Implementing an elastic-elastic contact algorithm for voxel models

Elliot Morley

INSIGNEO
Institute for *in silico* Medicine

An algorithm has been implemented to account for deformability of both bodies in contact when using voxel-based meshes. Preliminary verification produced unexpected results.

Background

Finite element analysis (FEA) allows non-invasive, computational experiments to be performed on results obtained from bone scans. The geometry of the bone can be mapped to a mesh, which usually comprises tetrahedral elements. This produces a smooth geometry but is inefficient to solve when the number of elements is in billions. Cartesian mesh models use regular voxels that are much more efficient to solve than smooth models. The recent **S**imulated **S**mooth **S**urface **C**ontact (SS-SC) formulation improved the accuracy of voxel-based models relative to smooth models [1]. Yet, SS-SC requires the "master" contact body to be rigid and static. This project aims to remove this limiting requirement by implementing the elastic–elastic formulation of Parisch (1989) [2] into the MATLAB code for SS-SC.

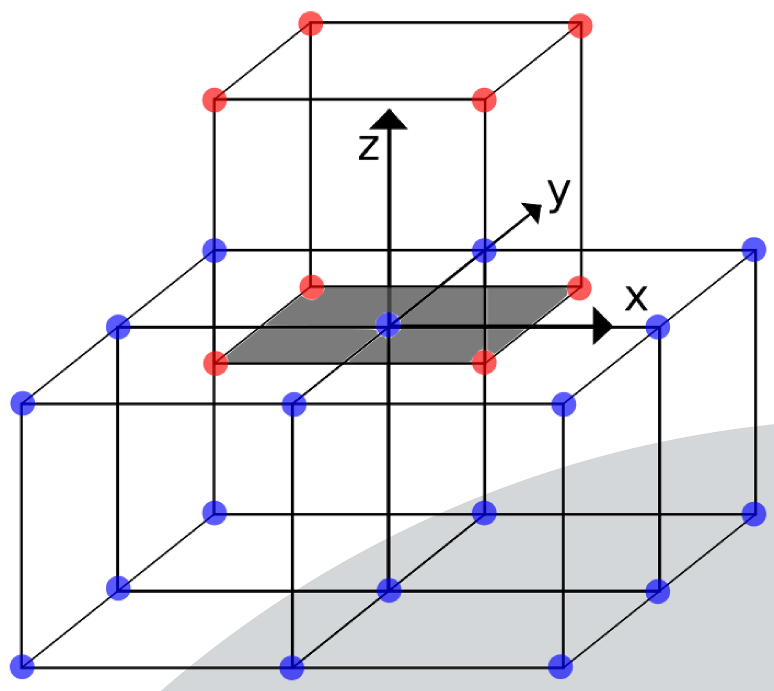

Figure 1. Model of elastic cube (top) indenting an elastic cuboid (bottom)

Methods

SS-SC is a node-to-surface (NTS), penalty contact method, and differs from the conventional approach (**S**tair-**C**ase, **S**liding **C**ontact, or SC-SC) in that it projects slave nodes to a smooth surface when computing the contact gap. The main changes I introduced in the MATLAB implementation following Parisch (1989) are: (a) an isoparametric transform of contacting surfaces; (b) a consistent definition of contact surface normal that changes with location on the surface; (c) a variational formulation of contact reactions accounting for variations of this normal.

The slave node / master surface dichotomy of NTS arises naturally when differences in stiffness and/or mesh refinement between the contact bodies are significant. Such differences are low in voxel-based models of bone, and hence the dichotomy is less relevant. Hence, in addition to above changes, I introduced the possibility of defining the contacting bodies as alternatively master and slave.

In the verification target problem considered here (Figure 1), an elastic cube indents a larger elastic cuboid resting on a rigid foundation. This problem choice minimises the improvement due to using SS-SC formulation over SC-SC. This ensures that any error we get is mainly due to the introduction of the method of Parisch (1989) [2] and not due to SS-SC, which could confound the verification.

Both bodies were linear elastic, homogeneous and isotropic with Young's 10 GPa and Poisson's ratio 0.3. All voxel edges were 1 unit long. Nodal constraints were applied to prevent uncontrolled rotation of the mesh, and the bottom surface of the bottom cuboid was fixed in the z-direction. The top surface of the top cube was displaced by -0.1 units in z-direction. The two-pass algorithm weighted both master/slave pairings equally, with produced results . Simulations of the same model were run in both MATLAB and Abaqus.

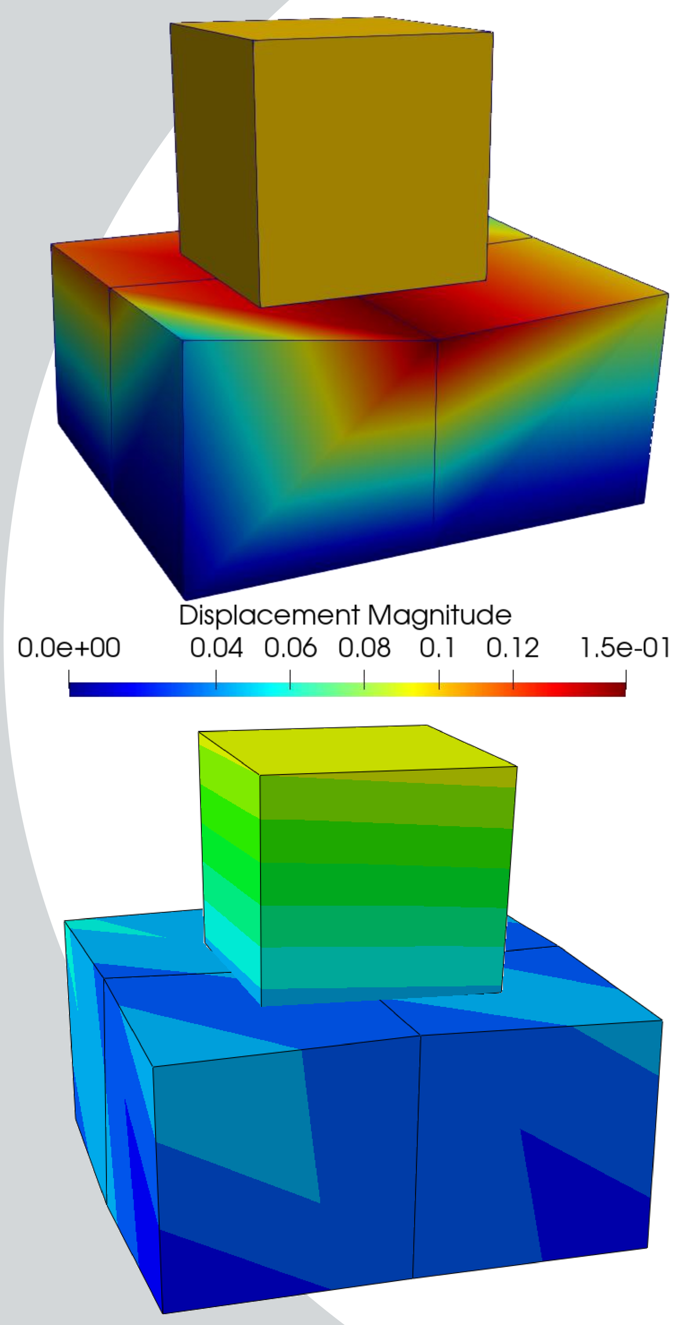

Figure 2. The visualised displacement magnitude from the MATLAB model (above) and the Abaqus model (below). The legend applied to both models.

Results

Magnitude of displacement (arbitrary units)	MATLAB	Abaqus	Percentage difference
Maximum	0.1504	0.1073	140%
Minimum (non-zero)	0.0422	0.0163	259%
Mean	0.0665	0.0368	181%

The single voxel in MATLAB did not experience either compression or a Poisson effect due to the applied boundary conditions, which is unexpected. The Abaqus model experienced both, which is realistic.

Conclusions

The MATLAB algorithm produces an unexpected result in this test – a constant displacement throughout the whole elastic cube. This is unrealistic, as compression along with Poisson expansion should occur. Validation of the MATLAB code needs to continue.

Acknowledgements

[1] P. Bhattacharya, D. Betts, and G. H. van Lenthe, "A novel contact interaction formulation for voxel based micro-finite-element models of bone," *International journal for numerical methods in engineering*, vol. 115, no. 4, pp. 411–426, 2018.
[2] H. Parisch, "A consistent tangent stiffness matrix for three-dimensional non-linear contact analysis," *International Journal for Numerical Methods in Engineering*, vol. 28, no. 8, pp. 1803–1812, 1989.

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