Decision-Maker Preference Modeling in Interactive Multiobjective Optimization

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Introduction

This work presents the results of the hybridization of the iTDEA with an enhanced version of the NN-DM.

- iTDEA - [Köksalan and Karahan(2010)]

The iTDEA (Interactive Territory Defining Evolutionary Algorithm) is a preference-based multiobjective evolutionary algorithm that interacts with the Decision Maker (DM) during the course of optimization.

- NN-DM - [Pedro and Takahashi(2011)]

The NN-DM model is a preference model encoded as a neural network, which is trained using ordinal information only, provided by queries to the DM.

Motivation

The iTDEA detaches preferred regions by obtaining denser solutions in those regions and generating a sparse set of solutions in other regions.

The NN-DM model makes the preference information available for being used in other decision processes. This methodology can be useful in those situations in which a recurrent decision process must be performed.

The main point raised here is: the information obtained from the DM should not be discarded, leading to a new complete interaction with the DM each time a new run of a problem of the same class is required.

iTDEA

The iTDEA algorithm is an interactive approach that aims to converge to the preferred solutions of the DM by progressively obtaining preference information from the DM.

At the minimum, the DM may only choose the solution s/he prefers among the several solutions provided at each interaction stage. Alternatively, the DM may set the values of some parameters.

A territory around each solution is created, where no other solutions are allowed. This property maintains the population diversity, with different sizes depending on the individual's location on the Pareto-optimal frontier.
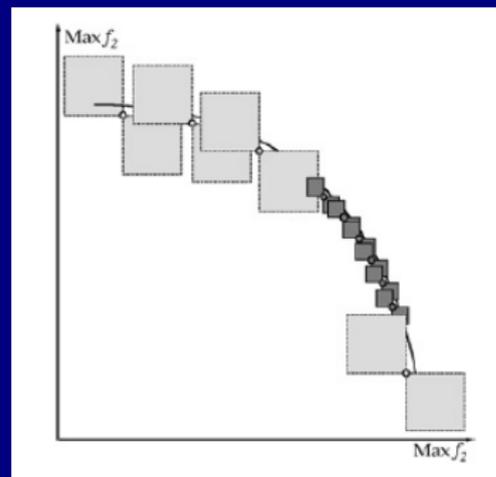
The algorithm aims to converge to the final preferred region of the DM.

Territories

Territory in two dimensions
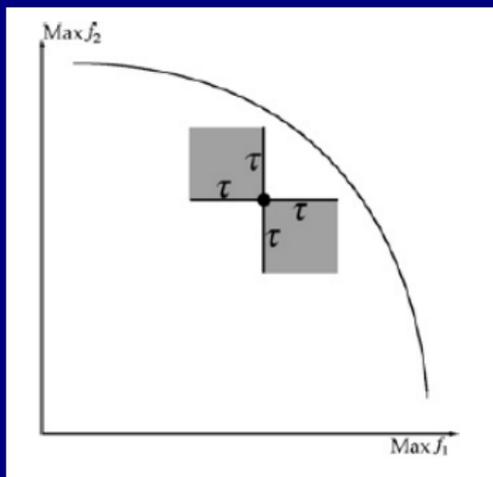
Different territory sizes

Discussion

The information extracted from the DM by the iTDEA is useful only within the scope of the optimization process in which such information is obtained.

However, it should be noticed that, very often, a multiobjective optimization problem might be solved for slightly different conditions, which makes the Pareto-front to become different from one run to the other, with the DM's preferences kept unchanged.

For instance, a product may be produced in different instances with different constraints in the resources availability, or with different parameters in some objective functions.

NN-DM Model

The NN-DM model is a representation for the DM's preferences using neural networks that has the purpose of reproducing the ordering of alternatives that would be delivered by the utility function $\mathcal{U}$ in a specific domain.

The NN-DM goal is to find a representation $\hat{\mathcal{U}}$ that models the DM's preferences using ordinal information obtained from queries presented to the DM.

The $\hat{\mathcal{U}}$ function must preserve the ordinal relationship between any two points, which is equivalent to stipulate that the level sets of $\mathcal{U}$ and $\hat{\mathcal{U}}$ must be the same.

The role of such a function is to replace the DM in new instances of the same multi-objective optimization problem, avoiding the formulation of new queries.

The NN-DM Methodology

Step 1: Choose the domain $\mathcal{A}$ for approximation.

*The available alternatives are used for this purpose.*

Step 2: Build a partial ranking, assigning a scalar value to each alternative and finding a partial sorting for the alternatives.

Select pivots and sort them in ascending order of the DM's preferences. Assign a rank to each pivot and therefore, a rank to each remaining alternative using comparisons with the pivots.
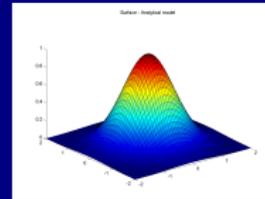
Step 3: Construct an artificial neural network $\hat{\mathcal{U}}$ which interpolates the results and approximates the DM utility function $\mathcal{U}$.

The alternatives within the domain are used as inputs and their ranking level as outputs.
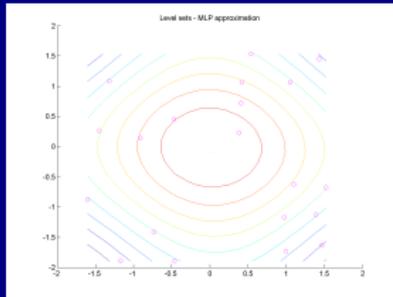
Underlying utility function
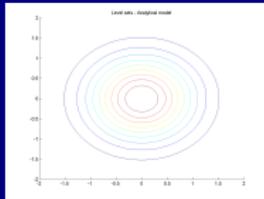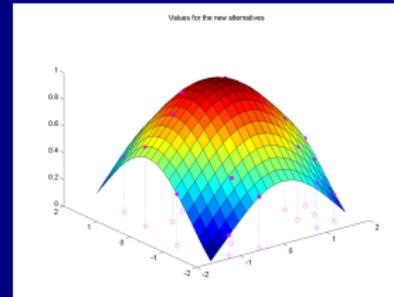

Level sets

Surface

Resulting estimated utility function

Level sets

Surface

DM Calls - iTDEA

In the iTDEA, the DM is required to interact with the optimization process at predetermined generations (4 or 6 times). In each interaction, the DM is required to find the best solution among those P filtered solutions. For m objectives, $P = 2m$, except in the first stage, in which $P = 4m$.

Considering that only ordinal information is available with binary comparisons, for each set of n elements, at least $n - 1$ queries are made to the DM. Thus, a lower bound for the number of queries presented to the DM would be $10m - 4$ in 4 interactions and $14m - 6$ in 6 interactions.

Those estimates were used as a reference for the number of queries that may be used to construct the model for the DM's preference.

DM Calls - NN-DM

In the NN-DM, the DM provides only ordinal information.

Let n be the total number of alternatives and $p = \log n$ the number of pivots for the partial sorting. The average (upper bound) total of queries to the DM is given by $n \cdot \log(\log n)$.

By considering the alternatives in a domain instead of the Pareto-optimal solutions only, the dominance also can be used to replace the DM in some decisions, making the process cost-effective.

Computational Experiments

	$2D$	$3D$
Ideal vector, f^*	$(0, 0)$	$(0, 0, 0)$
Number of interactions H	4	4
Population size	200	200
τ_0	0.1	0.1
τ_H	0.001	0.001
Number of iterations T	10 000	10 000
Number of replications	50	50
Number of training points	12	18
Estimate number of queries	20	44

Table : Test parameters

DM Utility Function

In all cases, the DM utility function is simulated considering the following function:

$$\mathcal{U}(p) = \exp(-\mathbf{f} \cdot A \cdot \mathbf{f}^t)$$

▶ Bi-objective, $\mathbf{f} = (f_1, f_2)$

$$A_{10} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad A_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

▶ Three-objective, $\mathbf{f} = (f_1, f_2, f_3)$

$$A_{100} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad A_{111} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Bi-Objective Optimization Problem

As a first example, the following bi-objective optimization problem with two decision variables is considered:

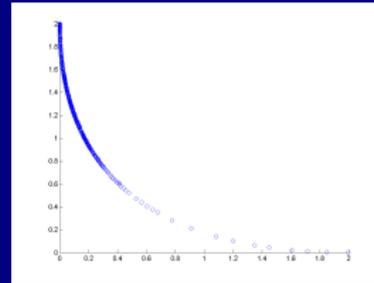$$\mathbf{p} = \{p_1, p_2\}, \quad \mathbf{f} = (f_1, f_2),$$

$$f_i(\mathbf{p}) = (\mathbf{p} - \mu_i) \cdot M \cdot (\mathbf{p} - \mu_i)^t, i = 1, 2$$

$$M = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right] \qquad \begin{array}{l} \mu_1 = [1\ 0] \\ \mu_2 = [0\ 1] \end{array}$$
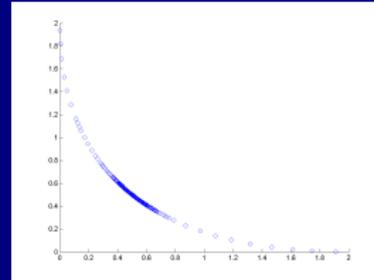
2D Results
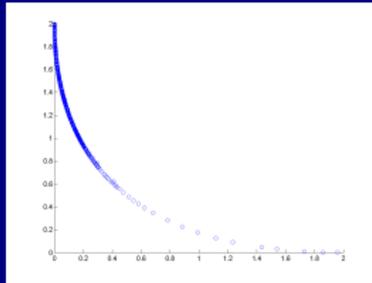
Territory (A_{10})

NN-DM (A_{10})
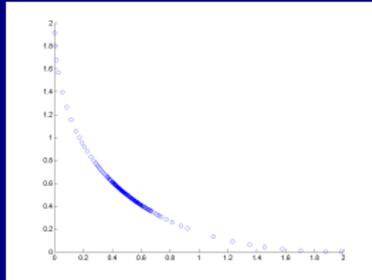
Territory (A_{11})

NN-DM (A_{11})
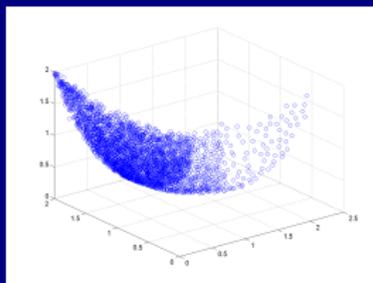
Three-Objective Optimization Problem

An optimization problem with three objectives and three variables is also considered:

$$\mathbf{p} = \{p_1, p_2, p_3\}, \quad \mathbf{f} = (f_1, f_2, f_3),$$
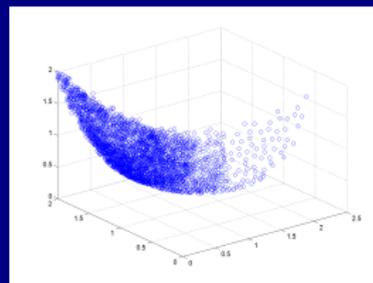
$$f_i(\mathbf{p}) = (\mathbf{p} - \mu_i) \cdot M \cdot (\mathbf{p} - \mu_i)^t, i = 1, 2, 3$$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{array}{l} \mu_1 = [1\ 0\ 0] \\ \mu_2 = [0\ 1\ 0] \\ \mu_3 = [0\ 0\ 1] \end{array}$$
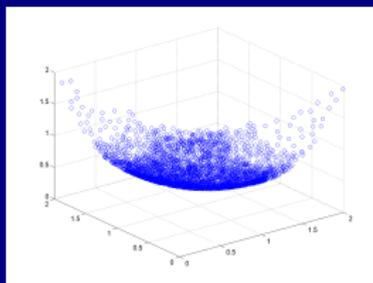
3D Results
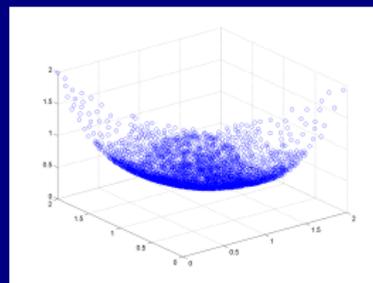

Territory (A_{100})

Model (A_{100})

Territory (A_{111})

Model (A_{111})

Conclusions

This work presented a methodology that allows to get the information concerning the Decision-Maker (DM) preferences.

The preference information is acquired from the DM in the course of one run of an interactive evolutionary multiobjective optimization algorithm (iTDEA) and stored in a neural network (NN-DM), which is trained using ordinal information only.

The proposed methodology can be useful in those situations in which a recurrent decision process must be performed.

The information obtained from the DM should not be discarded, leading to a new complete interaction each time a new run of this class of problems is required.

Bibliography

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Thank you!