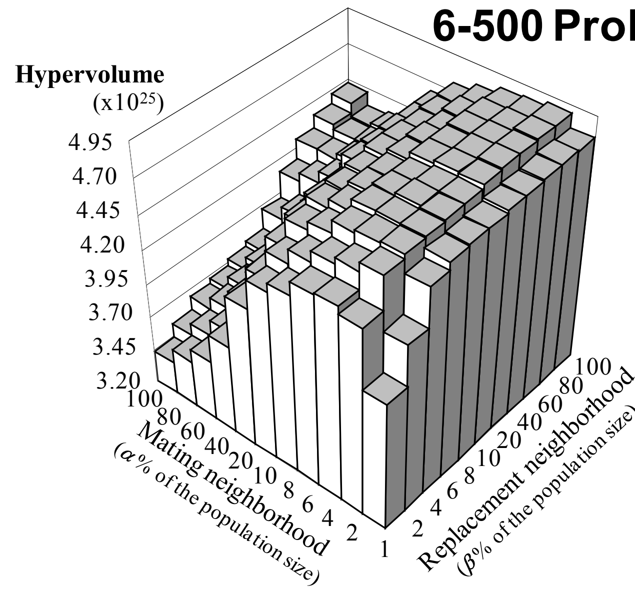




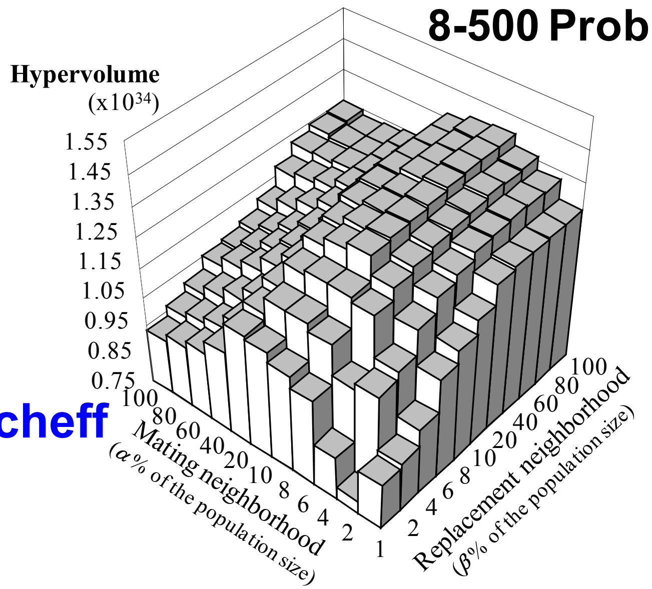
Weighted Tchebycheff and Weighted Sum



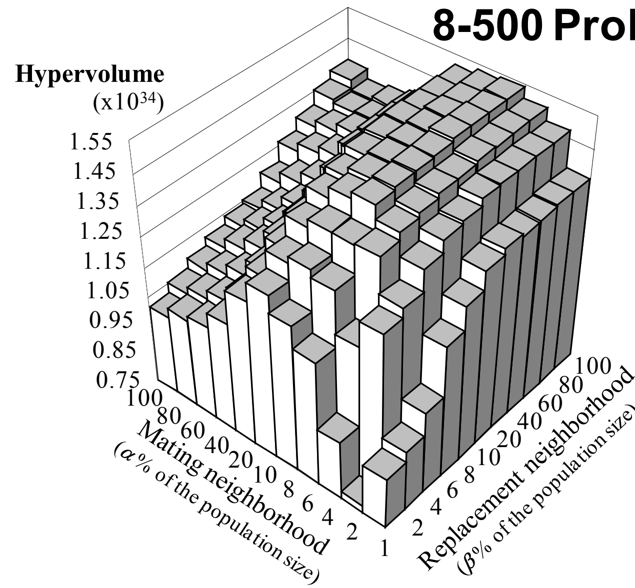
Tchebycheff



Weighted Sum



Tchebycheff

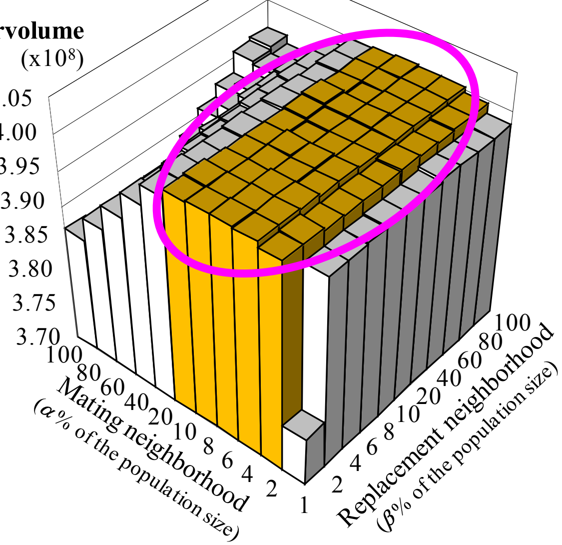


Weighted Sum

Knapsack Problems with 2-8 Objectives

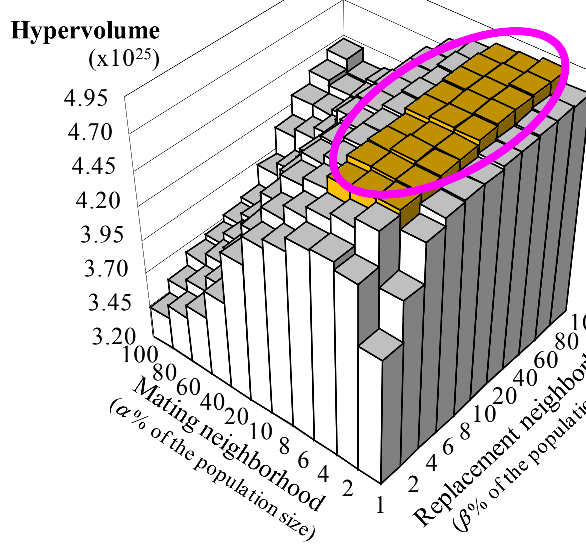
Average Hypervolume

2-500 Problem



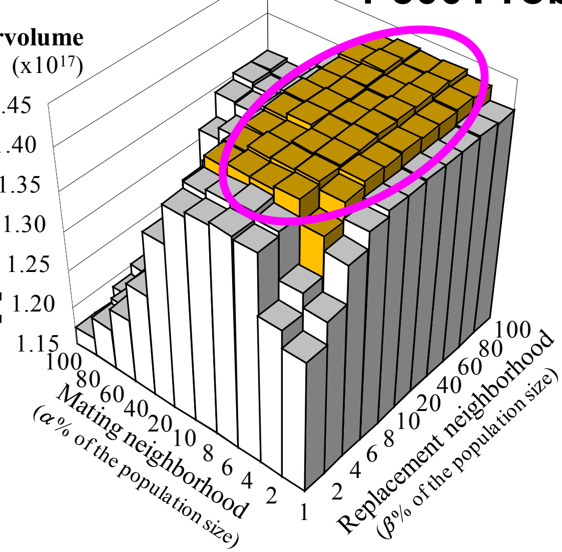
Pop size: 200

6-500 Problem



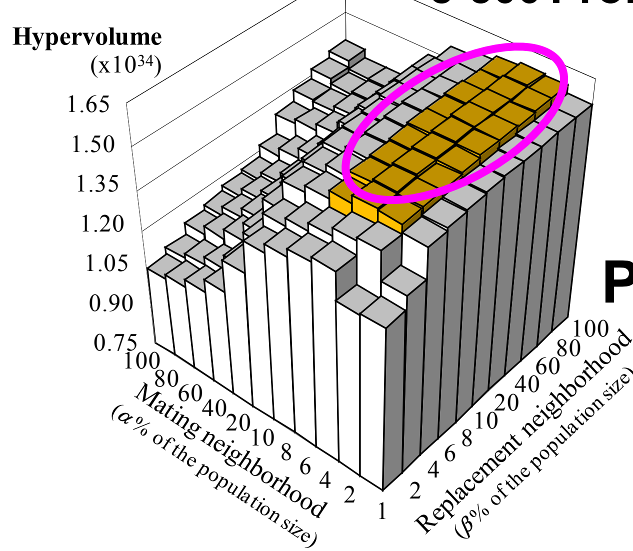
Pop size: 256

4-500 Problem



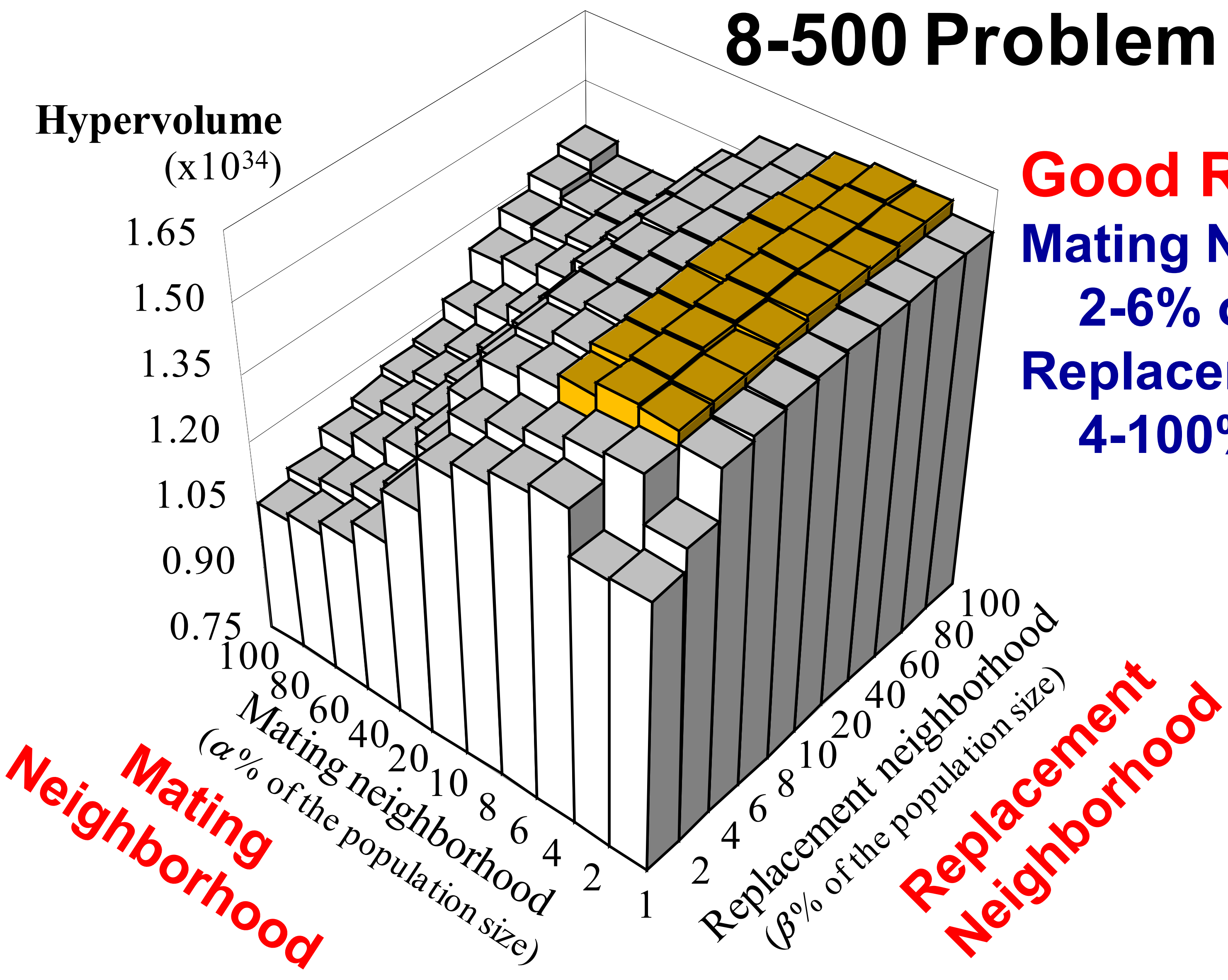
Pop size: 220

8-500 Problem



Pop size: 330

Eight-Objective 500-Item Problem
Population Size: 330



Good Results:

Mating Neighborhood:
2-6% of Pop size

Replacement:
4-100% of Pop size

Average Computation Time