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The Fundamental Surplus Revisited

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Abstract

To generate large responses of unemployment to productivity changes requires a high elasticity of the fundamental surplus with respect to productivity. When all deductions that enter the fundamental surplus are acyclical, and the fundamental surplus does not involve endogenous variables, then the elasticity of the fundamental surplus coincides with the inverse of the fundamental surplus fraction.

Keywords: the fundamental surplus, search frictions, labor market volatility, real wage rigidity (*JEL* E24, E32, J31, J41, J63)

1 Introduction

Understanding the causes of large fluctuations in unemployment is a key topic in macroeconomics. By reconfiguring the standard Diamond-Mortensen-Pissarides search and matching model of equilibrium unemployment, macroeconomists have identified two distinctive determinants of labor market volatility. These are real wage rigidity and small profits. Wage rigidity has been widely used in macroeconomic models to account for business cycle fluctuations. Its implications for labor market dynamics have been examined by numerous papers, including Hall (2005), Gertler and Trigari (2009), and Rogerson and Shimer (2011). Small profits, which refer to the small amount of resources a firm can allocate to vacancy creation, also play a quantitatively meaningful role in explaining business cycle fluctuations in vacancies and unemployment. Their importance is stressed by Hagedorn and Manovskii (2008).

An influential paper by Ljungqvist and Sargent (2017) points out that for many reconfigurations of the matching model that have been studied in the literature, we can actually go from these two determinants to a single more fundamental determinant, at least when it comes to necessity: Ljungqvist and Sargent (2017) argue that what really matters is the size of an object they call the *fundamental surplus*. To get large unemployment fluctuations over the

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business cycle requires that the *fundamental surplus fraction* (the ratio between the fundamental surplus and productivity) must be small.

In this paper, I show that while the fundamental surplus as a concept is insightful, one should shift the focus from the fundamental surplus fraction to the elasticity of the fundamental surplus, of which the fundamental surplus fraction is one of the determinants. To generate large responses of unemployment to productivity changes requires a high elasticity of the fundamental surplus with respect to productivity. When all deductions that enter the fundamental surplus are acyclical, and the fundamental surplus does not involve endogenous variables, then the elasticity of the fundamental surplus coincides with the inverse of the fundamental surplus fraction. This is the case Ljungqvist and Sargent (2017) consider (except in the discussion in their footnote 28, p.2664). Thus, their emphasis on the fundamental surplus fraction is due to the fact that, in their settings, it determines the elasticity of the fundamental surplus, but fundamentally it is the elasticity that matters. Indeed, Ljungqvist and Sargent (2017) point out that "a given change in productivity translates into a larger percentage change in the fundamental surplus when the fundamental surplus fraction is small".

The equality between the elasticity and the inverse fraction no longer holds when either deductions are allowed to be cyclical, or the fundamental surplus contains endogenous variables. However, it is still true that what matters is the elasticity of the fundamental surplus. Furthermore, to describe what determines the elasticity of the fundamental surplus is straightforward: the fundamental surplus fraction still matters, but now the elasticities of the different deductions with respect to productivity (or the elasticities of the coefficients of deductions with respect to productivity if these coefficients contain endogenous variables) come into play as well, together with the share of each deduction in productivity. There is also a special case in which the fundamental surplus fraction drops out of the formula for the elasticity of the fundamental surplus: when all deductions are proportional to productivity, the elasticity of the fundamental surplus is one. Ljungqvist and Sargent (2017) refer to this case as "arresting the fundamental surplus fraction".

To illustrate how the elasticity of the fundamental surplus is the essential determinant of labor market volatility, this paper presents three examples. The first example is based on the standard search and matching model with Nash bargaining. The second example uses Hall and Milgrom's (2008) model of alternating-offer wage bargaining (AOB) with the restriction that job destruction probabilities under bargaining and production are the same. In both examples, the opportunity cost of employment as a deduction varies positively with productivity, consistent with the empirical findings of Chodorow-Reich and Karabarbounis (2016). For both examples, the paper finds that the elasticity of the fundamental surplus is a reliable indicator of labor market volatility. Moreover, the paper considers different combinations of two parameters that enter the fundamental surplus fraction as a deduction from productivity in the AOB model, namely the opportunity cost of employment (which varies with productivity) and firms' cost of delay (which does not vary with productivity). The combinations are chosen such that all imply the same fundamental surplus fraction. Thus, across these combinations, the fundamental surplus is not informative about differences in labor market volatility. However, the elasticity of the fundamental surplus remains informative with regard to volatility. The emphasis on the cyclicality of the fundamental surplus is similar to the critique by Chodorow-Reich and Karabarbounis (2016) about the level of the opportunity cost of employment, where they argued that it is the cyclicality that really matters.

The third example continues with the examination of the AOB model while allowing the job destruction probabilities to be different under bargaining and production. In this setting the fundamental surplus fraction may contain endogenous variables, depending on which measure is adopted. In this paper I consider a "non-structural" version of the fundamental surplus fraction recently advocated by Ljungqvist and Sargent (2021) for this type of setting. Based on this measure, this paper aims to identify the key parameters that determine the elasticity of the fundamental surplus. The paper finds that the job destruction probability under bargaining has a profound impact on both the fundamental surplus fraction and the elasticity of the fundamental surplus. Moreover, the relative importance of firms' cost of delay for the elasticity of the fundamental surplus depends on the job destruction probability under bargaining. When this probability is reasonably high, the value of the cost of delay matters little with regard to labor market volatility. This result is in sharp contrast to Ljungqvist and Sargent (2021) who argue that the job destruction probability is just a "side-show". This paper also quantifies the difference between the elasticity of the fundamental surplus and the inverse of the fundamental surplus. The paper finds that, under reasonable parameterizations, the difference between the two is moderate, reflecting a moderate impact of market tightness on the fundamental surplus.

This paper is contemporaneous with two recent papers on the fundamental surplus. Christiano, Eichenbaum, and Trabandt (2021) study the multiple-measure issue of the fundamental surplus in their matching models. They show that depending on which measure is used, the fundamental surplus plays either a vital role or a minor role in accounting for labor market volatility. The third example I provide in this paper is inspired by the "non-structural" measure of the fundamental surplus in Christiano, Eichenbaum, and Trabandt (2021) for a similar setting. An important difference between Christiano, Eichenbaum, and Trabandt (2021) and this paper is that Christiano, Eichenbaum, and Trabandt (2021) emphasize the importance of wage rigidity for explaining unemployment fluctuations. While this paper agrees that some degree of wage rigidity is necessary to ensure that the amplification mechanism for productivity shocks in the standard matching model works in full force, the focus of this paper is on the elasticity of the fundamental surplus. Moreover, the paper shows that the elasticity of the fundamental surplus is a more reliable indicator of the magnitude of labor market volatility than the degree of wage rigidity.

Ljungqvist and Sargent (2021) study Christiano, Eichenbaum, and Trabandt's (2021) matching models. They argue that these models join a collection of models in which the key determinant of labor market volatility is the fundamental surplus. This paper agrees that the fundamental surplus unifies different forces generating high unemployment volatility over the business cycle across standard matching models. The paper differs from Ljungqvist and Sargent (2021) in what constitutes the critical parameters in the fundamental surplus for the AOB model. Ljungqvist and Sargent (2021) argue that only the opportunity cost of employment and firms' cost of delay are the key parameters in Hall and Milgrom's AOB model. By contrast, this paper shows that the probability of job destruction under bargaining is also an essential determinant of the fundamental surplus fraction because it determines the relative weight of different deductions. The paper also differs from Ljungqvist and Sargent (2021) by distinguishing between the inverse of the fundamental surplus fraction and the elasticity of the fundamental surplus, and by pointing out that what really matters is the latter.

The paper proceeds as follows. Section 2 presents a standard matching model. Section 3 illustrates how the elasticity of the fundamental surplus is the essential determinant of labor market volatility. Section 4 discusses the relationship between the elasticity of the fundamental surplus and wage elasticity. Section 5 offers concluding remarks.

2 Standard Search and Matching Model

The theoretical framework this paper builds upon is a standard discrete time search and matching model of equilibrium unemployment. The labor market is characterized by search frictions. Aggregate hiring is determined by the matching function

$$h_t = m u_t^{\alpha} v_t^{1-\alpha}. \tag{1}$$

The matching function shows hiring (h_t) is determined by the number of vacancies (v_t) and the number of unemployed workers actively searching for jobs (u_t). α is the matching elasticity with respect to unemployment. Using the matching function, the vacancy filling rate q_t can be defined as $q_t = h_t/v_t = m\theta_t^{-\alpha}$, where θ_t, the ratio of vacancies to unemployment, measures the tightness of the labor market.

Firms recruit unemployed workers by posting a vacancy at the flow cost of c. The value of an unfilled job vacancy is

$$V_t = -c + \frac{1}{1+r} E_t[q(\theta_{t+1})J_{t+1} + (1 - q(\theta_{t+1}))V_{t+1}]. \tag{2}$$

With the probability $q(\theta_t)$, a vacancy turns into a productive job match J_t. The value of a job match for a firm is defined as

$$J_t = y_t - w_t + \frac{1}{1+r} E_t[(1-s)J_{t+1} + sV_{t+1}]. \tag{3}$$

Once a job match is established, the employed worker produces y_t units of output and receives a wage payment w_t. The job match dissolves at the end of period with exogenous probability s.

Households supply a unit measure of infinitely lived workers. Each worker is either in a job match or actively searching for a job. The value function for a job-seeker is

$$U_t = z_t + \frac{1}{1+r} E_t[f(\theta_{t+1})L_{t+1} + (1 - f(\theta_{t+1}))U_{t+1}]. \tag{4}$$

During the job search, a job-seeker receives a flow value z_t per period. Here, I assume that $z_t = \bar{z} y_t^{\sigma}$, where σ is the elasticity of the opportunity cost of employment (z_t) with respect to productivity. $f(\theta_t)$ measures the probability of finding a job, which is defined as $f_t = h_t/u_t = m\theta_t^{1-\alpha}$. Once employed, a worker receives a value L_t from a job match. L_t is defined as

$$L_t = w_t + \frac{1}{1+r} E_t[(1-s)L_{t+1} + sU_{t+1}]. \tag{5}$$

For now, I assume the wage is a function of labor productivity and labor market tightness, $w_t = w(y_t, \theta_t)$. In section 3, the paper considers two leading bargaining protocols: the Nash bargaining protocol and AOB protocol. I assume that the total labor force is normalized to unity and inelastic. Thus, employment can be written as $n_t = 1 - u_t$. The law of motion for employment is

$$n_{t+1} = (1-s)n_t + h(u_t, v_t). \tag{6}$$

The model assumes free-entry to the labor market for the firms, therefore $V_t = 0$. Thus, the vacancy cost equals the firm's share of a match surplus, $c = \frac{1}{1+r} E_t q(\theta_{t+1}) J_{t+1}$. Substituting this into equation (3) yields the job creation condition:

$$y_t = w(y_t, \theta_t) + c[(1+r)\frac{1}{q(\theta_t)} - (1-s)E_t \frac{1}{q(\theta_{t+1})}]. \tag{7}$$

3 The Level vs. Cyclicality of the Fundamental Surplus

From this section onwards, I use comparative steady-state analysis to discuss the determinants of labor market volatility. In the standard matching model with exogenous job separation, variations in unemployment arise mostly from variations in outflows. Thus, to account for large unemployment volatility, the matching model requires a large response of market tightness to changes in productivity. Ljungqvist and Sargent (2017) show that various reconfigurations of the matching model that succeed in making market tightness very responsive to productivity shocks all have a common feature: a small fundamental surplus fraction. They argue that this small fraction is essential for generating large variations in market tightness and a high volatility of unemployment.

In this section, I show that a high elasticity of market tightness requires a high elasticity of the fundamental surplus with respect to productivity. When all deductions that enter the fundamental surplus are acyclical, the elasticity of the fundamental surplus coincides with the inverse of the fundamental surplus fraction, as long as the fundamental surplus is expressed solely in terms of parameters. This is the case Ljungqvist and Sargent (2017) consider (except in the discussion in their footnote 28, p. 2664). Thus, their emphasis on the fundamental surplus fraction is due to the fact that, in their settings, it determines the elasticity of the fundamental surplus, but ultimately it is the elasticity that matters.

The key observation on which Ljungqvist and Sargent (2017) is based is that for many reconfigurations of the matching model, one can write the equation which determines steady-state equilibrium market tightness in the following form:

$$y - \sum_{k=1}^{K} p_k d_k = c \cdot \Gamma(\theta). \tag{8}$$

The d_k's are so-called deductions from productivity that are measured in units of output. For now I follow Ljungqvist and Sargent (2017) by assuming that the deductions do not vary with productivity, and the parameters p_k's do not contain endogenous variables. On the right-hand side of the equation, the flow cost of posting a vacancy c (also measured in units of output) is multiplied by a function of market tightness. In most of the reconfigurations Ljungqvist and Sargent (2017) consider, the function $\Gamma(\theta)$ does not include any parameters that are measured in units of output. [1] It may include parameters such as the discount rate, the elasticity of the matching function, and the bargaining power of workers, and so on. Consequently, all parameters that are in units of productivity are on the left-hand-side, with the single exception of vacancy cost, which exclusively appears on the right-hand-side. For example, in the standard matching model with Nash bargaining, the opportunity cost of employment z is the single deduction, that is $K = 1$, $p_1 = 1$ and $d_1 = z$, and $\Gamma(\theta) = \frac{r+s+\phi\theta q(\theta)}{(1-\phi)q(\theta)}$ (see equation (16) in this paper).

According to Ljungqvist and Sargent (2017), the left-hand side $y - \sum_{k=1}^{K} p_k d_k$ is defined as the fundamental surplus, and $\frac{y-\sum_{k=1}^{K} p_k d_k}{y}$ is what they call the fundamental surplus fraction. When the equilibrium condition can be written in format (8), then there is typically only one way to do this, making the fundamental surplus well defined. [2]

Defining $F(y) \equiv y - \sum_{k=1}^{K} p_k d_k$, equation (8) implies that the elasticity of market tightness with respect to productivity is

[1] There are two exceptions: one is the matching model with a financial accelerator (see Ljungqvist and Sargent (2017, equation (43) on p. 2650)). Another exception is the matching model with a fixed matching cost before bargaining (see Ljungqvist and Sargent's (2017) online appendix, equation (85)). In both reconfigurations, deductions that enter the fundamental surplus also appear in $\Gamma(\theta)$. Ljungqvist and Sargent (2017) show that in both cases the fundamental surplus fraction is still the essential determinant of the elasticity of market tightness.

[2] One important exception is the alternating-offers bargaining (AOB) model without the restriction imposed by Ljungqvist and Sargent (2017). In this setting, the endogenous variable market tightness enters p_k and the fundamental surplus fraction is not well defined. See Section 3.2.2 for the discussions on the multiple-measures of the fundamental surplus.

$$\eta_{\theta,y} \equiv \Upsilon \cdot \eta_{F,y}, \tag{9}$$

where $\Upsilon \equiv 1/\eta_{\Gamma,\theta}$. This equation highlights that what matters is the elasticity of the fundamental surplus. The intuition for this finding is similar to the intuition in Chodorow-Reich and Karabarbounis (2016) who argue that it is the cyclicality of the opportunity cost of employment that matters for labor market volatility. Ljungqvist and Sargent (2017) argue that for all the reconfigurations they considered, the value of Υ has a fairly small upper bound, coming from a consensus about values of the elasticity of matching with respect to unemployment. Given this, a large value of $\eta_{\theta,y}$ necessarily requires a large value of $\eta_{F,y}$. With acyclical and exogenous deductions

$$\eta_{F,y} = \frac{y}{y - \sum_{k=1}^{K} p_k d_k}, \tag{10}$$

that is, $\eta_{F,y}$ is the inverse of the fundamental surplus fraction. Consequently, a high value of $\eta_{\theta,y}$ requires a low value of the fundamental surplus fraction.

There are two important cases under which the equality between the elasticity $\eta_{F,y}$ and the inverse fraction no longer holds. What remains true is that what matters is the elasticity of the fundamental surplus. The first case is when deductions vary with productivity. In this case equation (9) remains valid. The only thing that changes is the expression for the elasticity of the fundamental surplus:

$$\eta_{F,y} = \frac{y}{y - \sum_{k=1}^{K} p_k d_k}\left(1 - \sum_{k=1}^{K} \eta_{d_k,y} \cdot \frac{p_k d_k}{y}\right). \tag{11}$$

Thus, the elasticity of the fundamental surplus now depends on the fundamental surplus fraction $y - \sum_{k=1}^{K} p_k d_k$, the elasticities $\{\eta_{d_k,y}\}_{k=1}^{K}$ of the deductions with respect to productivity, and the shares $\{p_k d_k / y\}_{k=1}^{K}$ for each deduction in productivity. This equation reduces to

$$\eta_{F,y} = \frac{y - \eta_{d,y} \sum_{k=1}^{K} p_k d_k}{y - \sum_{k=1}^{K} p_k d_k} \tag{12}$$

when all deductions have the same elasticity $\eta_{d,y}$. Consequently, $\eta_{F,y}$ only depends on the fundamental surplus fraction and this common elasticity. If one also imposes $\eta_{d,y} = 1$, then $\eta_{F,y} = 1$. This is the case under which the fundamental surplus fraction drops out of the formula for the elasticity of the fundamental surplus. Ljungqvist and Sargent (2021) refer to this outcome as "disarming the fundamental surplus channel".

The second case is when endogenous variables enter the formula for the fundamental surplus. While equation (9) remains valid, the expression for the elasticity of the fundamental surplus becomes

$$\eta_{F,y} = \frac{y}{y - \sum_{k=1}^{K} p_k(\theta) d_k}\left(1 - \eta_{\theta,y} \sum_{k=1}^{K} \eta_{p_k(\theta),\theta} \cdot \frac{p_k(\theta) d_k}{y}\right). \tag{13}$$

Consequently, the determinants of the elasticity of the fundamental surplus now include the fundamental surplus fraction, the elasticities $\{\eta_{p_k(\theta),y}\}_{k=1}^{K}$ of the coefficients for each deduction with respect to productivity, and the shares for each deduction in productivity.

It is useful to combine equation (9) and (13) to eliminate $\eta_{\theta,y}$ from the formula for $\eta_{F,y}$. This yields

$$\eta_{F,y} = \frac{y}{y - \sum_{k=1}^{K}[1 - \Upsilon \eta_{p_k,\theta}]p_k(\theta)d_k}. \tag{14}$$

Next, I will derive some closed-form solutions for the elasticity of labor market tightness in two popular setups. The first setup is based on the standard matching model with Nash bargaining. The second setup uses the alternating-offers bargaining (AOB) model of Hall and Milgrom (2008). These will further highlight that what matters for the volatility of market tightness is the elasticity of the fundamental surplus.

3.1 Nash Bargaining

Under Nash bargaining, the steady-state version of the surplus sharing rule is $(1-\phi)(L-U) = \phi J$, where ϕ is the worker's relative bargaining power. The resultant wage is given by

$$w = (1-\phi)\bar{z}y^{\sigma} + \phi(y + \theta c). \tag{15}$$

The steady-state version of the job creation condition (7) and the wage equation (15) jointly determine the equilibrium value of θ:

$$y - \bar{z}y^{\sigma} = \frac{r + s + \phi\theta q(\theta)}{(1-\phi)q(\theta)}c. \tag{16}$$

After implicit differentiation of equation (16), I derive the following decomposition of the elasticity of labor market tightness:

$$\eta_{\theta,y} = \frac{(r+s) + \phi\theta q(\theta)}{\alpha(r+s) + \phi\theta q(\theta)} \frac{y - \sigma\bar{z}y^{\sigma}}{y - \bar{z}y^{\sigma}} \equiv \Upsilon^{Nash} \frac{y - \sigma\bar{z}y^{\sigma}}{y - \bar{z}y^{\sigma}}. \tag{17}$$

Here, the fundamental surplus is what remains after deducting the opportunity cost of employment $\bar{z}y^{\sigma}$ from productivity. The elasticity of the fundamental surplus depends on the fundamental surplus fraction and the elasticity of the opportunity cost of employment with respect to productivity. Equation (17) reduces to

$$\eta_{\theta,y} = \Upsilon^{Nash} \frac{y}{y - \bar{z}} \tag{18}$$

when the opportunity cost of employment is acyclical (i.e., $\sigma = 0$). Under this assumption, the elasticity of the fundamental surplus coincides with the inverse of the fundamental surplus fraction. This is why Ljungqvist and Sargent (2017) emphasize the size of the fundamental surplus fraction in their setting.

As the opportunity cost of employment is the single deduction from productivity, its cyclicality plays a vital role in defining the elasticity of the fundamental surplus. The literature often assumes that the opportunity cost is constant over the business cycle. This assumption, however, has been challenged by Chodorow-Reich and Karabarbounis (2016). They present evidence to show that the opportunity cost is in fact strongly procyclical. They argue that "across specifications, [the] elasticity [of the opportunity cost] exceeds 0.8 and is typically close to simply 1. Importantly, z comoves roughly proportionally with y over the business cycle irrespective of whether the level of z is high or low". This finding poses a significant challenge to the literature that addresses the volatility puzzle. Chodorow-Reich and Karabarbounis (2016) demonstrate that models with either Nash bargaining or AOB fail to generate a large elasticity of market tightness when the opportunity cost is procyclical. Here, I review their arguments through the lens of the fundamental surplus.

Assuming that the opportunity cost is proportional to productivity (i.e., $\sigma = 1$), equation (17) reduces to

$$\eta_{\theta,y} = \Upsilon^{Nash}. \tag{19}$$

This is a special case in which the fundamental surplus fraction drops out of the formula for the elasticity of the fundamental surplus: when the opportunity cost is proportional to productivity, the elasticity of the fundamental surplus is equal to one, regardless of how small the fundamental surplus is. The unit elasticity of the fundamental surplus implies that there is no chance of obtaining high volatility here, e.g., by recalibrating other parameters. For example, under Nash bargaining, one may get the idea to try to offset the procyclicality of opportunity costs by reducing the bargaining power of workers in an attempt to make wages more rigid. The unit elasticity of the fundamental surplus tells us that this will not help, and offers some insight into why this will be so.

3.2 Alternating-Offers Bargaining (AOB)

Hall and Milgrom (2008) replaced standard Nash bargaining with AOB. Under this bargaining protocol, a firm and a worker take turns making wage offers. Both parties understand that they obtain a strictly higher payoff from reaching an agreement rather than breaking up and accepting the outside option. In each bargaining round, each party either accepts the counterparty's offer or rejects it and proposes a counteroffer in the following bargaining round. After a delay, the firm incurs a cost of delay $\gamma > 0$ while the worker enjoys the value of leisure $\bar{z}y^{\sigma}$. There is a probability δ that the wage negotiation is exogenously terminated between bargaining rounds, in which case the worker goes back to the unemployment pool.

The optimal wage offer proposed by each party is such that the counterparty is indifferent with regard to accepting the wage offer or rejecting it and waiting until the next round to make a counteroffer. As a result, the initial wage offer will be accepted. Following Ljungqvist and Sargent (2017), the steady-state wage offer w^f from the firm and wage offer w^w from the worker must satisfy the following two indifference conditions:

$$\frac{w^f + \beta sU}{1-\beta(1-s)} = \bar{z}y^\sigma + \beta[(1-\delta)\frac{w^w + \beta sU}{1-\beta(1-s)} + \delta U], \tag{20}$$

$$\frac{y - w^w}{1-\beta(1-s)} = -\gamma + \beta(1-\delta)\frac{y - w^f}{1-\beta(1-s)}. \tag{21}$$

Solving equation (20) and (21) simultaneously for w^f yields

$$w^f = \beta_1 y + \beta_2 \bar{z} y^\sigma + \beta_3 \gamma + \beta_4 U, \tag{22}$$

where $\beta_1 = \frac{\beta(1-\delta)}{1+\beta(1-\delta)}$, $\beta_2 = \frac{1-\beta(1-s)}{1-\beta^2(1-\delta)^2}$, $\beta_3 = \frac{\beta(1-\delta)[1-\beta(1-s)]}{1-\beta^2(1-\delta)^2}$, and $\beta_4 = \frac{\beta(1-\beta)(\delta-s)}{1-\beta^2(1-\delta)^2}$. Equation (22) describes the wage a firm would offer a worker in the first round of bargaining which would be accepted.

Next, solving equation (4) and (5) for U and substituting the solution into equation (22) for U, the wage offer can be rewritten as

$$w^f = \frac{\beta_1}{1-\beta_4\tau_1(\theta)} y + \frac{\beta_2 + \beta_4\tau_2(\theta)}{1-\beta_4\tau_1(\theta)} \bar{z} y^\sigma + \frac{\beta_3}{1-\beta_4\tau_1(\theta)} \gamma, \tag{23}$$

where $\tau_1(\theta) = \frac{\beta f(\theta)}{(1-\beta)[1-\beta(1-s-f(\theta))]}$ and $\tau_2(\theta) = \frac{1}{1-\beta} - \tau_1(\theta)$.

Combining the wage equation (23) with the steady-state version of the job creation condition gives the following equilibrium condition for θ:

$$y - \frac{\beta_2 + \beta_4\tau_2(\theta)}{1-\beta_1-\beta_4\tau_1(\theta)} \bar{z} y^\sigma - \frac{\beta_3}{1-\beta_1-\beta_4\tau_1(\theta)} \gamma = \frac{1-\beta_4\tau_1(\theta)}{1-\beta_1-\beta_4\tau_1(\theta)} \frac{r+s}{q(\theta)} c. \tag{24}$$

To derive the elasticity of market tightness, I consider two versions of the AOB model. Following Ljungqvist and Sargent (2017), the first version is based on the assumption that exogenous job destruction probabilities under bargaining and production are the same (i.e., $\delta = s$). In this what Ljungqvist and Sargent (2017) call the approximating version of the AOB model, the fundamental surplus fraction does not contain endogenous variables. The second version allows the probability of job destruction to be different under bargaining and production (i.e., $\delta \neq s$). In this setting the fundamental surplus fraction is not well-defined and may contain endogenous variables.

3.2.1 $\delta = s$

After imposing the parameter restriction $\delta = s$, the equilibrium condition for θ (24) reduces to

$$y - \bar{z} y^\sigma - \beta(1-s)\gamma = \frac{1}{1-\beta_1} \frac{r+s}{q(\theta)} c. \tag{25}$$

Totally differentiating (25) and rewriting yields

$$\eta_{\theta,y} = \frac{1}{\alpha} \frac{y - \sigma \bar{z} y^{\sigma}}{y - \bar{z} y^{\sigma} - \beta(1-s)\gamma}. \tag{26}$$

Here, the fundamental surplus is what remains after deducting the opportunity cost of employment $\bar{z}y^{\sigma}$ and firms' discounted cost of delay $\beta(1-s)\gamma$ from productivity. Similar to (17), the elasticity of the fundamental surplus is determined by both the fundamental surplus fraction and the elasticity of the opportunity cost with respect to productivity. If the opportunity cost is assumed to be acyclical (i.e., $\sigma = 0$), then equation (26) reduces to

$$\eta_{\theta,y} = \frac{1}{\alpha} \frac{y}{y - \bar{z} - \beta(1-s)\gamma}. \tag{27}$$

Not surprisingly, the elasticity of the fundamental surplus is now the inverse of the fundamental surplus fraction. If instead the opportunity cost is assumed to be proportional to productivity, equation (26) becomes

$$\eta_{\theta,y} = \frac{1}{\alpha} \frac{y - \bar{z} y}{y - \bar{z} y - \beta(1-s)\gamma}. \tag{28}$$

To get a sense of the effects of the cyclicality of the opportunity cost on the elasticity of the fundamental surplus, consider the following numerical example. The elasticity of matching with respect to unemployment is set at $\alpha = 0.5$, and productivity is normalized to unity. The steady-state opportunity cost of employment is $\bar{z} = 0.71$. This is the mid-point of the empirical estimates by Chodorow-Reich and Karabarbounis (2016). The model period is a month, and the discount factor is $\beta = 0.95^{1/12}$, i.e., an annual interest rate of 5 percent. The exogenous monthly separation rate is $s = 0.0333$.

The value of firms' cost of delay is chosen to let $\eta_{\theta,y}$ in (28) meet the target value of 20. By doing so I obtain $\gamma = 0.197$. With acyclical opportunity cost of employment, both the elasticity of the fundamental surplus and the inverse of the fundamental surplus fraction are equal to 10. By contrast, if the opportunity cost is proportional to productivity, the inverse of the fundamental surplus fraction is still equal to 10, but the elasticity of the fundamental surplus reduces to 2.9. The elasticity of market tightness, $\eta_{\theta,y}$, then falls from 20 to 5.8. To restore the target value of $\eta_{\theta,y}$, one has to increase the value of γ from 0.197 to 0.271. While this lets the elasticity of the fundamental surplus retain its target value of 10, the inverse of the fundamental surplus fraction is now equal to 34.5. Clearly, this example shows that the elasticity of the fundamental surplus is a more reliable indicator of $\eta_{\theta,y}$ compared with the inverse of the fundamental surplus fraction, and it is deeply influenced by the cyclicality of the opportunity cost.

Finally, if one also assumes that firms' cost of delay γ is proportional to productivity, i.e., $\gamma = \bar{\gamma} y$, [3] equation (28) reduces to

$$\eta_{\theta,y} = \frac{1}{\alpha}. \tag{29}$$

[3]For a quantitative assessment of the impact of a procyclical γ on the volatility of unemployment, see Wang (2022). One important difference between Wang (2022) and this paper is that Wang (2022) does not analyze the impact of δ and z on the elasticity of market tightness. A second important difference is that Wang (2022) does not discuss the fundamental surplus.

This equation is similar to the earlier elasticity under Nash bargaining in (19). This is not surprising as the elasticity of the fundamental surplus is one when all deductions are proportional to productivity.

3.2.2 $\delta \neq s$

Things become complicated when the probability of job destruction is different under bargaining and production (i.e., $\delta \neq s$). According to Christiano, Eichenbaum, and Trabandt (2021), there are at least two different decompositions of the elasticity of market tightness in their estimated AOB model, which give rise to two measures of the fundamental surplus. In the decomposition Christiano, Eichenbaum, and Trabandt (2021) call "structural", the fundamental surplus does not contain endogenous variables. In their "non-structural" decomposition, the fundamental surplus contains endogenous variable θ. Christiano, Eichenbaum, and Trabandt (2021) found that on average the non-structural decomposition attributes a large role to the fundamental surplus in accounting for labor market volatility. This non-structural decomposition is also advocated by Ljungqvist and Sargent (2021) as it "[identifies] critical parameters that determine the sensitivity of unemployment to productivity".

The multiple-decomposition issue also applies to Hall and Milgrom's (2008) AOB model. [4] In light of the findings in Christiano, Eichenbaum, and Trabandt (2021) and Ljungqvist and Sargent (2021), I provide a non-structural decomposition of the elasticity of market tightness under the assumption that $\delta \neq s$. To simplify the analysis, I assume that $\sigma = 0$.

The Technical Appendix provides the derivation of the following non-structural decomposition:

$$\eta_{\theta,y} = \Upsilon^{AOB} \frac{y}{y - \tau_z \bar{z} - \tau_\gamma \gamma}, \tag{30}$$

where

$$\Upsilon^{AOB} = \frac{(1-\beta_4\tau_1(\theta))(1-\beta_1-\beta_4\tau_1(\theta))}{\alpha(1-\beta_4\tau_1(\theta))(1-\beta_1-\beta_4\tau_1(\theta)) + \beta_1\beta_4\tau_3(\theta)}$$

$$\tau_3(\theta) = \frac{\beta(1-\alpha)[1-\beta(1-s)]f(\theta)}{(1-\beta)[1-\beta(1-s-f(\theta))]^2}$$

$$\tau_z = \frac{\alpha(1-\beta_4\tau_1(\theta))(\beta_2+\beta_4\tau_2(\theta)) + \beta_4(1-\beta_2-\beta_4\tau_1(\theta)-\beta_4\tau_2(\theta))\tau_3(\theta)}{\alpha(1-\beta_4\tau_1(\theta))(1-\beta_1-\beta_4\tau_1(\theta)) + \beta_1\beta_4\tau_3(\theta)}$$

[4] A key difference between Christiano, Eichenbaum, and Trabandt's (2021) AOB model and Hall and Milgrom's AOB model is that Christiano, Eichenbaum, and Trabandt (2021) assume that bargaining proceeds within a period. They further assume there are an even number of bargaining rounds within a period and the number of bargaining rounds is not restricted by the probability of bargaining breakdown δ. By contrast, Hall and Milgrom assume that alternating offers are made in successive periods. Under this assumption, the number of bargaining rounds is restricted by δ. Breaking the linkage between wages and worker's outside value U requires $\delta = 0$ in Christiano, Eichenbaum, and Trabandt's (2021) AOB model and $\delta = s$ in Hall and Milgrom's AOB model. For other differences between the two models, see Christiano, Eichenbaum, and Trabandt (2021, p. 7, footnote 5).

$$\tau_\gamma = \frac{\alpha\beta_3(1-\beta_4\tau_1(\theta)) - \beta_3\beta_4\tau_3(\theta)}{\alpha(1-\beta_4\tau_1(\theta))(1-\beta_1-\beta_4\tau_1(\theta)) + \beta_1\beta_4\tau_3(\theta)}.$$

The second component on the right-hand side of equation (30) defines the elasticity of the fundamental surplus with respect to productivity. The corresponding fundamental surplus is given by the left-hand side of equation (24). While this may not be the unique measure of the fundamental surplus, the decomposition in (30) meets an essential criterion set by Ljungqvist and Sargent (2017, 2021). This is that the first factor must have a relatively low upper bound. It is easy to verify that the upper bound of Υ^{AOB} is $1/\alpha$. In fact, if $\delta = s$, it follows that $\beta_4 = 0$. This implies that Υ^{AOB} is reduced to $1/\alpha$. The elasticity of the fundamental surplus is reduced to $y/[y - \bar{z} - \beta(1-s)\gamma]$, and it is identical to the inverse of the fundamental surplus fraction.

3.2.3 Quantitative Analysis

In this section, I perform two experiments on the AOB model. The first experiment, based on the assumption that $\delta \neq s$, serves two purposes. First, it aims to identify the key primitives in the AOB model that determine the elasticity of the fundamental surplus. What these primitives are is the subject of an ongoing debate. Using dynamic analyses of their estimated DSGE models, Christiano, Eichenbaum, and Trabandt (2021) show that the value of δ is as important as the value of $\bar{z}$ and γ in accounting for high labor market volatility. Ljungqvist and Sargent (2021) argue that any difference between δ and s "acts like nuisances". They further argue that "either the value of leisure or a firm's cost of delay in bargaining can be calibrated high enough to generate any market tightness elasticity regardless of the calibration of other parameters". To test Ljungqvist and Sargent's (2021) claims, I consider a set of admissible values for δ, and study their implications for the elasticity of market tightness. I find that the value of δ has a profound impact on both the elasticity of the fundamental surplus and the inverse of the fundamental surplus fraction. Moreover, when the value of δ is reasonably high, simply raising the value of γ cannot help. In this case the only parameter that can deliver a high $\eta_{\theta,y}$ is $\bar{z}$. Thus, the importance of γ for $\eta_{\theta,y}$ depends on the value of δ.

The second purpose of the first experiment is to quantify the difference between the elasticity of the fundamental surplus and the inverse of the fundamental surplus fraction when all deductions are acyclical. I find that if $\delta = s$, the two objects are identical, since no endogenous variables enter the formula of the fundamental surplus. Once δ is different to s, the two objects are no longer the same. However, under reasonable parameterizations, the difference between the two is moderate, reflecting a relatively small impact of θ on the coefficients of deductions.

The second experiment, based on the assumptions that $\sigma = 1$ and that $\delta = s$, studies the AOB model when the opportunity cost of employment is proportional to productivity. Here I consider different constellations of (z, γ) that give rise to the same size of the fundamental surplus fraction, but different elasticities of the fundamental surplus, and study their impact on the elasticity of market tightness. I show that across these parameter configurations, the fundamental surplus fraction is not informative about differences in labor market volatility. However, the elasticity of the fundamental surplus remains informative with regard to volatility.

For both experiments, I standardize a time period to be one month. Model parameters are summarized in Table 1. For the matching function, I set $\alpha = 0.5$; this value lies in the plausible range of estimates surveyed by Petrongolo and Pissarides (2001). The discount factor is set as $\beta = 0.996$, so the annual discount rate is approximately 5%. The average monthly job separation rate is $\tau = 0.0333$. I follow Hall and Milgrom (2008) and assume that the steady-state opportunity cost of employment is $\bar{z} = 0.71$. This is also the mid-point of the empirical estimates by Chodorow-Reich and Karabarbounis (2016). Following Hall and Milgrom (2008), the cost of posting a vacancy is set as $c = 0.433$.

Table 1: Calibrated Parameter Values

Parameter	Definition	Value	
		1st Experiment	2nd Experiment
α	Elasticity of the Matching Function	0.5	0.5
β	Discount Factor	0.996	0.996
s	Monthly Job Separation Rate	0.033	0.033
c	Cost of Vacancy When $y = 1$	0.433	0.433
$\bar{z}$	Opportunity Cost of Employment	0.71	0.41 - 0.71
γ	Cost of Delay to the Firm	0.27	0.2 - 0.51
m	Matching Efficiency	0.46 - 0.78	0.43
δ	Probability of Bargaining Breakdown	0.033 - 0.5	0.033

In the first experiment, the most critical parameter is the probability of job destruction during bargaining δ. The literature often considers small values for δ. In Ljungqvist and Sargent (2017), the daily probability of job destruction during bargaining ranges from 0.14 to 0.55 percent. Christiano, Eichenbaum, and Trabandt (2021) and Ljungqvist and Sargent (2021) consider an even smaller range of values for δ. The daily rate of δ is up to 0.3 percent in Christiano, Eichenbaum, and Trabandt (2021) and 0.25 in Ljungqvist and Sargent (2021). Within those ranges, the value of δ is seemingly unimportant in accounting for the elasticity of market tightness, especially when the fundamental surplus fraction is small. This leads to Ljungqvist and Sargent (2021) to conclude that "differences in job destruction probability during bargaining versus production are a side-show".

However, the common practice of assigning a small value to δ has been questioned by Kehoe et al. (2022). They argue that this implies that duration of a job opportunity is long, which in turn implies implausible cyclicality of the user cost of labor. To meet the observed cyclicality, they show that the duration of a job opportunity has to be 2.6 months, therefore monthly δ is approximately 39 percent. By contrast, by setting daily $\delta = 0.0055$, which is the maximum daily rate considered in the literature, the corresponding monthly δ is roughly 15 percent. Kehoe et al. (2022) further argue that small δ implies low efficiency of allocations. They demonstrate that in order for the resulting allocations to converge to the constrained efficient allocations, δ has to be close to one.

In light of these findings, I consider a wide range of values for δ. In particular, I focus on the following three cases: $\delta = 0.0333$, $\delta = 0.15$, and $\delta = 0.39$. The first case implies that $\delta = s$. The second case is considered in Ljungqvist and Sargent (2017) and Hall and Milgrom (2008). The third case is the empirically plausible rate suggested by Kehoe et al. (2022). For each value of δ, I adjust the efficiency parameter of the matching function to make the equilibrium unemployment rate 5.5 percent. The cost of delaying bargaining to the firm is set as $\gamma = 0.27$, a value taken from Hall and Milgrom (2008).

Table 2: Results From the Calibrated AOB Model

δ	Elasticity, $\eta_{\theta,y}$	Elasticity, $\eta_{F,y}$	Inverse FS	Υ_{AOB}	Elasticity, $\eta_{w,y}$
0.033	66.6	33.3	33.3	2	0.5
0.15	14.4	8	10.13	1.8	0.84
0.39	7.3	4.66	5.36	1.57	0.9

The main results from the first experiment can be summarized as follows. First, the elasticity of the fundamental surplus is a reliable indicator of labor market volatility. As shown in Figure 1, there is a nearly perfect comovement between the elasticity of labor market tightness and the elasticity of the fundamental surplus when δ changes. Second, the value of the probability of job destruction during bargaining has a profound impact on the elasticity of the fundamental surplus. This results in a strong influence on the elasticity of market tightness. Table 2 shows that simply raising the value of δ from 0.033 to 0.15 leads to a sharp decline in the elasticity of the fundamental surplus. As a consequence, the elasticity of market tightness decreases by almost 80%. Third, the elasticity of the fundamental surplus and the inverse of the fundamental surplus fraction are no longer identical once the job destruction probability is different under bargaining and production. Table 2 shows that the elasticity of the fundamental surplus is smaller than the inverse of the fundamental surplus fraction. This is because the derivative of the fundamental surplus with respect to productivity is smaller than one when the endogenous variable θ enters the coefficients of deductions. A positive change in productivity leads to an increase in labor market tightness, and therefore increases the coefficients of deductions. This explains why the derivative is smaller than one.

The focus of the analysis thus far has been the impact of δ on $\eta_{\theta,y}$ and $\eta_{F,y}$ conditional on the given value of $\bar{z}$ and γ. As mentioned earlier, Ljungqvist and Sargent (2021) argue that the value of δ is not important as either $\bar{z}$ or γ can be calibrated high enough to offset the impact of δ on $\eta_{\theta,y}$. To examine this claim, I perturb a firm's cost of delay in bargaining γ while keeping the value of δ fixed at 0.39. [5] Figure 2 shows how such perturbations affect the elasticity of market tightness and its components.

[5] I also keep the value of m fixed for the following reason. The value of γ affects the elasticity of the fundamental surplus through two channels: γ enters the fundamental surplus fraction as a deduction from productivity; γ affects the coefficients of deductions through its impact on market tightness. Adjusting m will also affect the coefficients of deductions and therefore makes it difficult to observe the impact of γ on the elasticity of the fundamental surplus.

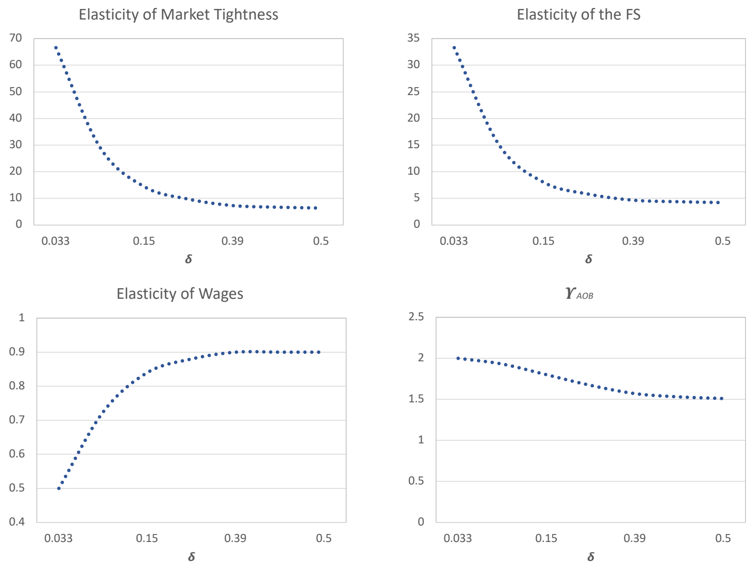

Figure 1: Outcomes in the AOB model for Different Values of δ

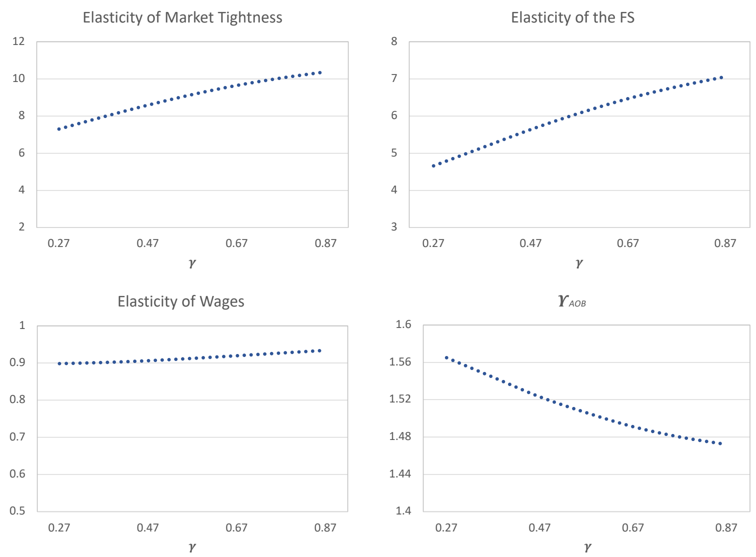

Figure 2: Outcomes in the AOB model for Different Values of γ

The striking finding is that perturbations of γ toward higher values cannot raise $\eta_{\theta,y}$ to the target value of 20. A threefold increase in γ only leads to a mild increase in the elasticity of market tightness. This is because, when δ=0.39, wages are affected more by the worker's outside value $U(\theta)$ than by the disagreement payoff. Thus, γ has only a limited influence on the elasticity of market tightness. By contrast, Ljungqvist and Sargent (2021) demonstrate how γ shapes $\eta_{\theta,y}$ when δ is sufficiently small (see their Figure 2). This example clearly shows that the importance of a firm's cost of delay is conditional on δ. The value of γ matters little when the probability of job destruction under bargaining is reasonably high.

In conclusion, the first exercise shows that the elasticity of the fundamental surplus is a reliable indicator of labor market volatility. It also shows that the job destruction probability under bargaining is a key primitive that determines the elasticity of the fundamental surplus.

Unlike γ, z affects both the worker's outside value and the disagreement payoff. This is part of the reason why the cyclicality of z has such a large impact on the wage elasticity. In the second experiment, I consider the AOB model when z is proportional to productivity. To be specific, I consider different constellations of (z, γ) that give rise to the same size of the fundamental surplus fraction but with different elasticities of the fundamental surplus. For each pair (z, γ), the implied elasticity of market tightness is 20 if $z = \bar{z}$. Figure 3 shows how different constellations of (z, γ) affect the elasticity of market tightness.

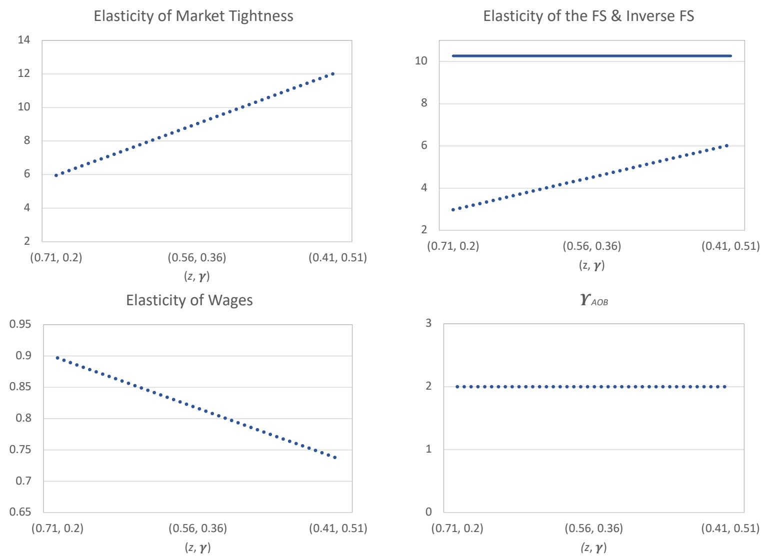

Figure 3: Outcomes in the AOB model for Different Constellations of (z, γ)

As shown in Table 2, the elasticity of market tightness jumps to 66.6 in the baseline parameterization when $\delta = s$. In order to lower $\eta_{\theta,y}$ to the target value of 20, I set $\gamma = 0.2$. The efficiency parameter of the matching function is adjusted so that the equilibrium unemployment is 5.5 percent. Under this alternative parameterization, $\eta_{\theta,y} = 20.5$ when z is acyclical. The inverse of the fundamental surplus fraction is just above 10. By allowing z to co-move proportionally with productivity, $\eta_{\theta,y}$ falls from 20.5 to 5.95.

Along the horizontal axis of Figure 3, I reduce the level of the opportunity cost of employment and increase the firm's cost of delay. Not surprisingly, this leads to a steady rise in $\eta_{\theta,y}$. By construction, the fundamental surplus fraction does not change (see the solid line in the upper right panel). Clearly, across these parameter configurations, the fundamental surplus fraction is not informative about differences in labor market volatility. The key point here is that the elasticity of the fundamental surplus remains informative about volatility (see the dotted line in the upper right panel). This simple exercise shows that the elasticity of the fundamental surplus is a more reliable indicator of labor market volatility compared with the fundamental surplus fraction.

4 Further Topics

4.1 The Elasticity of the Fundamental Surplus vs. Wage Elasticity

Before Ljungqvist and Sargent (2017) publish their seminal paper on the fundamental surplus, the literature normally decomposed the elasticity of labor market tightness into the elasticity of wages, the level of profits and the constant elasticity of matching with respect to unemployment. This three-factor decomposition can be expressed as

$$\eta_{\theta,y} = \frac{1}{\alpha}[1 - \frac{w(y,\theta)}{y}\eta_{w,y}]\frac{y}{y - w(y,\theta)}. \tag{31}$$

Rearranging the terms in equation (31) yields

$$\eta_{\theta,y} = \frac{1}{\alpha}\frac{y - w(y,\theta)\eta_{w,y}}{y - w(y,\theta)}. \tag{32}$$

This simplified decomposition captures the relationship between the elasticity of market tightness and the elasticity of wages. The second factor, $\frac{y - w(y,\theta)\eta_{w,y}}{y - w(y,\theta)}$, denotes the elasticity of firm's profits with respect to productivity. In order to generate a high elasticity of firm's profits, which leads to a high $\eta_{\theta,y}$, one has to reconfigure matching models, either to lower the wage's response to productivity changes, or to suppress a firm's profits, or to achieve both.

The role of wage rigidity in determining $\eta_{\theta,y}$ is somewhat controversial and has sparked a debate. Ljungqvist and Sargent (2017) argue that "the low wage elasticity ... is incidental to and neither necessary nor sufficient to obtain a high volatility of unemployment". By contrast, Christiano, Eichenbaum, and Trabandt (2021) argue that "wage inertia does in fact play a crucial role in allowing variants of standard search and matching models to account for the large countercyclical response of unemployment to shocks". Different views on wage rigidity reflect the fact that the elasticity of wages by itself is not sufficient to determine the elasticity of market tightness: the level of firm's profits also matters, as shown in equation (32). However, this does not mean that wage rigidity is of no importance. To understand the relationship between the elasticity of the fundamental surplus and the wage elasticity, I derive the following expression by combining (9) and (32):

$$\eta_{F,y} = \frac{1}{\alpha\Upsilon}\frac{y - w(y,\theta)\eta_{w,y}}{y - w(y,\theta)}. \tag{33}$$

This equation conveys two important messages. First, the elasticity of the fundamental surplus is a more fundamental indicator of the magnitude of labor market volatility when compared to wage elasticity. The reason is that the elasticity of the fundamental surplus is a single determinant of labor market volatility. It not only unifies the wage rigidity channel and the firm's profits channel, but also identifies the key primitives that determine the combined effects of these two channels. For example, the elasticity of the fundamental surplus under Nash bargaining tells us that it is the level and cyclicality of the opportunity cost of employment that ultimately determines the elasticity of market tightness. Low wage elasticity generated by suppressing the worker's bargaining power is not sufficient to deliver a large elasticity of the fundamental surplus.

Figure 1,2 and 3 in section 3.2.3 confirm the first message. There we observe a negative comovement between the elasticity of market tightness and the elasticity of wages in Figures 1 and 3, but a positive comovement between the two objects in Figure 2. Figure 2 confirms that the wage elasticity per se is not sufficient to determine the elasticity of market tightness. In contrast, a strong positive comovement between the elasticity of market tightness and the elasticity of the fundamental surplus holds across all three figures.

Second, while excessive wage rigidity is not necessary for generating a large elasticity of market tightness, to ensure that the fundamental surplus channel works in full force, the wage elasticity has to be smaller than one. Equation (33) shows that, if the wage elasticity is equal to or larger than one, the elasticity of the fundamental surplus is bounded from above by $1/(\alpha\Upsilon)$. If $\Upsilon = 1/\alpha$ as in many reconfigured models, the upper bound of the elasticity of the fundamental surplus is unity. To see this, consider a constant wage $w = \hat{w}$ as in Hall (2005). In this case, the elasticity of the fundamental surplus is given by

$$\eta_{F,y} = \frac{y}{y - \hat{w}}. \tag{34}$$

Here, the fundamental surplus is what remains after deducting the wage from productivity. It is identical to a firm's profits as the invisible hand cannot allocate the resources assigned to workers by wage norm to vacancy creation. If one assumes that wage norm entitles workers to receive a fixed proportion of productivity, i.e., $w = ky$, then the elasticity of the fundamental surplus is given by

$$\eta_{F,y} = 1. \tag{35}$$

Not surprisingly, the fundamental surplus channel is disarmed when the elasticity of wages with respect to productivity is equal to one. Similar consequences would flow when wages are formed under AOB and Nash bargaining. It is easy to verify that, when both z and γ are proportional to productivity and $\delta = s$, the wage elasticity in the AOB model is equal to one. Recall that when $\delta = s$, $\Upsilon^{AOB} = 1/\alpha$. Given these, equation (33) implies that $\eta_{F,y} = 1$.

Under Nash bargaining, if one assumes $z = \bar{z}y$, the wage elasticity is given by

$$\eta_{w,y} = \frac{(1-\phi)\bar{z}y + \phi y + \phi c\theta\eta_{\theta,y}}{w} = 1 + \frac{\phi c\theta(\Upsilon^{Nash} - 1)}{w}, \tag{36}$$

where the second equality is obtained after using equation (15) and (19) to simplify the numerator. Because $\Upsilon^{Nash} > 1$, equation (36) shows that the wage elasticity is no less than one. When $\phi = 0$, $\eta_{w,y} = 1$, $\Upsilon^{Nash} = 1/\alpha$ and $\eta_{F,y} = 1$. In this case, the wage is identical to that set by the wage norm considered in equation (34). When $\phi > 0$, $\eta_{w,y} > 1$ and $\Upsilon^{Nash} < 1/\alpha$. This again leads to $\eta_{F,y} = 1$. Consequently, $\eta_{\theta,y} < 1/\alpha$.

What is the economic intuition behind these results? Suppose that the steady-state productivity changes by one percent after a permanent shock to productivity. Equation (32) makes it clear that, if the percentage change in wages is larger than (or equal to) one, the percentage change in firms' profits must be smaller than (or equal to) one, irrespective of how small these profits are, otherwise the sum of the change in wages and profits would exceed the change in productivity. Therefore, the small profits channel has been disarmed when the elasticity of wages reaches one. Similarly, because the size of the resources allocated to vacancy creation is irrelevant, the amount of resources the invisible hand can distribute no longer matters.

4.2 Wage Elasticity vs. Firms' Profits

This section sheds further light on why some degree of wage rigidity is necessary for generating a large response of unemployment to changes in productivity. As shown in equation (31), firms' profits are a distinctive channel that complements wage rigidity in order to generate large elasticity of market tightness. Such complementarity means that, for a given degree of wage rigidity, the smaller the firms' profits, the more responsive market tightness is to a change in productivity. This complementarity also implies that there are a sequence of parameterizations which give rise to different degrees of wage rigidity and sizes of firms' profits, but generate the same magnitude of the elasticity of market tightness. Therefore it is an open question as to which channel the model should primarily rely on to account for labor market volatility.

To answer this question, I draw an isoquant for the elasticity of market tightness, using the three-factor multiplicative decomposition of the elasticity of market tightness shown in equation (31). The target elasticity of market tightness is 20, and I set $\alpha = 0.5$. The result is reported in Figure 4. Because equation (31) uses only the job creation condition, the result holds regardless of how wages are determined. Along the isoquant, a higher wage elasticity is associated with smaller profits. To ascertain the implications of small profits for hiring, I calculate the corresponding vacancy-filling rate for each given value of firm's profits, using the job creation condition and the parameterization presented in Table 1. (See the dotted line in Figure 4.)

Next, to select a plausible range of the isoquant, I use the vacancy-filling rate to pin down a firm's profits, and then use the isoquant to identify the corresponding wage elasticity. I follow Hall and Schulhofer-Wohl (2018) in measuring the vacancy-filling rate. Based on data from the Job Openings and Labor Turnover Survey, they report that vacancy duration varies between 0.64 and 0.94 months. Thus, the monthly vacancy-filling rate is in the range of 1.06 to 1.56. Mapping

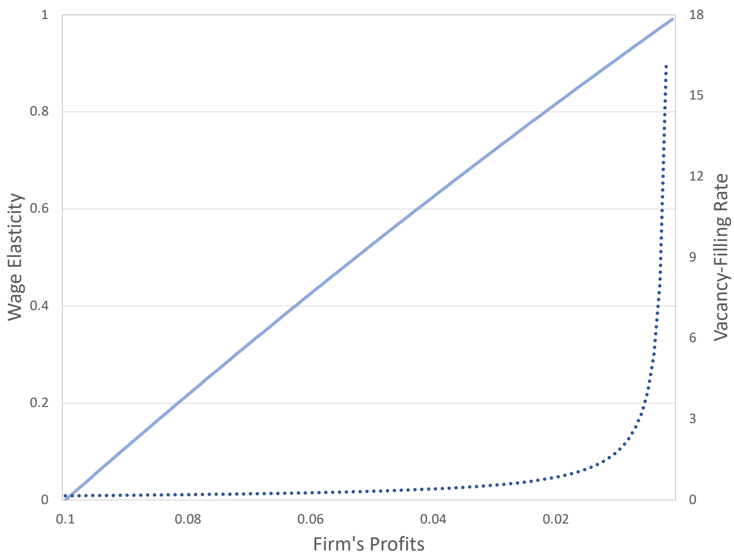

Figure 4: Isoquant for the Elasticity of Market Tightness

Notes: The solid line refers to the isoquant for the elasticity of market tightness. The dashed line refers to the vacancy filling rate $q(\theta)$.

from the vacancy-filling rates to firm's profits, and then to wage elasticities, the wage elasticity implied by the isoquant ranges from 0.85 to 0.9. Notably, this range is close to the estimated wage elasticity of 0.8 in Haefke et al (2013). It therefore seems that the standard parameterization supports a moderate degree of wage rigidity with reasonably sized small profits. This finding is consistent with the Christiano, Eichenbaum, and Trabandt (2021) narrative. They argue that "conditional on having some wage inertia (i.e., $1 - [dw/dy] > 0$), additional wage inertia is neither necessary nor sufficient to have a high value of $[\eta_{\theta,y}]$".

Figure 4 also highlights the danger of relying too heavily on small profits to increase the elasticity of market tightness. If firms' profits are too small, the total number of vacancies posted will also be very small under a realistic calibration of the vacancy cost. This implies that the model would require an extremely high vacancy-filling rate to generate the unemployment rate observed in the data. The resultant equilibrium labor market tightness would be very low and the amplification mechanism relies on highly efficient matching. This is another reason for why some degree of wage rigidity is essential for generating large unemployment volatility in standard matching models.

5 Concluding Remarks

This paper used variants of search and matching models to show that the fundamental surplus as a concept remains insightful. One simply has to shift from looking at the fundamental surplus fraction towards looking at the elasticity of the fundamental surplus of which the fundamental surplus fraction is one of the determinants. When all deductions that enter the fundamental surplus are acyclical, and the fundamental surplus does not involve endogenous variables, then the elasticity of the fundamental surplus coincides with the inverse of the fundamental surplus fraction. When all deductions are proportional to productivity, the elasticity of the fundamental surplus is no larger than one, and wages become fully flexible, meaning that the standard search and matching model loses its mechanism to generate large responses of unemployment to productivity changes. In this sense some degree of wage rigidity (i.e., $\eta_{w,y} < 1$) is essential to ensure that the fundamental surplus channel works in full force.

Technical Appendix

Elasticity of Market Tightness in Hall and Milgrom's (2008) AOB Model

The equilibrium expression for market tightness can be written as

$$\frac{1-\beta_1-\beta_4\tau_1(\theta)}{1-\beta_4\tau_1(\theta)}y = \frac{\beta_2+\beta_4\tau_2(\theta)}{1-\beta_4\tau_1(\theta)}\bar{z} + \frac{\beta_3}{1-\beta_4\tau_1(\theta)}\gamma + \frac{r+s}{q(\theta)}c. \tag{A.1}$$

Totally differentiating equation (A.1) with respect to y and θ, using $f(\theta) = \theta q(\theta) = m\theta^{1-\alpha}$, we have

$$\eta_{\theta,y} = \frac{(1-\beta_4\tau_1(\theta))(1-\beta_1-\beta_4\tau_1(\theta))y}{\beta_1\beta_4\tau_3(\theta)y + \beta_4[(\beta_2+\beta_4\tau_1(\theta)+\beta_4\tau_2(\theta)-1)\tau_3(\theta)]\bar{z} + \beta_3\beta_4\tau_3(\theta)\gamma + \frac{\alpha(r+s)(1-\beta_4\tau_1(\theta))^2}{q(\theta)}c}, \tag{A.2}$$

where

$$\tau_3(\theta) = \frac{\beta(1-\alpha)[1-\beta(1-s)]f(\theta)}{(1-\beta)[1-\beta(1-s-f(\theta))]^2}.$$

Using equation (A.1) to substitute for $(r+s)/q(\theta)$ in the denominator of equation (A.2), after rearranging terms, equation (A.2) becomes:

$$\eta_{\theta,y} = \Upsilon^{AOB}\frac{y}{y-\tau_z\bar{z}-\tau_\gamma\gamma}, \tag{A.3}$$

where

$$\Upsilon^{AOB} = \frac{(1-\beta_4\tau_1(\theta))(1-\beta_1-\beta_4\tau_1(\theta))}{\alpha(1-\beta_4\tau_1(\theta))(1-\beta_1-\beta_4\tau_1(\theta))+\beta_1\beta_4\tau_3(\theta)}$$

$$\tau_z = \frac{\alpha(1-\beta_4\tau_1(\theta))(\beta_2+\beta_4\tau_2(\theta)) + \beta_4(1-\beta_2-\beta_4\tau_1(\theta)-\beta_4\tau_2(\theta))\tau_3(\theta)}{\alpha(1-\beta_4\tau_1(\theta))(1-\beta_1-\beta_4\tau_1(\theta))+\beta_1\beta_4\tau_3(\theta)}$$

$$\tau_\gamma = \frac{\alpha\beta_3(1-\beta_4\tau_1(\theta))-\beta_3\beta_4\tau_3(\theta)}{\alpha(1-\beta_4\tau_1(\theta))(1-\beta_1-\beta_4\tau_1(\theta))+\beta_1\beta_4\tau_3(\theta)}.$$

An alternative way to compute the coefficients τ_z and τ_γ is using equation (14) and (24). According to equation (24), $p_z(\theta) = (\beta_2+\beta_4\tau_2(\theta))/(1-\beta_1-\beta_4\tau_1(\theta))$,

$p_\gamma(\theta) = \beta_3/(1 - \beta_1 - \beta_4\tau_1(\theta))$ and $\Gamma(\theta) = (r + s)(1 - \beta_4\tau_1(\theta))/[q(\theta)(1 - \beta_1 - \beta_4\tau_1(\theta))]$. It is easy to verify that

$$\Upsilon^{AOB} = 1/\eta_{\Gamma,\theta}$$

$$\tau_z = [1 - \Upsilon^{AOB}\eta_{p_z,\theta}]p_z(\theta)$$

$$\tau_\gamma = [1 - \Upsilon^{AOB}\eta_{p_\gamma,\theta}]p_\gamma(\theta).$$

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