An Overview of Structural Model Uncertainty

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The world of structural model uncertainty

model averaging

decision−making without data

model discrepancy

data−driven model structure

deterministic model

reification

value of information

statistical model

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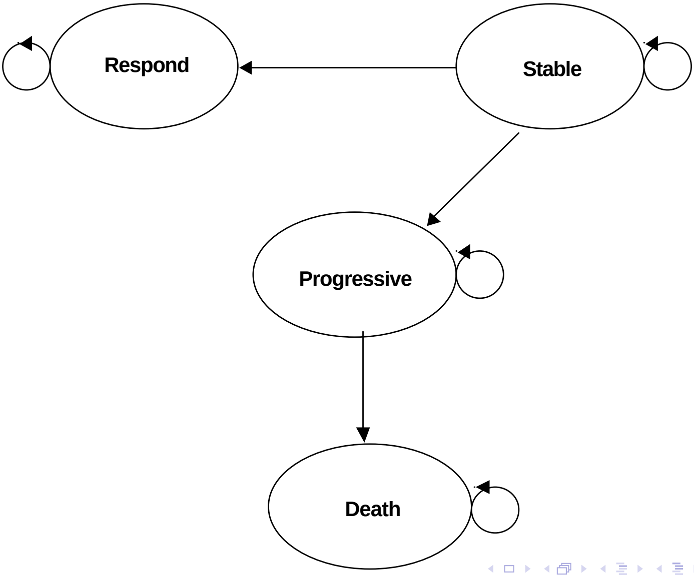
  - Quantifies uncertainty about Y, not Y^*

Structural model uncertainty: an example



Perspectives on model uncertainty

From Bernardo & Smith (1994). We have set of models $\{M_i, i \in I\}$, with $M_i = \{\eta_i(x_{(i)}), p_i(X_{(i)})\}$.

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2. The $\mathcal{M}-$ *open* view:
   None of the models in $\{M_i, i \in I\}$ are correct. Not meaningful to consider $p(M_i)$

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or just

$$M_0 : response = \alpha + \beta age + \varepsilon,$$

with $p(\beta = 0) \neq 0$?

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  - But statistical formulation equivalent to model averaging (with associated pitfalls)?

$\mathcal{M} - \textit{open}$ view and model discrepancy

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    - Involves notion of model discrepancy, potential for dealing with multiple (conflicting) models.

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   - ...probably less practical here, given data requirements

References

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