Engineering Maths First Aid Kit

2.18

Graphical solution of inequalities

Introduction

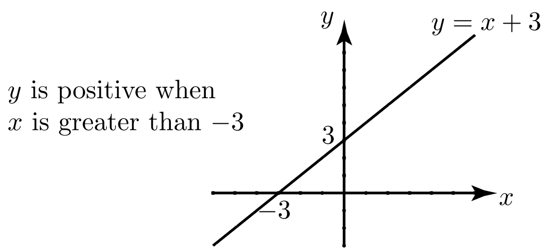
Graphs can be used to solve inequalities. This leaflet illustrates how.

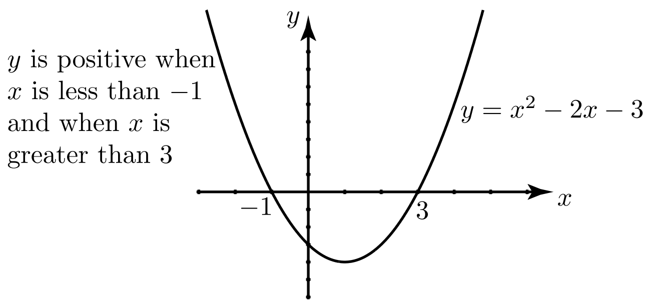
1. Solving inequalities

We start with a very simple example which could be solved very easily using an algebraic method.

Example
Solve the inequality $x + 3 > 0$.

Solution
We seek values of x which make $x + 3$ positive. There are many such values, e.g. try $x = 7$ or $x = -2$. To find all values first let $y = x + 3$. Then the graph of $y = x + 3$ is sketched as shown below. From the graph we see that the y coordinate of any point on the line is positive whenever x has a value greater than -3. That is, $y > 0$ when $x > -3$. But $y = x + 3$, so we can conclude that $x + 3$ will be positive when $x > -3$. We have used the graph to solve the inequality.





Example
Solve the inequality $x^2 - 2x - 3 > 0$.

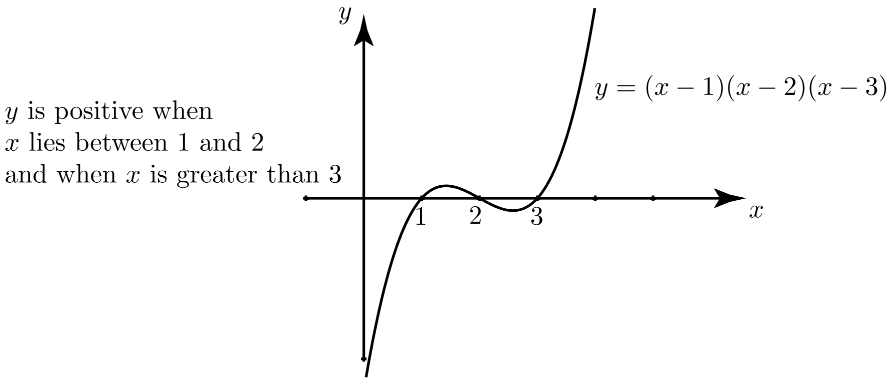
Solution
We seek values of x which make $x^2 - 2x - 3$ positive. We can find these by sketching a graph of $y = x^2 - 2x - 3$. To help with the sketch, note that by factorising we can write y as $(x+1)(x-3)$. The graph will cross the horizontal axis when $x = -1$ and when $x = 3$. The graph is shown above on the right. From the graph note that the y coordinate of a point on the graph is positive

when either x is greater than 3 or when x is less than -1. That is, $y > 0$ when $x > 3$ or $x < -1$ and so:

$$x^2 - 2x - 3 > 0 \qquad \text{when} \qquad x > 3 \quad \text{or} \quad x < -1$$

Example
Solve the inequality $(x-1)(x-2)(x-3) > 0$.

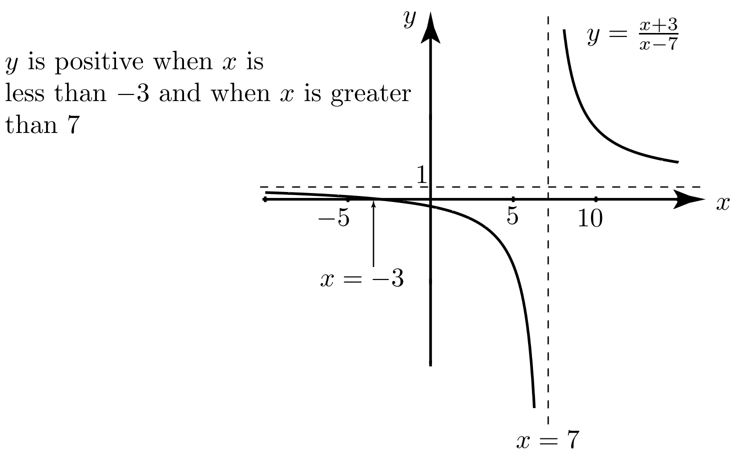
Solution
We consider the graph of $y = (x-1)(x-2)(x-3)$ which is shown below. It is evident from the graph that y is positive when x lies between 1 and 2 and also when x is greater than 3. The solution of the inequality is therefore $1 < x < 2$ and $x > 3$.



Example
For what values of x is $\frac{x+3}{x-7}$ positive ?

Solution
The graph of $y = \frac{x+3}{x-7}$ is shown below. We can see that the y coordinate of a point on the graph is positive when $x < -3$ or when $x > 7$.

$$\frac{x+3}{x-7} > 0 \qquad \text{when} \quad x < -3 \quad \text{or when} \quad x > 7$$



For drawing graphs like this one a graphical calculator is useful.