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Hypothesis Testing

Learning outcomes

By studying this Workbook you will learn how to apply statistical techniques to test the validity, on the basis of available evidence, of a given hypothesis. For example, a motor engineer may be interested in testing the expected life of a given set of tyres ("the mean life is 2,000 miles") against an alternative ("the mean life is less than 2,000 miles"). You will learn about techniques which will enable you to answer such questions.

This Workbook will introduce you to the basic ideas of hypothesis testing in a non-mathematical way by using a problem solving approach to highlight the concepts as they are needed.

Once you have learned how to apply the basic ideas, you will be capable of applying hypothesis testing to a very wide range of practical problems and learning about methods of hypothesis testing which are not covered in this Workbook.

Statistical Testing

41.1

Introduction

If you are applying statistics to practical problems in industry, you may find that much of your work is concerned with making decisions concerning populations and population parameters on the basis of available evidence. For example you may be asked to decide whether one production process is preferable to another or whether to repair or continue to use a machine that is producing a certain proportion of defective components. In order to make such decisions, you will find that you have to make certain assumptions which will determine the statistical tools that you may legitimately use. Any assumptions made may or may not be true but you must always be sure of your grounds for using a given statistical tool. Effectively you will find that you will be asked to decide which of two statements, each called an hypothesis, is the more likely to be true. Note the choice of words. You should be clear from the outset that the statistical tools you will study here will not allow you to prove anything, but they will allow you to measure the strength of the evidence against the hypothesis.

Prerequisites

Before starting this Section you should ...

- understand the term 'sample'
- be able to differentiate between statements which are a matter of opinion and those which are of a numerical nature and as such can be challenged

Learning Outcomes

On completion you should be able to ...

- understand what is meant by the terms hypothesis and hypothesis testing
- understand the what is meant by the terms one-tailed test and two-tailed test
- understand what is meant by the terms type I error and type II error
- understand the term level of significance
- apply a variety of statistical tests to problems based in engineering

1. Types of statements

Almost every time we read a magazine or newspaper we see claims made by manufacturers about their products. Such claims can take many forms, they may for example be subjective:

'Luxcar, makers of the best luxury cars'

'Burnol, the finest fuel you can buy'

'ConstructAll, designers of beautiful buildings'

Such claims do not need to be backed up by facts and figures, they are a matter of opinion.

Many claims do contain information which is open to question and can be investigated statistically:

'the expected life of these tyres is 20,000 miles'

'on average, low energy light bulbs can be expected to last at least 8000 hours'

'average bottle contents 330 ml.'

The validity of claims which contain information of a numerical nature can often be investigated by taking random samples of the objects or quantities in question and investigating the likelihood that a statement or hypothesis concerning them is true.

As stated in the introduction, it should be noted that hypothesis testing can never prove that a statement is either true or false, it can only give a measure of the truth or otherwise of a given statement. Statements which are investigated statistically are normally called hypotheses and we usually try to establish a pair of hypotheses, called a **null hypothesis** and an **alternative hypothesis** and then investigate how the evidence that we have supports one hypothesis more than the other. For example, a demolition engineer might be interested in the burn rate of fuses connected to explosive devices and on the basis of experience hypothesize that the mean burn rate (say μ) is 600 mm/sec. A colleague may disagree and claim that the mean burn rate is greater than 600 mm/sec.

We can describe this situation by setting up the null hypothesis:

$H_0:\ \mu = 600$

and test this against the alternative hypothesis:

$H_1:\ \mu > 600$

2. Types of errors

Since we cannot be 100% sure that a hypothesis is true or false it is possible that:

(a) a correct hypothesis will be rejected;

(b) a false hypothesis will be accepted.

Rejecting a correct hypothesis is called a Type I error and accepting a false hypothesis is called a Type II error.

By working in a logical manner and developing a set of rules or guide-lines, it is possible to minimise the occurrence of such errors.

This will introduce you to the basic ideas of hypothesis testing in a non-mathematical way by using a problem solving approach to highlight the concepts as they are needed.

Once you have learned how to apply the basic ideas, you will be capable of applying hypothesis testing to a very wide range of practical problems and learning about methods of hypothesis testing which are not covered in this Workbook.