Oscillatory response of the solar atmosphere to the leakage of photospheric motion

Viktor Fedun, Robertus Erdélyi, S. Shelyag

The University of Sheffield,
Department of Applied Mathematics
v.fedun@sheffield.ac.uk

Outline of the talk

- Waves in the Solar Atmosphere
- Geometry, equilibrium and equations
- Numerics (SAC)
- 3D full HD and MHD simulations
- Magnetic field in MHD modelling
- 2D full MHD simulations (self-similar magnetic field)
- Conclusions

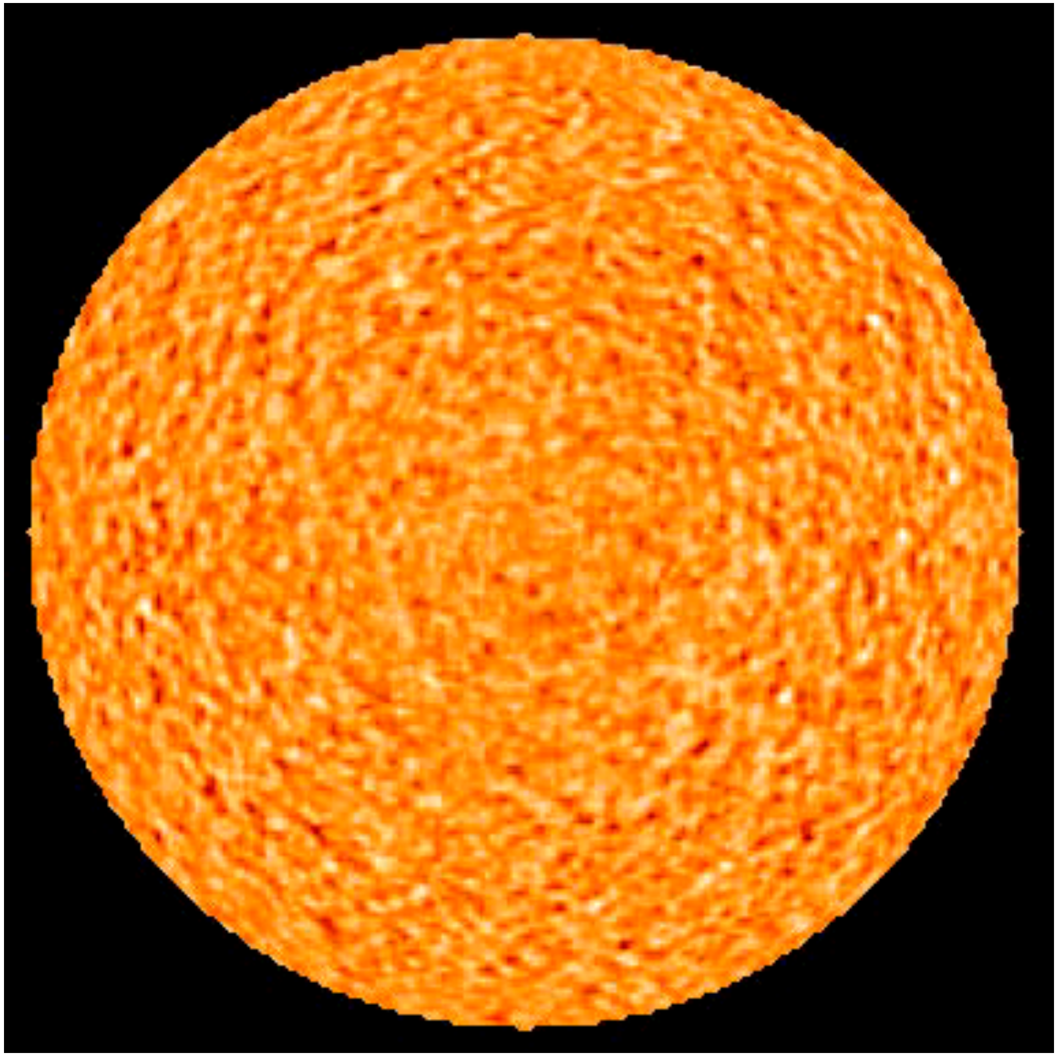
Waves in the Solar Atmosphere

Recent high-resolution satellite observations clearly show the existence of a wide range of previously theoretically predicted MHD waves in solar atmospheric magnetic structures. See Aschwanden (2004), Wang (2004).

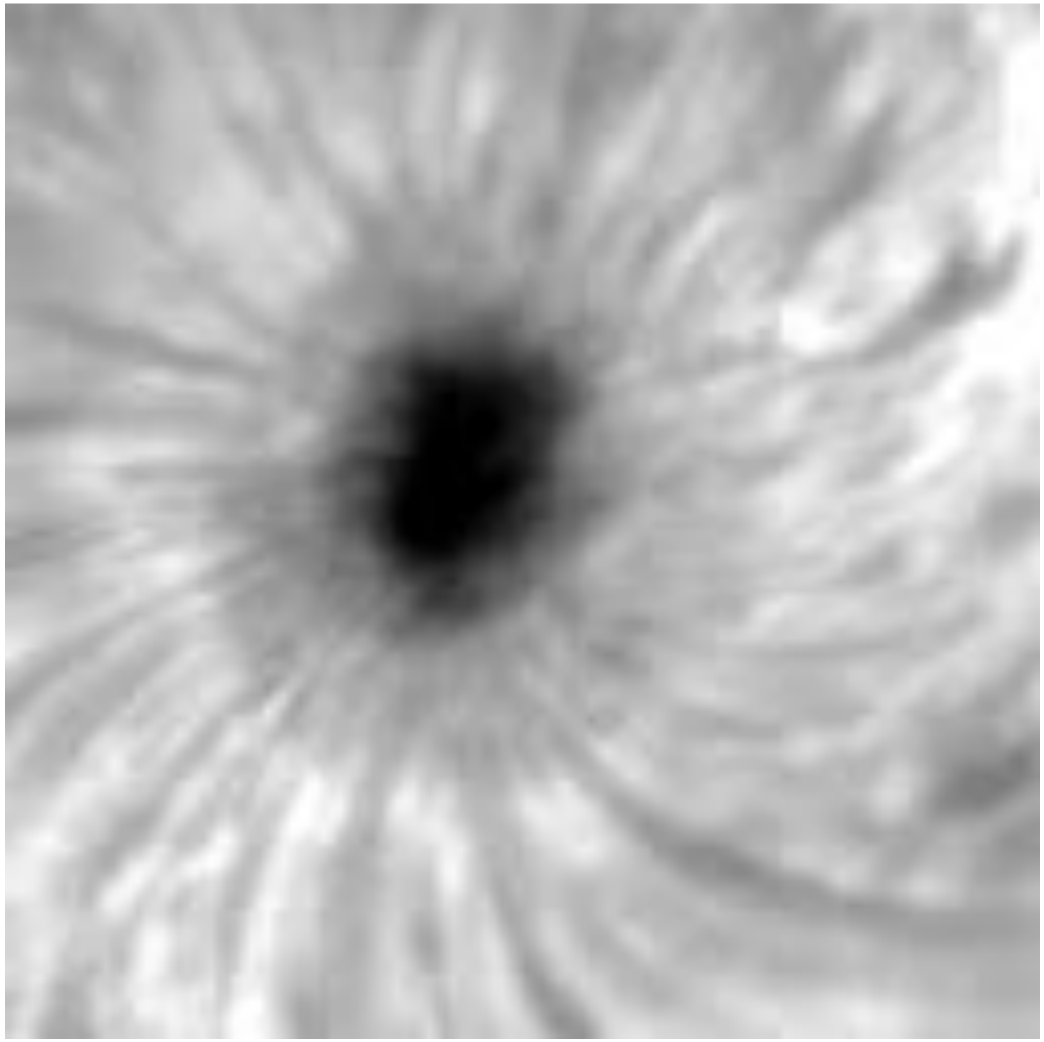
These waves include:

- **global *p*-mode oscillations** observed in the photosphere,
- **umbral oscillations** observed in sunspots,
- **transverse oscillations** in coronal loops
- and more (plumes, fibrils, prominences, etc).

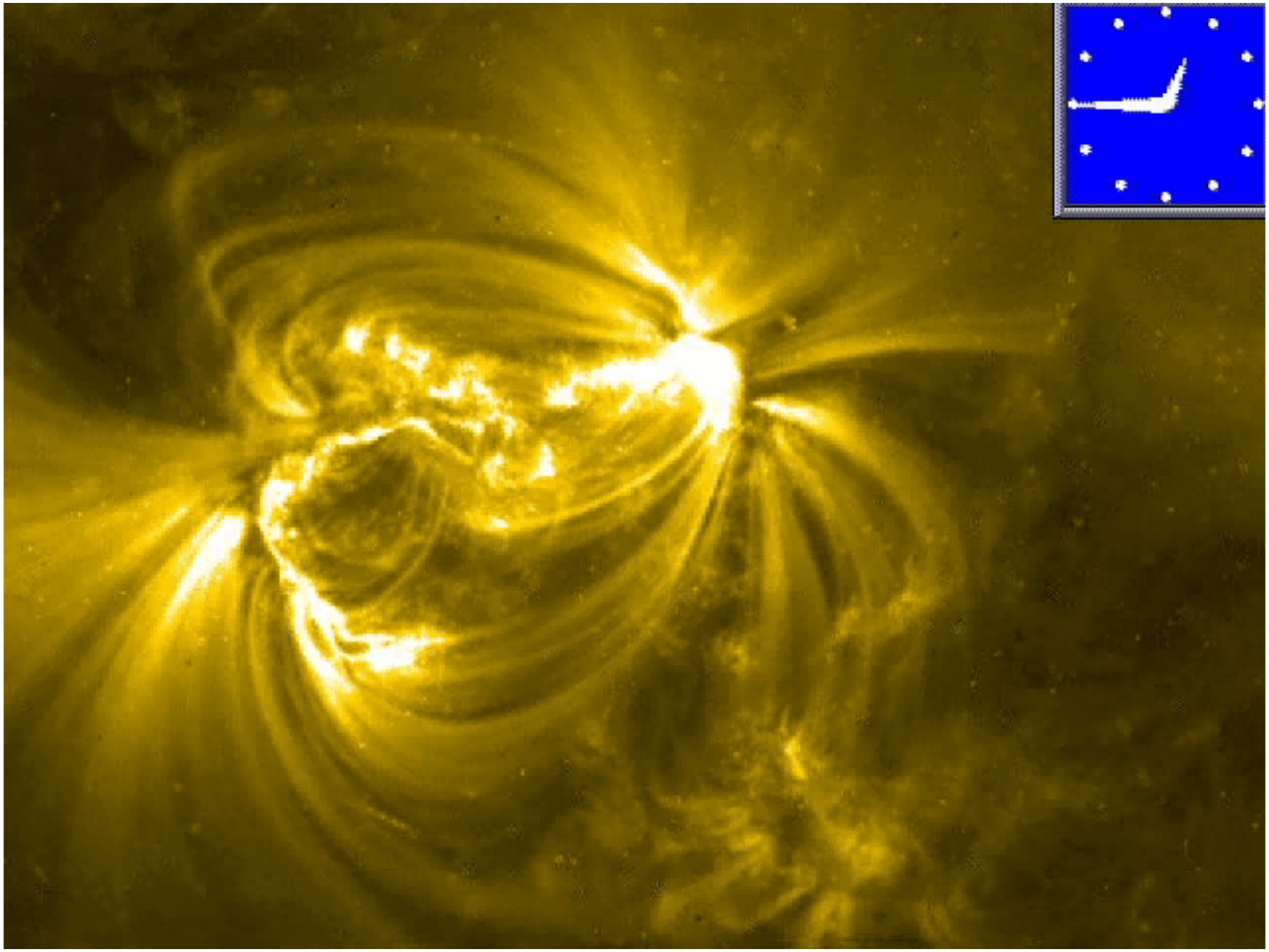
Global *p*-mode oscillations



Umbral oscillations



Transverse oscillations in coronal loops

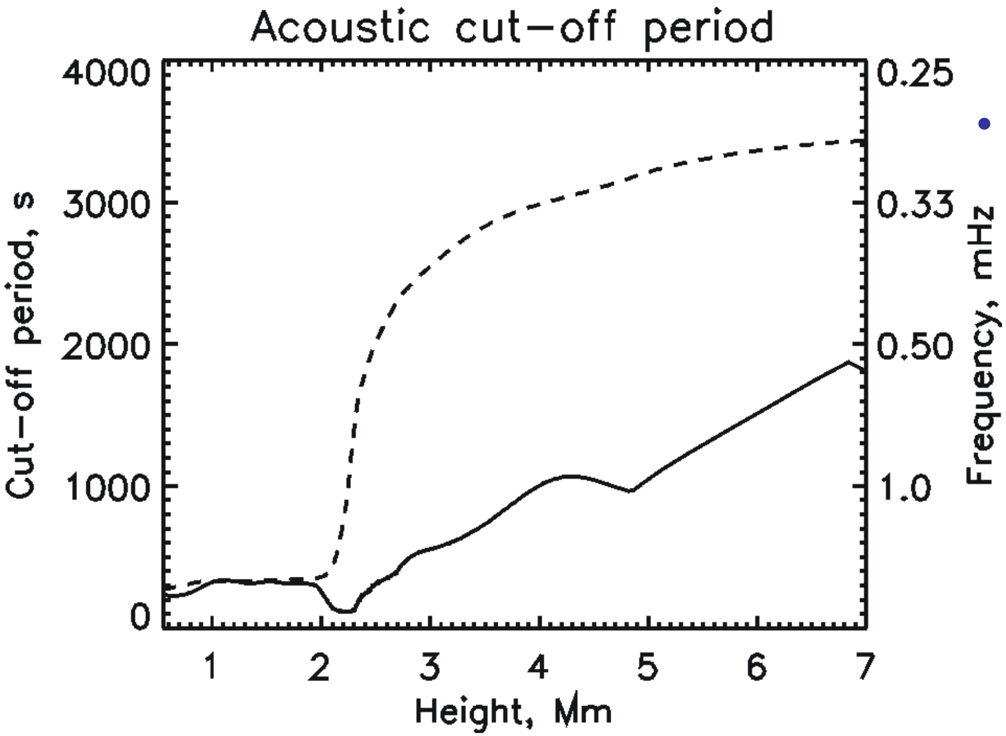


The waves in the photosphere have a **peak in power** at around **5 minutes**.

There are also various observations of phenomena with ~ **5 minute period in the chromosphere and corona** (eg De Moortel 2004, De Pontieu, Erdelyi & De Moortel 2005).

The photosphere however has a temperature minimum which presents an apparent **barrier to the 5 minute signals**, as the cut-off period at this point is somewhat below 5 minutes.

Cut-off frequency



- A key parameter for upwards propagation is the **acoustic cut-off frequency**.
- For an isothermal case this would be:

$$\Omega = \frac{\gamma g}{2V_S}$$

γ - the ratio of the specific heats

$g = g_0 \cos\theta$ - effective gravity

$g_0 = 274 m \cdot s^{-2}$ - effective gravity

θ - inclination angle

- In a non-isothermal case, the relation is:

$$\Omega = \frac{\gamma g}{2V_S} \cdot \sqrt{\left(1 + 2\Lambda'\right)}$$

Λ - density scale height

Our newly developed code SAC (**Sheffield Advanced Code**) is used to carry out our simulations.
The code can solve the full system if ideal hydrodynamic or magnetohydrodynamic equations in one, two or three dimensional Cartesian geometry.

- For HD and MHD we use Central Difference forth order Spatial discretisation (CD4) scheme, and forth order Runge-Kutta time discretisation.
- The code uses variable separation and hyperdiffusivity and hyperresistivity techniques to advance the solution

Set of Governing MHD Equations for the stratified plasma (classical)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\mathbf{V}\rho) = 0,$$

$$\frac{\partial (\mathbf{V}\rho)}{\partial t} + \nabla \cdot (\mathbf{V}\rho\mathbf{V} - \mathbf{B}\mathbf{B}) + \nabla p_{\mathrm{tot}} = -(\nabla \cdot \mathbf{B})\mathbf{B} + \rho\mathbf{g},$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (\mathbf{V}e - \mathbf{B}\mathbf{B}\cdot\mathbf{V} + \mathbf{V}p_{tot}) = -(\nabla \cdot \mathbf{B})\mathbf{B}\cdot\mathbf{V} + \rho\mathbf{g}\cdot\mathbf{V},$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{V}\mathbf{B} - \mathbf{B}\mathbf{V}) = -(\nabla \cdot \mathbf{B})\mathbf{V},$$

$$p = (\gamma - 1)\left(e - \frac{\rho\mathbf{V}^2}{2} - \frac{\mathbf{B}^2}{2}\right),$$

where $p_{\mathrm{tot}} = p + \dfrac{\mathbf{B}^2}{2}$ denotes the total pressure.

Neo-classical MHD system:

$$\frac{\partial\tilde{\rho}}{\partial t} + \nabla\cdot\left[\mathbf{v}\left(\rho_b + \tilde{\rho}\right)\right] = 0 + D_\rho\left(\tilde{\rho}\right),$$

$$\frac{\partial[(\rho_b + \tilde{\rho})\,\mathbf{v}]}{\partial t} + \nabla\cdot\left[\mathbf{v}\left(\rho_{\mathbf{b}} + \tilde{\rho}\right)\mathbf{v} - \tilde{\mathbf{B}}\tilde{\mathbf{B}}\right] -$$
$$\nabla\left[\tilde{\mathbf{B}}\mathbf{B}_b + \mathbf{B}_b\tilde{\mathbf{B}}\right] + \nabla\tilde{p}_t = \tilde{\rho}\mathbf{g} + \mathbf{D}_{\rho v}\left[(\tilde{\rho} + \rho_b)\,\mathbf{v}\right],$$

$$\frac{\partial\tilde{e}}{\partial t} + \nabla\cdot\left[\mathbf{v}\left(e + e_b\right) - \tilde{\mathbf{B}}\tilde{\mathbf{B}}\cdot\mathbf{v} + \mathbf{v}\tilde{p}_t\right] -$$
$$\nabla\left[\left(\tilde{\mathbf{B}}\mathbf{B}_b + \mathbf{B}_b\tilde{\mathbf{B}}\right)\cdot\mathbf{v}\right] + p_{tb}\nabla\mathbf{v} - \mathbf{B}_b\mathbf{B}_b\nabla\mathbf{v} =$$
$$= \tilde{\rho}\mathbf{g}\cdot\mathbf{v} + D_e\left(\tilde{e}\right),$$

$$\frac{\partial\tilde{\mathbf{B}}}{\partial t} + \nabla\cdot\left[\mathbf{v}(\tilde{\mathbf{B}} + \mathbf{B}_b) - (\tilde{\mathbf{B}} + \mathbf{B}_b)\mathbf{v}\right] = 0 + \mathbf{D}_B\left(\tilde{\mathbf{B}}\right),$$

Where

background pressures

$$p_{tb} = p_{kb} + \frac{\mathbf{B}_b^2}{2},$$

$$p_{kb} = (\gamma - 1)\left(e_b - \frac{\mathbf{B}_b^2}{2}\right),$$

$$p_{tb} = (\gamma - 1)\, e_b - (\gamma - 2)\, \frac{\mathbf{B}_b^2}{2}$$

perturbed pressures

$$\tilde{p}_t = \tilde{p}_k + \frac{\tilde{\mathbf{B}}^2}{2} + \mathbf{B}_b \tilde{\mathbf{B}},$$

$$\tilde{p}_k = (\gamma - 1)\left(\tilde{e} - \frac{(\rho_b + \tilde{\rho})\,\mathbf{v}}{2} - \mathbf{B}_b \tilde{\mathbf{B}} - \frac{\tilde{\mathbf{B}}^2}{2}\right),$$

$$\tilde{p}_t = (\gamma - 1)\left[\tilde{e} - \frac{(\rho_b + \tilde{\rho})\,\mathbf{v}^2}{2}\right] - (\gamma - 2)\left(\mathbf{B}_b \tilde{\mathbf{B}} + \frac{\tilde{\mathbf{B}}^2}{2}\right)$$

Numerics 1D-3D HD/MHD

CD4 !

The derivatives can be represented as their central difference approximations:

$$\frac{df}{dx} = \frac{1}{12}\frac{\left(f_{i-2} - 8f_{i-1} + 8f_{i+1} - f_{i+2}\right)}{\Delta x}$$

But ….. We don't want to use flux limiters anymore…

What we are going to use is called **hyperdiffusion (numerical diffusion or sub-grid diffusion)**:

!!!! not like flux limiter (intrinsic property of numerical scheme), but an additional term in the MHD equations

Numerics 3D HD, MHD

The periodic driver

$$E = A\sin\left(\frac{2\pi t}{\Delta t}\right)\cdot e^{-\left(\left(\frac{x}{\Delta x}\right)^2+\left(\frac{y}{\Delta y}\right)^2+\left(\frac{z-z_0}{\Delta z}\right)^2\right)}$$

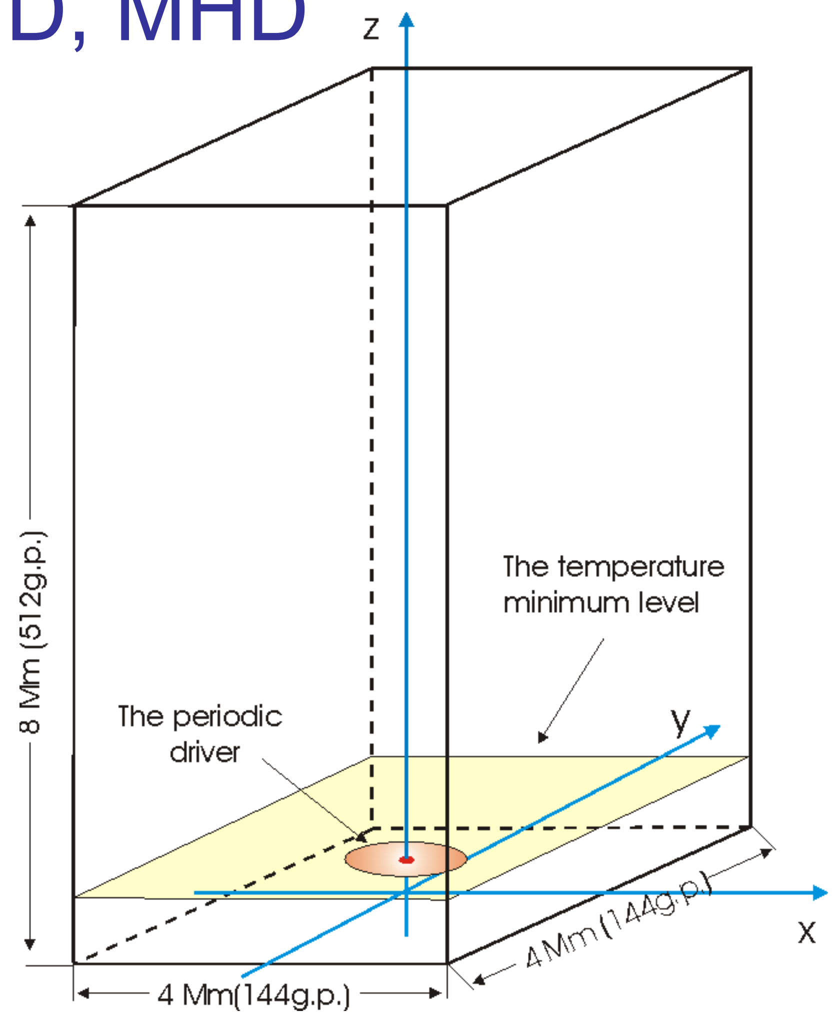
The half width of the Gaussian

$$\Delta x = 0.32\mathrm{Mm}$$
$$\Delta y = 0.32\mathrm{Mm}$$
$$\Delta z = 0.12\mathrm{Mm}$$

for $\Delta t = 30\sec$ driver

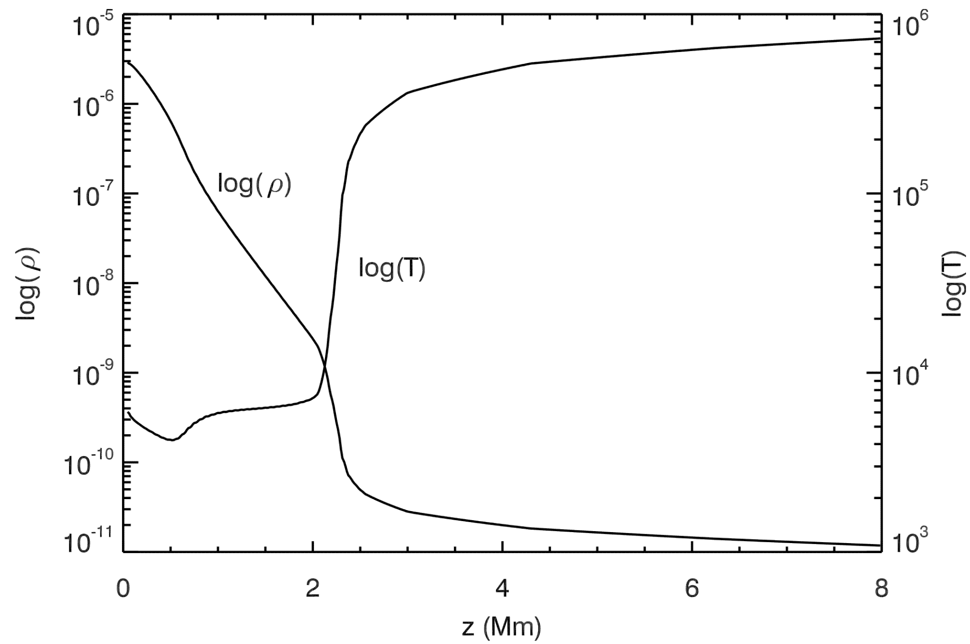
$$\Delta x = 0.6\mathrm{Mm}$$
$$\Delta y = 0.6\mathrm{Mm}$$
$$\Delta z = 0.12\mathrm{Mm}$$

for $\Delta t = 300\sec$ driver



Numerics HD, MHD

- We have started by considering a case with **realistic temperature stratification**, but ignoring the magnetic field.
- The non-magnetic case can be considered as an **approximation to the quiet sun**, and a preliminary to including magnetism.
- We are using the equations of **ideal hydrodynamics**.
- The temperature profile of our model is based on the **VALIIIc** (Vernazza et al 1981) model atmosphere below the transition region, and the McWhirter (McWhirter et al 1975) model atmosphere above it.



High frequency signal

Before looking at the propagation of the 5-minute signal which suffers from evanescence both at the transition region, and in the upper photosphere, we considered a **30 second driver**.

This driver is well **below the acoustic cut-off period** at any point in our atmosphere, and therefore allows us to look at the simple case of strong propagation. In the real Sun, the propagation into the corona of high frequency waves such as this would be **suppressed by radiative damping**, but it is useful to look at them as a point of comparison.

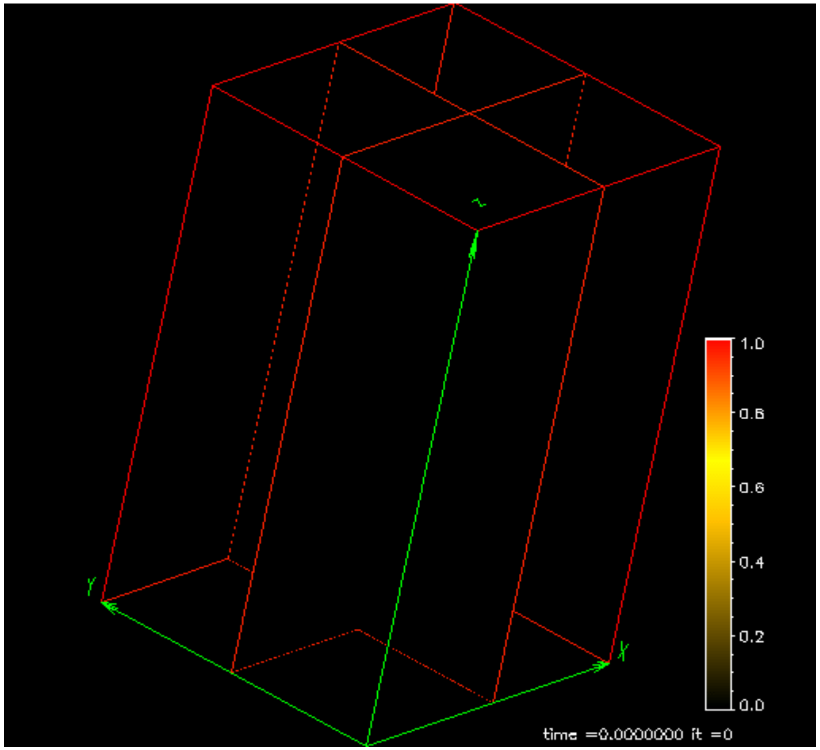
300 second / 5 minute point driver

- The signal is **evanescent in parts of the atmosphere** and also experiences stronger reflection at the transition region
- We see some leakage of the waves to the corona for HD case and mainly slow magnetosonic waves for MHD simulations
- Key features are a **standing wave** which forms vertically in the between the chromosphere and TR
- Formation of a **surface wave on the transition region.** The surface waves cause a **'granulation'** of the transition region.

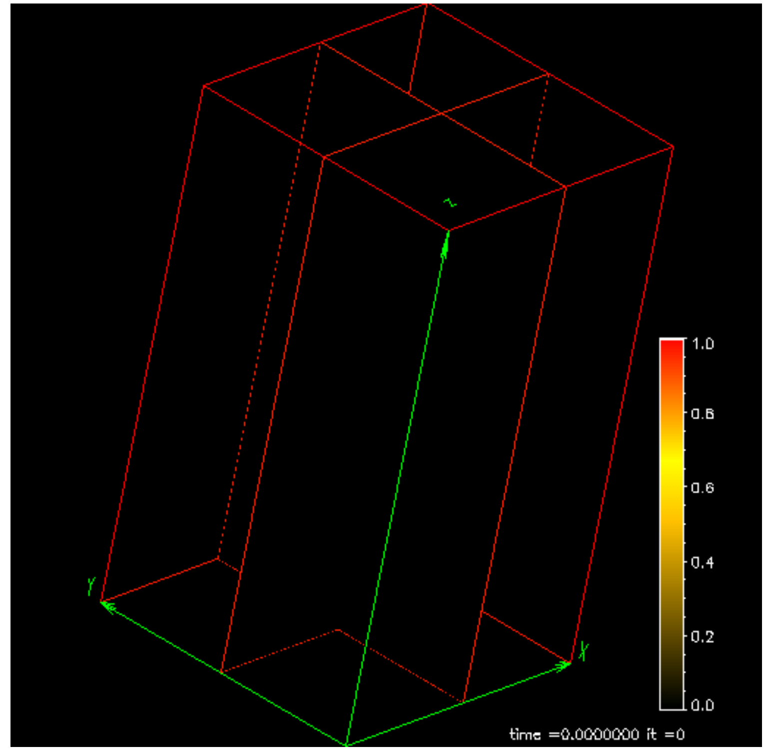
Results: 3D HD 30sec periodic driver

initial amplitude 200m/sec

Vz

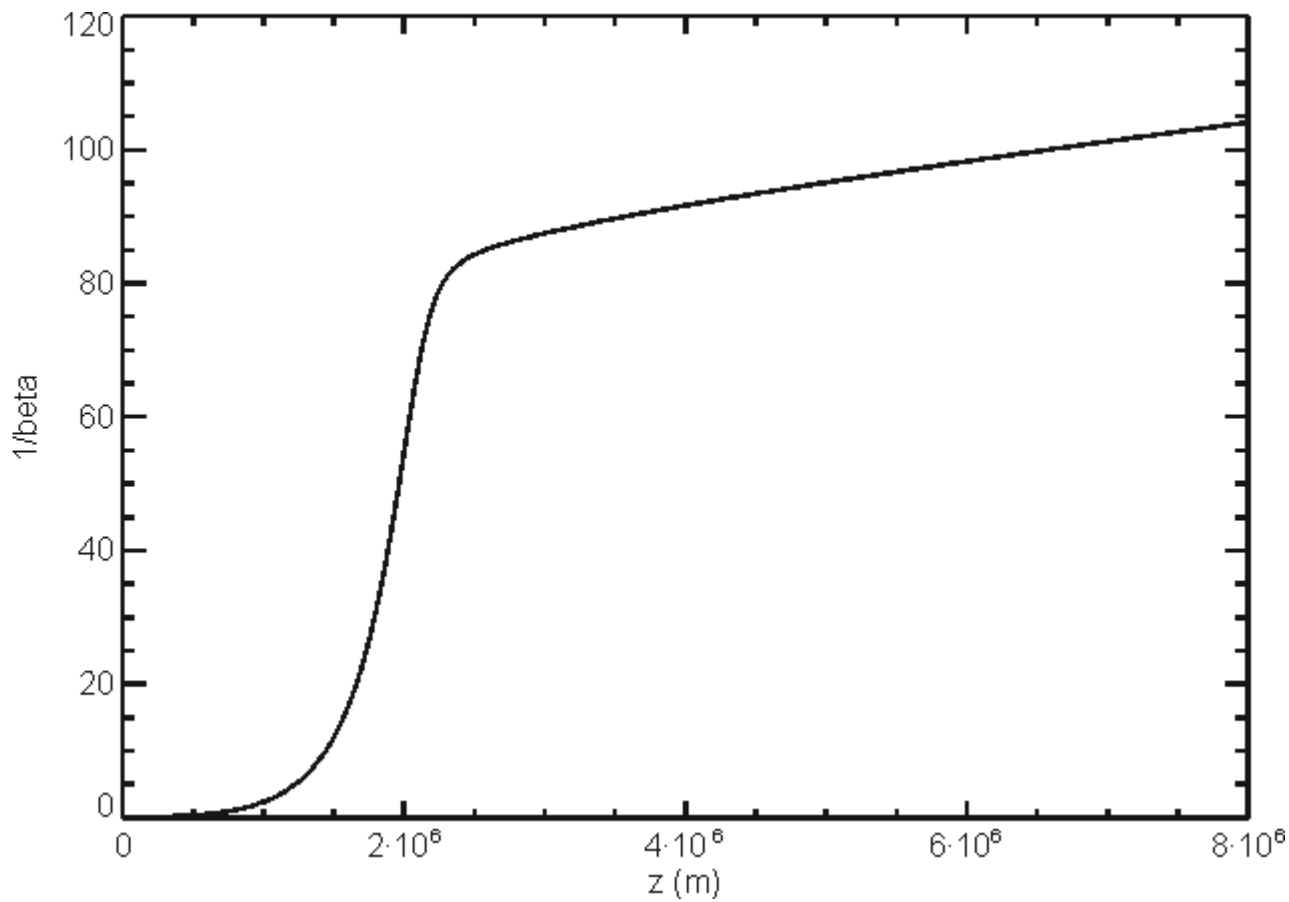


Vx



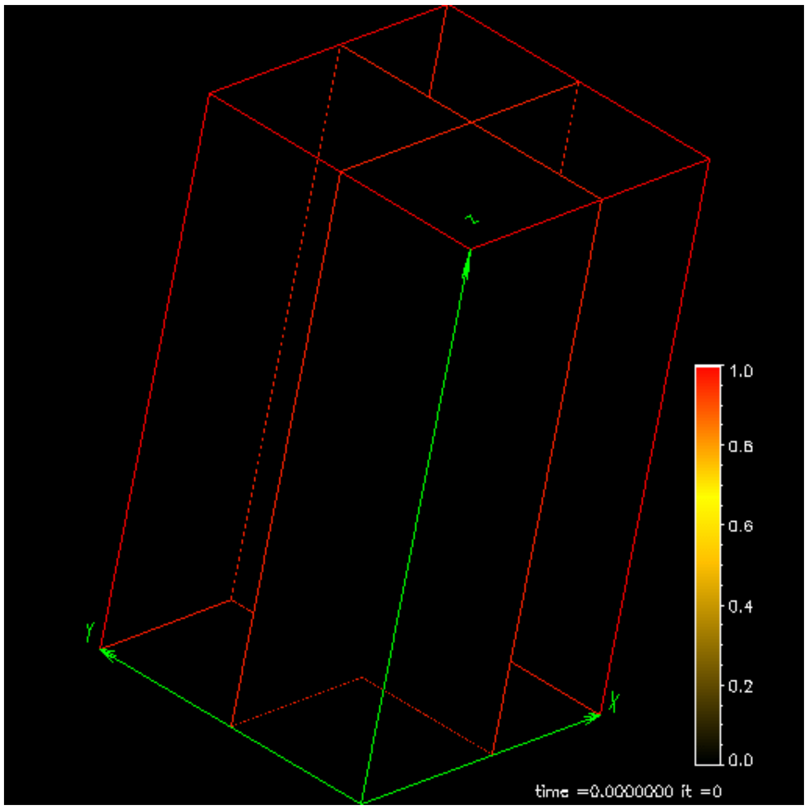
The interference of the incoming and the reflected by the transition region waves also produces a standing wave pattern in the region between about 1000 and 2000 km height in the solar atmosphere. This result is comparable to the previous observational results by Fleck and Deubner 1989 and 2D numerical simulations by Erdéliy and Malins 2006.

Results: 3D MHD 30sec periodic driver (uniform magnetic field ~ 80G)

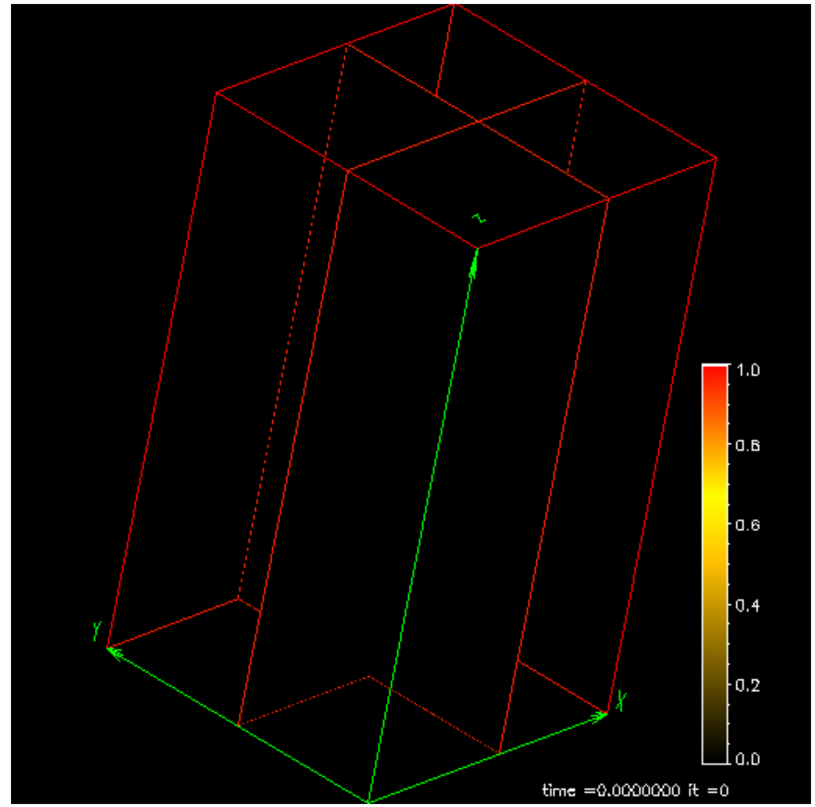


Results: 3D MHD 30sec periodic driver (uniform magnetic field ~ 80G)

Vz



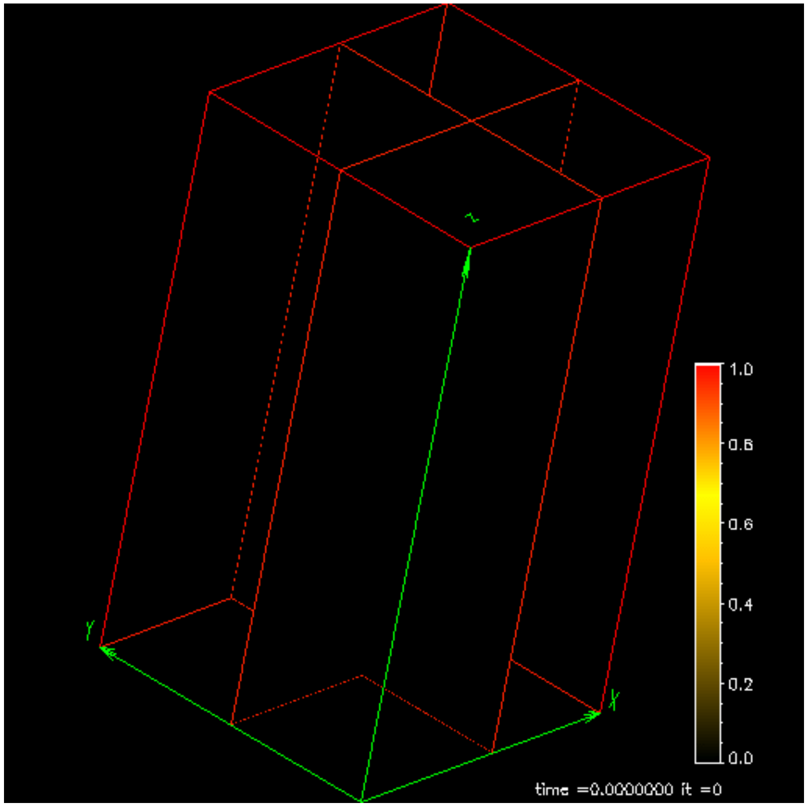
Vx



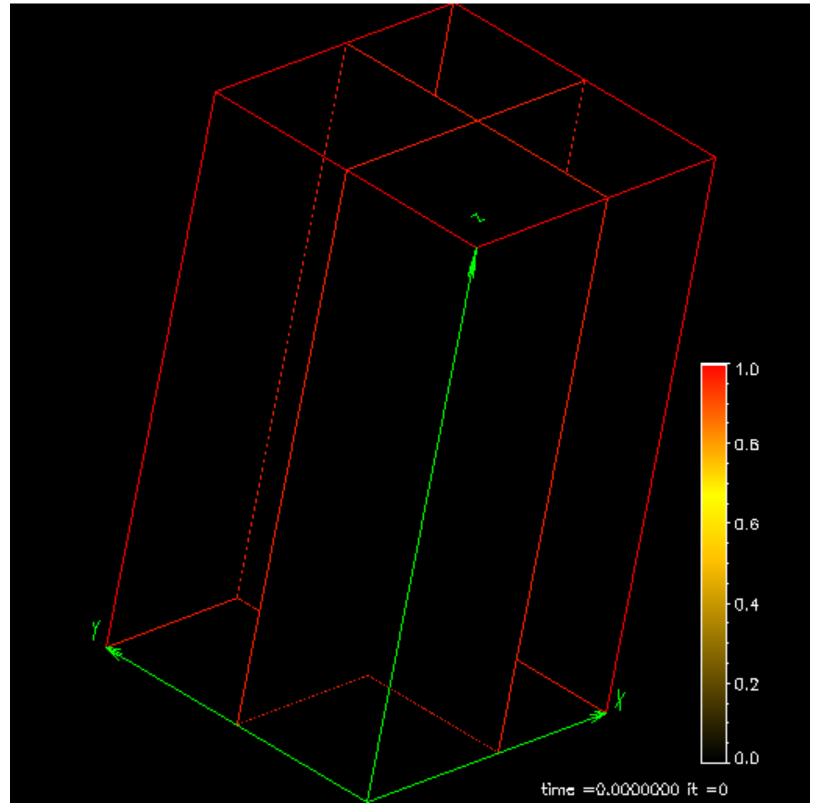
initial amplitude 200m/sec

Results: 3D MHD 30sec periodic driver (uniform magnetic field ~ 80G)

Bz



Bx



Units: $B = \sqrt{\mu} \cdot B_{VAC} \cdot 10^4 (G)$ $\quad \mu = 1.257 \cdot 10^{-6}$

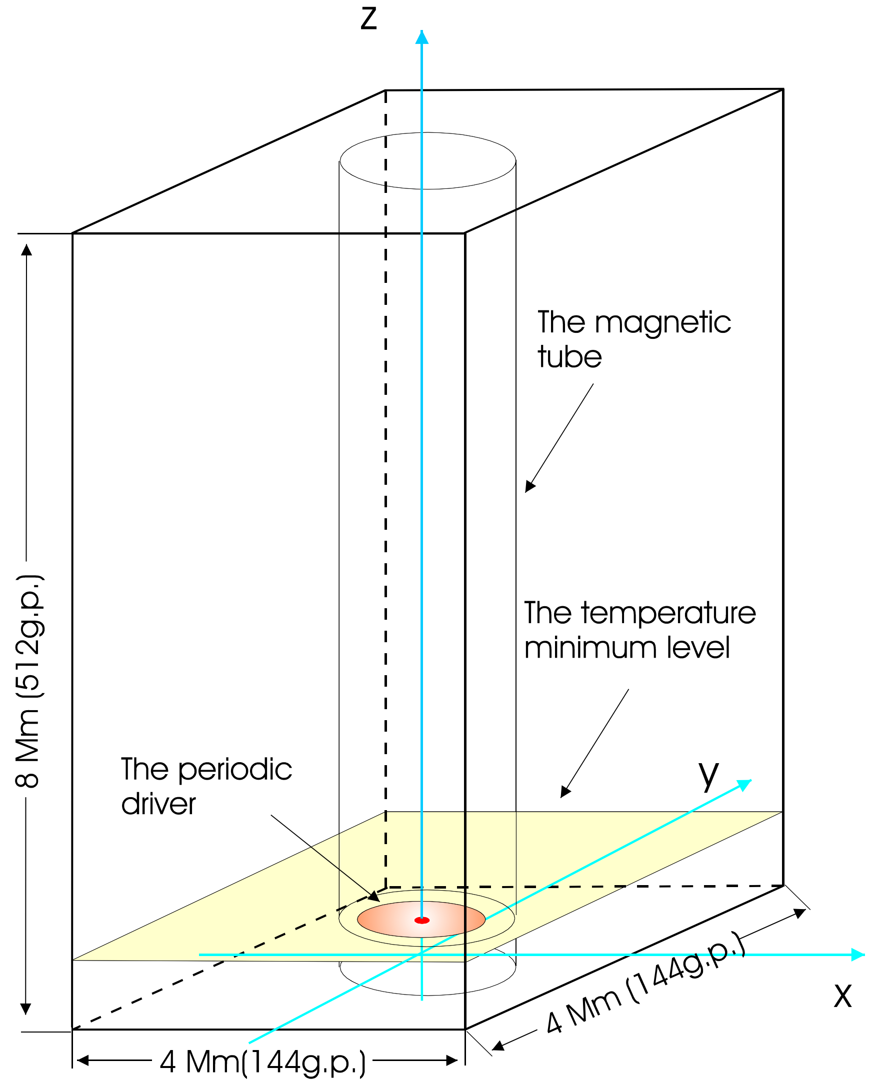
Results: 3D MHD tube 30sec periodic driver

Here we show the numerical solutions of the full MHD equations for a magnetic tube (~ 15G) embedded in a vertically stratified atmosphere. The initial equilibrium is similar to the one for the case of an uniform magnetic field.

$$p_{Ti} = p_{Tp} - \frac{B^2}{2}$$

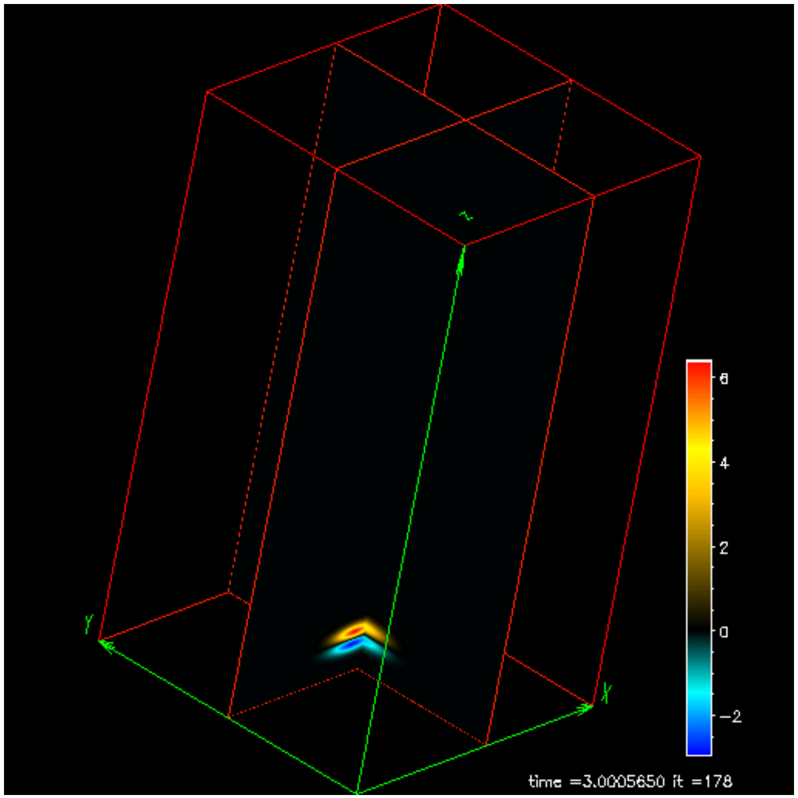
$$p_{Te} = p_{Tp}$$

$$p_{Ti} + \frac{B^2}{2} = p_{Te}$$

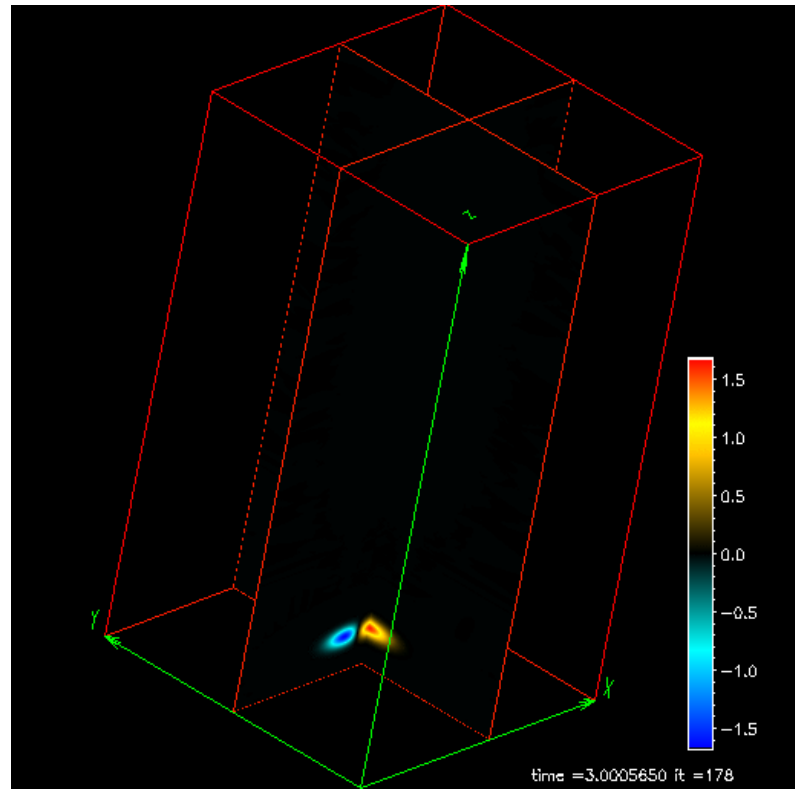


Results: 3D MHD tube 30sec periodic driver

Vz

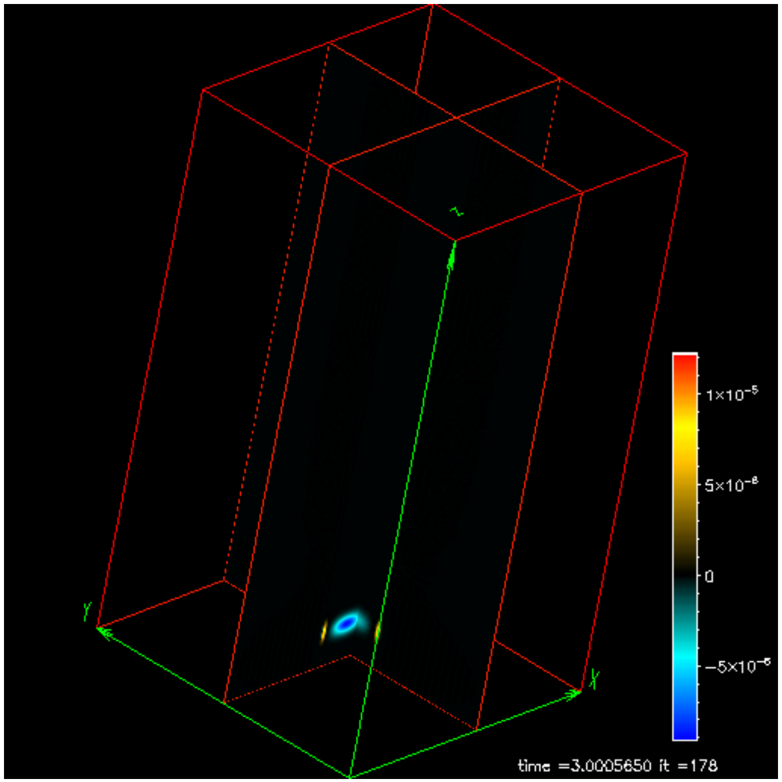


Vx

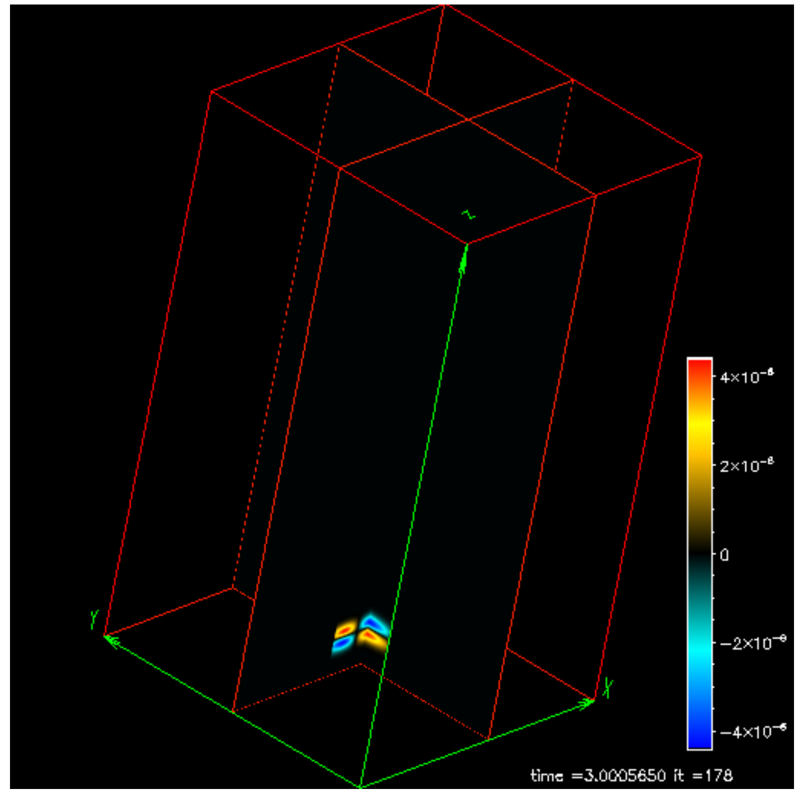


Results: 3D MHD tube 30sec periodic driver (continue)

Bz



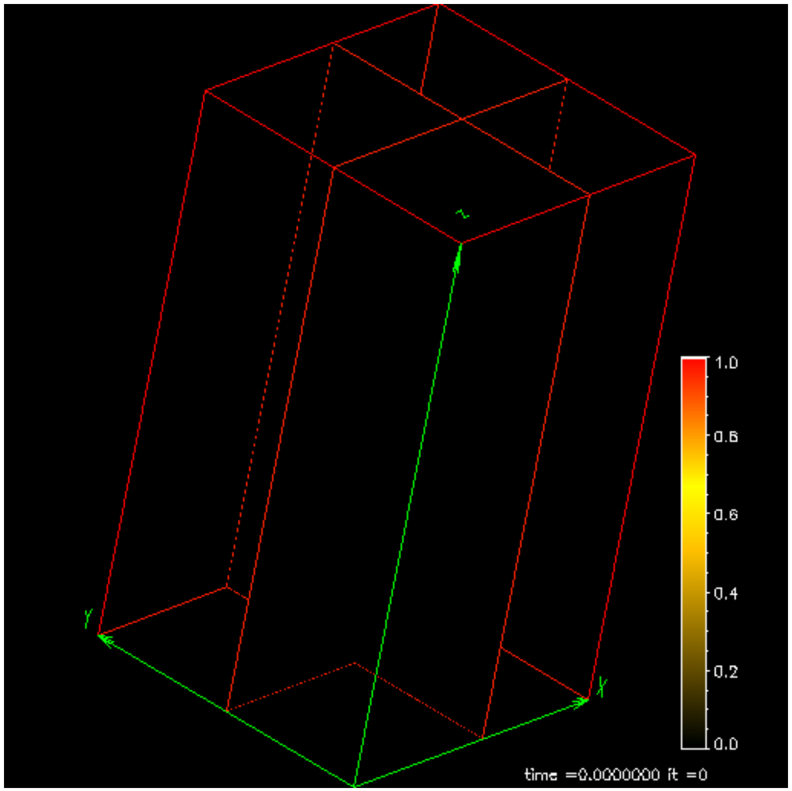
Bx



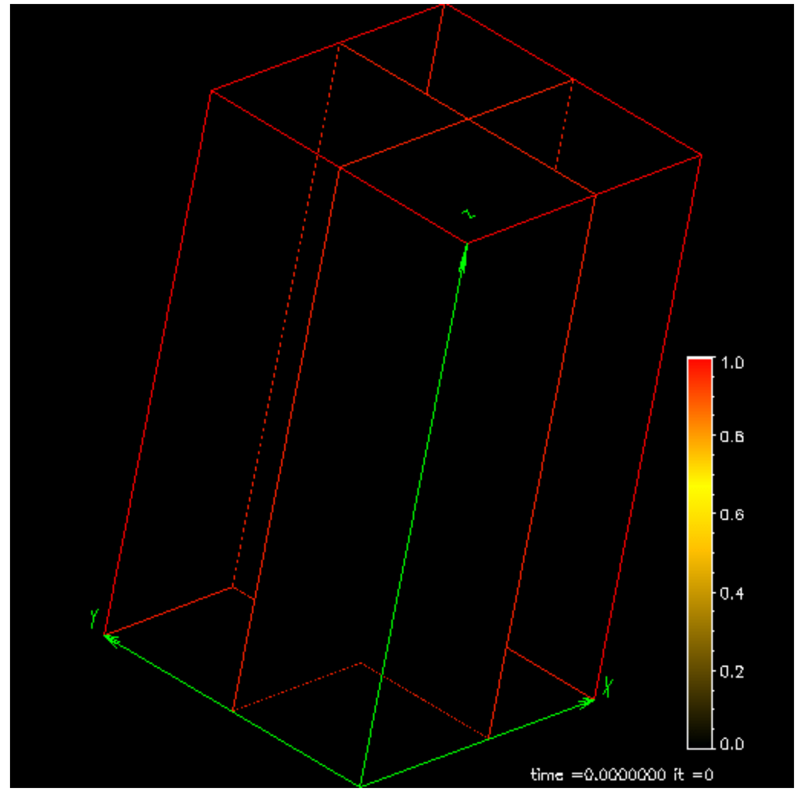
Units: $\mathrm{B} = \sqrt{\mu} \cdot \mathrm{B}_{\mathrm{VAC}} \cdot 10^{4}(\mathrm{G}) \qquad \mu = 1.257 \cdot 10^{-6}$

Results: 3D HD 300sec periodic driver

Vz



Vx

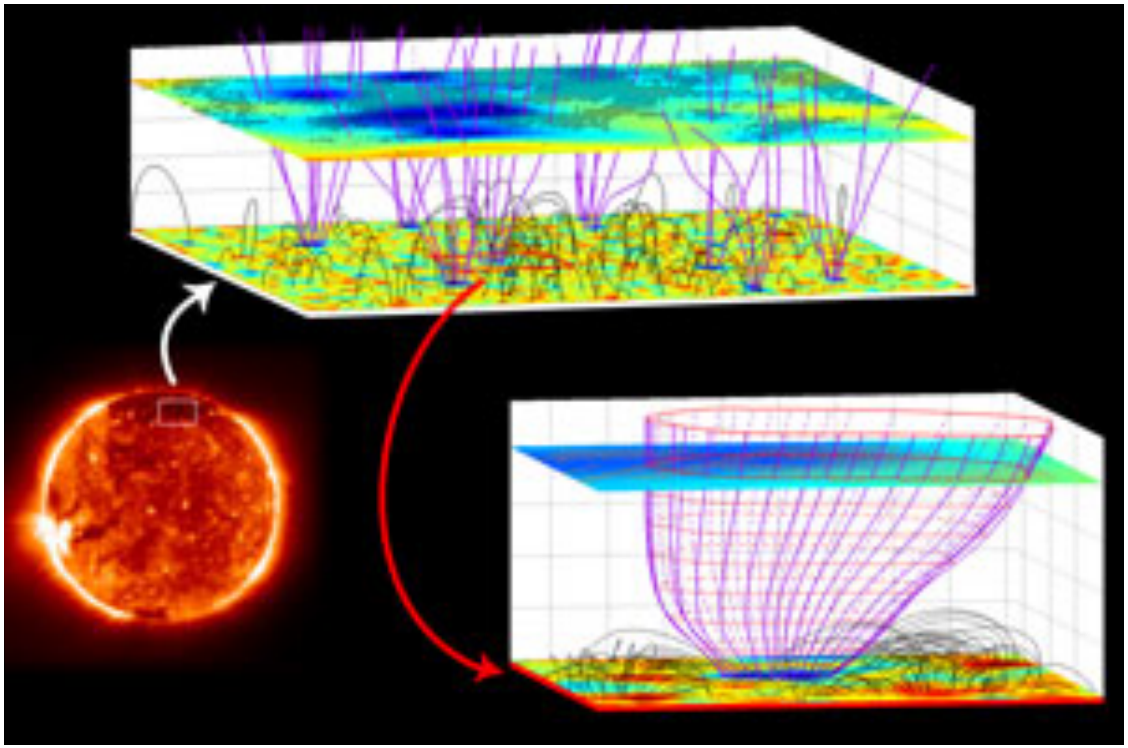


Units: $B = \sqrt{\mu} \cdot B_{VAC} \cdot 10^4 (G)$ $\mu = 1.257 \cdot 10^{-6}$

Results: 3D MHD 300sec periodic driver

The image in the quicktime format (3D_all)

The magnetic field current improvement



The picture shows a potential field extrapolation based on a high resolution magnetogram from the MDI instrument onboard the Solar and Heliospheric Observatory. Opposite polarities are shown in red and yellow.
(see http://www.astro.uio.no)

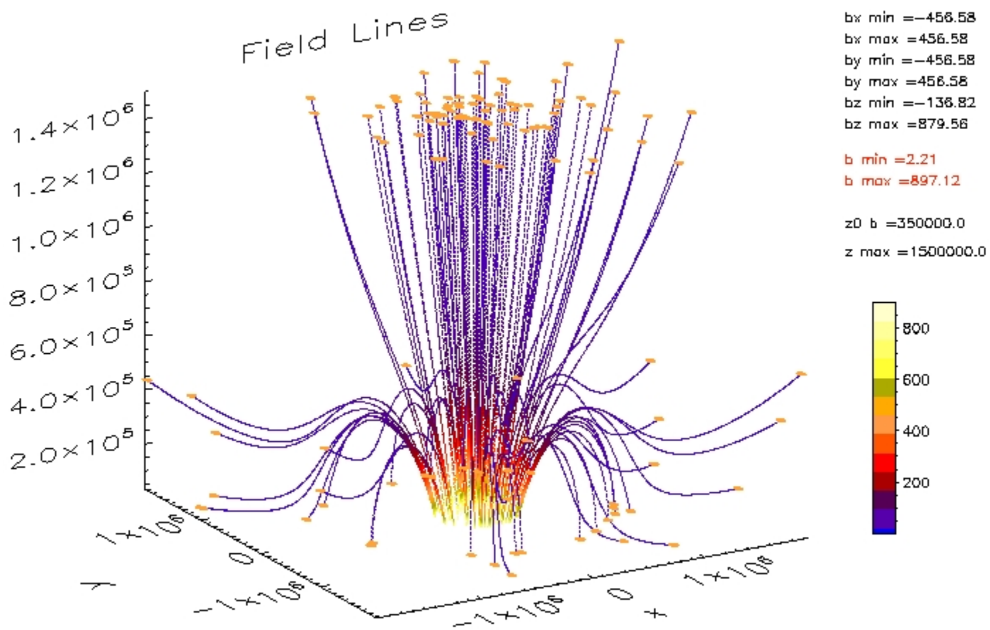
The magnetic field current improvement

Relaxations methods

Extrapolation

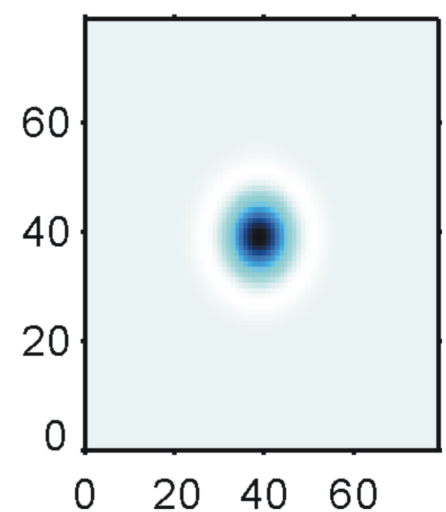
For a planar photosphere unbounded above, the scalar potential is

$$\varphi(x,y,z)=\frac{1}{2\pi}\int\frac{B_z(x',y')}{\sqrt{(x-x')^2+(y-y')^2+z^2}}dx'dy'$$



by analogy to Coulomb's law.

$B_z(x,y)$



The magnetic field current improvement

$$\cancel{f(\xi) \approx Sin(\xi)}$$

For Cartesian coordinates

$$\mathrm{B}_x = -\xi_z' f(\xi),$$
$$\mathrm{B}_z = \xi_x' f(\xi),$$
$$\xi = \mathrm{B}_0(z) \cdot x$$

$$\Rightarrow \quad div\mathbf{B} = 0$$

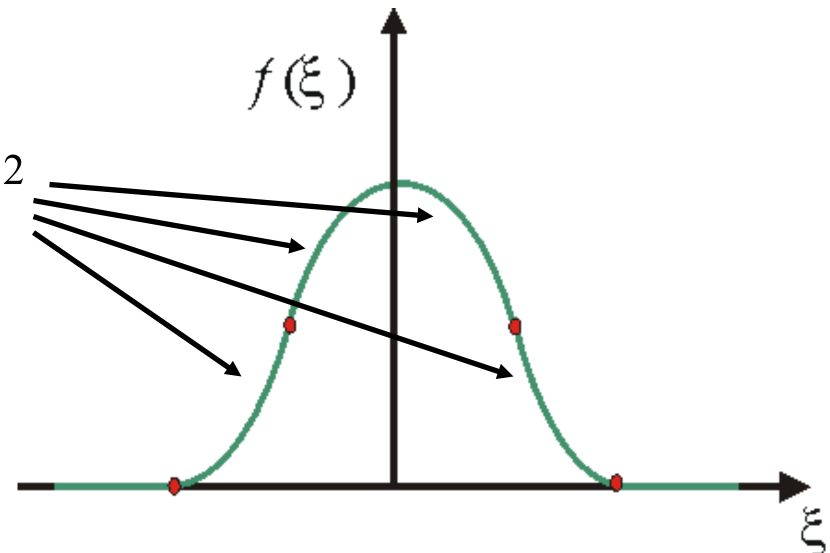
$$\cancel{f(\xi) \approx e^{-(\xi-\xi_0)^2}}$$

For cylindrical coordinates

$$\mathrm{B}_r = -\frac{1}{2} B_0 \left(1 + \frac{\mathrm{z}}{\mathrm{D}}\right) f\left(\frac{\mathrm{r}^2}{\mathrm{R}^2}\left(1 + \frac{\mathrm{z}}{\mathrm{D}}\right)\right),$$

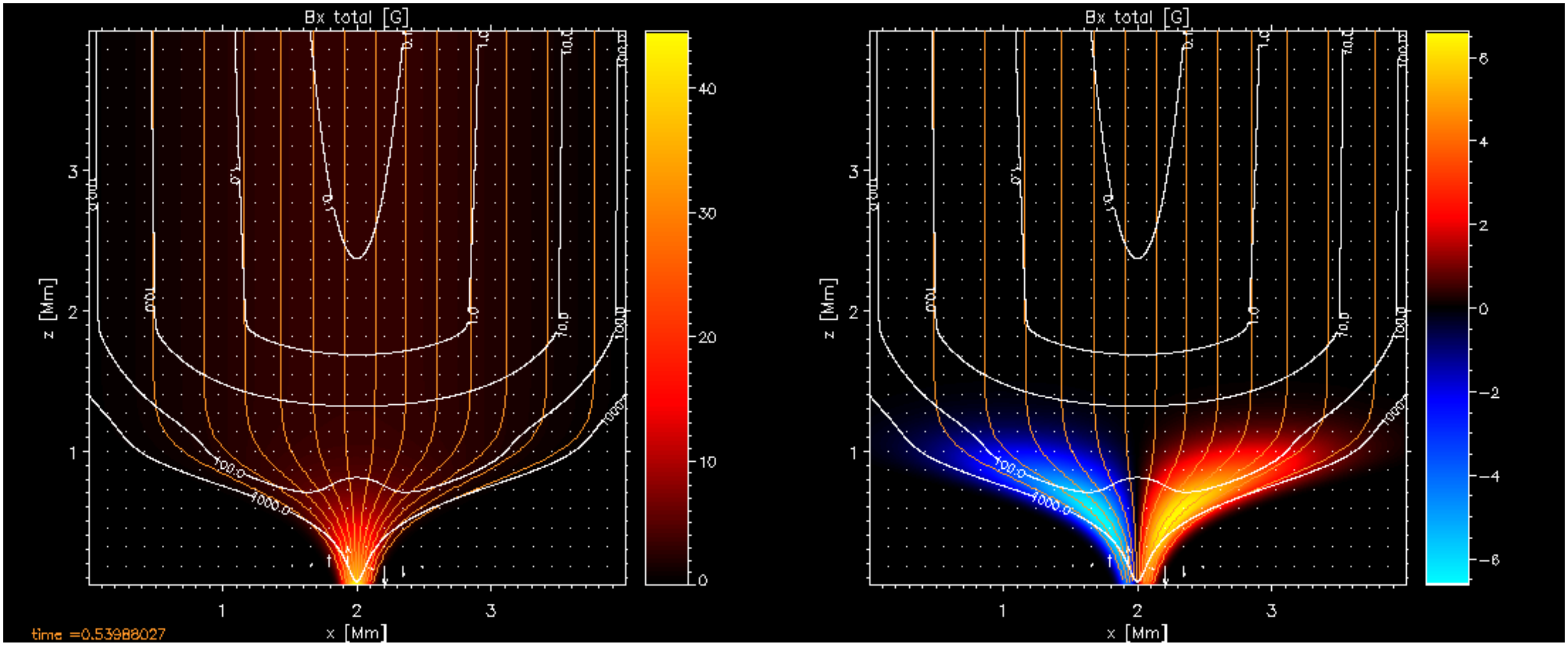
$$\mathrm{B}_z = B_0 \left(1 + \frac{\mathrm{z}}{\mathrm{D}}\right) f\left(\frac{\mathrm{r}^2}{\mathrm{R}^2}\left(1 + \frac{\mathrm{z}}{\mathrm{D}}\right)\right)$$

$\approx \xi^2$



Gordovskyy and Jain, 2007

2D numerical simulations with non-uniform magnetic field



2D numerical simulations with non-uniform magnetic field

$$V_{\parallel} = \frac{V_x B_{0x} + V_z B_{0z}}{\sqrt{B_{0x}^2 + B_{0z}^2}}, \quad V_{\perp} = \frac{V_x B_{0x} - V_z B_{0z}}{\sqrt{B_{0x}^2 + B_{0z}^2}}$$

- Kink driver
- High period (P=30s)
- Driver amplitude (A=500 m/s)
- Thin flux tube (R=180 km)
- Footpoint magnetic field (B=45G)

1. **magnetic flux tube, single source** (V parallel and V perpendicular to the magnetic field, pressure and temperature movies)
   VpVd_P30_R100_A500_B45_Dx4_Dzx4_4_1976_400
   DPDT_P30_R100_A500_B45_Dx4_Dz2_6_1023_400
   DPDT_P30_R100_A500_B45_Dx4_Dzx4_4_1976_400
2. **magnetic flux tube, shifted single source**
   VpVd_P30_R100_A500_B45_Dx4_Dz2_6_1023_400_shift
   DPDT_P30_R100_A500_B45_Dx4_Dz2_6_1023_400_shift
3. **magnetic flux tube, multiple sources**
   VpVd_P30_R100_A500_B45_Dx4_Dz4_1976_400_multi

Conclusions

- We have shown that high period signals may well be able to **leak significantly into the chromosphere**.
- We have shown that low frequency signals which are evanescent crossing the transition region are likely to set up **transition layer surface waves**. These surface waves are **generated by various types of drivers**
- We have shown that in the MHD case, the main part of energy carried by the slow magnetosonic waves
- We have shown that the **standing modes** may be set up by the waves propagating upwards and the waves, reflected from the TR waves
- We have shown that sufficient driven amplitudes can generate **fine structures extending into the corona** for both uniform and non-uniform magnetic field.