Data provided: Statistics Tables, H.R. Neave

The University Of Sheffield.

MAS5051

SCHOOL OF MATHEMATICS AND STATISTICS **June 2014**

Probability and Probability Distributions **2 hours**

RESTRICTED OPEN BOOK EXAMINATION.
Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.
Candidates should attempt ***ALL*** *questions.*
The maximum marks for the various parts of the questions are indicated.
The paper will be marked out of 80.

Please leave this exam paper on your desk
Do not remove it from the hall

Registration number from U-Card (9 digits)
to be completed by student

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1 Let X be a continuous random variable with probability density function given by

$$f(x) = \begin{cases} k/(x+1) & 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of k. *(4 marks)*

(b) Find the distribution function of X. *(5 marks)*

(c) Find $E(X)$. *(3 marks)*

(d) Let $Y = 1/X$. Find the probability density function of Y. *(5 marks)*

2 A fair coin is tossed three times. Assume the tosses are independent of each other.

(a) Describe an appropriate sample space to use for this experiment. *(3 marks)*

(b) Let X be the random variable which counts the number of heads observed in the experiment.

(i) For each $i \in \{0, 1, 2, 3\}$, give the elements of the sample space which are in the event $\{X = i\}$ and the probability that $X = i$. *(8 marks)*

(ii) Find the mean and variance of the random variable X. *(7 marks)*

(c) Let E be the event that there are more heads than tails, and let F be the event that the number of heads is odd. Are these events independent? Give a reason for your answer. *(3 marks)*

3 Let R be the region defined by $R = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 1$, and let X and Y be random variables with joint probability density function given by

$$f_{X,Y}(x, y) = \begin{cases} \frac{3}{4}(x-y)^2 & (x, y) \in R \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $P(X \leq 1, Y \leq 1/2)$. *(4 marks)*

(b) Find the marginal probability density functions of X and Y. *(5 marks)*

(c) Are X and Y independent? Give a reason for your answer. *(3 marks)*

(d) Find $E(XY)$. *(4 marks)*

4 (a) State which standard distribution might serve as a model for each of the following.

(i) The number of meteorites above a certain size hitting a particular area of the Earth in a year. ***(2 marks)***

(ii) The number of items in a batch of fixed size which are faulty. ***(2 marks)***

(iii) The length of time until the next meteorite above a certain size hits a particular area of the Earth. ***(2 marks)***

(b) In (a) parts (i) and (ii), under what circumstances might a Normal approximation to the distribution you suggested be appropriate? ***(4 marks)***

5 A patient takes a test to see whether they have a disease. If the patient has the disease, then the test will be positive with probability p and negative with probability $1 - p$, while if the patient does not have the disease the test will be positive with probability q and negative with probability $1 - q$. According to the patient's doctor, the prior probability that they have the disease is d.

(a) Find the posterior probability that the patient has the disease, in terms of p, q and d, if

(i) the test result is positive;

(ii) the test result is negative. ***(8 marks)***

(b) It is considered that it is worthwhile to treat the patient for the disease if the posterior probability of them having the disease is at least 0.4.

(i) If $p = 0.9$ and $q = 0.1$, for what values of d will a positive test indicate that it will be worthwhile for the patient to be treated for the disease? ***(4 marks)***

(ii) If $p = 0.9$ and $q = 0.1$, for what values of d will it be worthwhile to treat the patient for the disease even if the test result is negative? ***(4 marks)***

End of Question Paper