Solving linear equations

Introduction

Equations always involve one or more unknown quantities which we try to find when we **solve** the equation. The simplest equations to deal with are **linear equations**. On this leaflet we describe how these are solved.

A linear equation

Linear equations are those which can be written in the form

$$ax + b = 0$$

where x is the unknown value, and a and b are known numbers. The following are all examples of linear equations.

$$3x + 2 = 0, \qquad -5x + 11 = 0, \qquad 3x - 11 = 0$$

The unknown does not have to have the symbol x, other letters can be used.

$$3t - 2 = 0, \qquad 7z + 11 = 0, \qquad 3w = 0$$

are all linear equations.

Sometimes you will come across a linear equation which at first sight might not appear to have the form $ax + b = 0$. The following are both linear equations. If you have some experience of solving linear equations, or of transposing formulas, you will be able to check that they can all be written in the standard form.

$$\frac{x - 7}{2} + 11 = 0, \qquad 6x - 2 = 9$$

Solving a linear equation

To solve a linear equation it will be helpful if you know already how to transpose or rearrange formulas.

When solving a linear equation we try to make the unknown quantity the subject of the equation. To do this we may

- add or subtract the same quantity to or from both sides
- multiply or divide both sides by the same quantity

Example
Solve the equation $x + 7 = 18$.

Solution
We try to obtain x on its own on the left hand side.

$$\begin{aligned} x+7 &= 18 \\ x &= 18-7 \qquad \text{by subtracting 7 from both sides} \\ x &= 11 \end{aligned}$$

We have solved the equation and found the solution: $x = 11$. The solution is that value of x which can be substituted into the original equation to make both sides the same. You can, and should, check this. Substituting $x = 11$ in the left-hand side of the equation $x + 7 = 18$ we find $11 + 7$ which equals 18, the same as the right-hand side.

Example
Solve the equation $5x + 11 = 22$.

Solution

$$\begin{aligned} 5x+11 &= 22 \\ 5x &= 22-11 \qquad \text{by subtracting 11 from both sides} \\ x &= \frac{11}{5} \qquad \text{by dividing both sides by 5} \end{aligned}$$

Example
Solve the equation $13x - 2 = 11x + 17$.

Solution

$$\begin{aligned} 13x-2 &= 11x+17 \\ 13x-11x-2 &= 17 \qquad \text{by subtracting } 11x \text{ from both sides} \\ 2x-2 &= 17 \\ 2x &= 17+2 \qquad \text{by adding 2 to both sides} \\ 2x &= 19 \\ x &= \frac{19}{2} \qquad \text{by dividing both sides by 2} \end{aligned}$$

Exercises
1. Solve the following linear equations.

a) $4x + 8 = 0$, b) $3x - 11 = 2$, c) $8(x + 3) = 64$, d) $7(x - 5) = -56$, e) $3c - 5 = 14c - 27$.

Answers
1. a) $x = -2$, b) $x = \frac{13}{3}$, c) $x = 5$, d) $x = -3$, e) $c = 2$.

