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Alessandro Flamini* Costas Milas†

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1 Introduction

A pervasive feature of monetary policymaking is the uncertainty about the state of the economy, the economy's structure and the inferences that the public will draw from policy actions or economic developments (Bernanke 2007). Policymakers have long recognized that coping with these forms of uncertainty poses complicated issues in real-time monetary decisions. Uncertainty on the output gap, for example, corresponds to uncertainty on where the economy is located with respect to the business cycle. Thus real-time decisions, which are necessarily based on output gap estimates, can turn out to be wrong from an ex-post perspective. This problem is well documented by Orphanides and van Norden (2002) who show that the reliability of the output gap measure is quite low. Uncertainty on the structure of the economy is another key cause of policy errors. This uncertainty stems from the limited knowledge of the critical forces that govern the economic system at any point in time. Clearly, the more these forces are missed in modeling the economy, the larger the policy errors. The problem raised by the uncertainty on the state and structure of the economy is further aggravated by the difficulty to relate observable changes in the volatility of key state variables –as inflation and output gap– to these sources of uncertainty. As noted by Bernanke (2007), "Apart from issues of measurement, policymakers face enormous challenges in determining the sources of variation in the data."

This paper studies optimal monetary policy under three fairly realistic assumptions. The first is that in real-time policymakers do not know with certainty the exogenous disturbances to the economy and the parameters of their model. Furthermore, they are unable to distinguish the impact of these sources of uncertainty on the inflation and output gap processes. Under these assumptions some natural questions arise. Given the importance that modern monetary policy attributes to the distribution forecast of inflation and output gap for policy actions, should the difficulty to tell in real-time the impact of the various sources of uncertainty on these variables be considered by policymakers? And to what extent, if any, and how perceived changes in the level of undistinguishable uncertainty should be considered? Given the current economic conditions, these are timely questions. The US, for instance, witnessed average real-time output gap volatility rising from 2.1% per annum over the 1997-2006 period to 2.6% per annum afterwards, and average GDP price inflation volatility rising from 0.3% per annum in 1997-2006 to 0.5% per annum afterwards.

To address these questions, we introduce a final assumption. We let policymakers use their judgment to form an opinion on the level of exogenous volatility surrounding the demand and supply side of the economy, which can be interpreted as the perceived level of exogenous volatility for the inflation and output gap processes. Then, policymakers take this volatility as a proxy for the level of undistinguishable uncertainty in the exogenous disturbances and the model parameters.

These assumptions get reflected in a stylized two-step representation of the central bank decision process. First, central banks estimate at a certain time frequency the state and model of the economy being aware that the estimates are surrounded by uncertainty. Second, at a higher frequency, policymakers assess the amount of uncertainty of the estimates using as proxies of the undistinguishable uncertainty the exogenous volatility of the inflation and output gap processes and take better-informed policy decisions.

Methodologically, the analysis is performed by introducing a New-Keynesian model that conveniently accounts for undistinguishable uncertainty in the state and structure of the economy. We then estimate the model and use the Markov Jump-Linear-Quadratic approach developed by Svensson and Williams (2007) to study how optimal monetary policy responds to changes in the exogenous output gap and inflation volatilities as proxies of uncertainty about the state and structure of the economy. Finally, we test the theoretical results on US data.

Previous literature has studied optimal monetary policy in presence of multiplicative uncertainty. The initial idea that model uncertainty requires some cautiousness in policy decisions, as suggested by the Brainard's (1967) seminal contribution, has been revised by Söderström (2002) who first shows, in a backward-looking model, that in presence of uncertainty on the inflation persistence monetary policy should be more aggressive. Kimura and Kurozumi (2007) extend the analysis to a New-Keynesian model with a hybrid Phillips curve and where the uncertainty on the inflation persistence makes also uncertain the social welfare loss that the central bank wants to minimize. The results of Kimura and Kurozumi are in line with Söderström's work and both papers mark an important deviation from the conventional wisdom originated by Brainard's article and corroborated by the subsequent literature[1]. Our contribution, with respect to this line of research, lies in studying optimal monetary policy when the central bank considers the undistinguishable uncertainty on the model parameters and the exogenous disturbances proxied by the perceived level of uncertainty surrounding the demand and supply sides

[1] See, among others, Estrella and Mishkin (1999), Hall et al. (1999), Martin and Salmon (1999), Svensson (1999), Sack (2000).

of the economy. Within a Markov jump linear quadratic framework, this paper shows that when the exogenous volatility surrounding a state variable increases, the optimal policy response to that variable should increase too, while the optimal response to the remaining state variables should attenuate or be unaffected. Empirically, when a test is carried out on the US economy the model predictions tend to be consistent with the data. This result matters for policy in that when policymakers have real-time limited information on the sources of uncertainty, nonetheless perceive shifts in the relative volatility of inflation and output gap, they may exploit this information to fine-tune the policy response to the state variables.

The remaining of the paper is structured as follows. Section 2 introduces the theoretical model. In section 3 we estimate the aggregate demand and supply, (henceforth AD and AS) using US data. In section 4 we use the estimates of the AD and AS along with a standard calibration for the central bank preferences to find the optimal monetary policy when central bank decisions consider the volatility level on the inflation and output gap process. Our theoretical predictions are then tested on US monetary policy data. Section 5 provides some robustness analysis and Section 6 summarizes our findings and offers some conclusions.

2 Theoretical model

The behavior of the private sector is captured by a New-Keynesian model with realistic monetary policy transmission lags and inertia in the behavior of households and firms. The model is derived from microfoundations[2] and is similar in spirit to Boivin and Giannoni (2006) and Flamini (2007). Specifically, while it shares with both a special attention in describing accurately the dynamics of the private sector, it mainly differs from the former in considering optimal monetary policy instead of a forward-looking Taylor rule and from the latter in focusing on a closed economy rather than an open economy. Furthermore, the current model finds the optimal monetary policy in presence of multiplicative uncertainty following the Markov-jump linear quadratic approach developed by Svensson and Williams (2007). The AD and AS are respectively described by the following relations

$$y_{t+1} = \left[\alpha_y y_t + (1-\alpha_y)\, y_{t+2|t} - \alpha_r r_t + \varepsilon^y_{t+1}\right] \phi^y_{t+1}, \tag{1}$$

$$\pi_{t+2} = \left[\beta_\pi \pi_{t+1} + (1-\beta_\pi)\, \pi_{t+3|t} + \beta_y y_{t+2|t} + \varepsilon^\pi_{t+2}\right] \phi^\pi_{t+2} \tag{2}$$

[2]See Appendix A for details.

where for any variable x the expression $x_{t+\tau|t}$ denotes the rational expectation of x in period $t+\tau$ conditional on the information available in period t, all the variables are in terms of log deviations from constant steady state values, y_t, r_t, π_t denote output gap, real short term interest rate, and inflation rate, respectively, with $r_t \equiv i_t - \pi_{t+1|t}$ and i_t denoting the short term nominal interest rate, and finally, ε^y and ε^π are exogenous disturbances allowing the economy to depart from the steady state.

The presence of the factors ϕ_t^y and ϕ_t^π is an innovation with respect to the previous literature. ϕ_t^y and ϕ_t^π are random variables capturing the uncertainty on the state and structure of the economy assuming that in real-time it is impossible for the central bank to distinguish the impact of these sources of uncertainty on the volatility of the output gap and inflation processes. This assumption is motivated by the fact that in real-time policymakers face difficulties in observing specific disturbances hitting the economy and have limited knowledge on their properties too. Furthermore, the true model of the economy is not known with certainty so that even if there were full knowledge of the disturbances, difficulties would arise in nowcasting and forecasting their impact on the economy.

To model the assumption of undistinguishable uncertainties, we let ϕ_t^y and ϕ_t^π have a symmetric distribution, expected value equal to one and variance proportional to the exogenous volatility of the inflation and output gap processes as we will discuss below. At any point in time, a value of the factors different from one can capture uncertainty on the stochastic properties of the exogenous disturbances hitting the economy and/or uncertainty in the central bank estimates of the model parameters. Regarding the former, an example can be a preference or a technology shock which occurred in the previous period and that turns out to be more persistent than expected. On the other hand, uncertainty in the estimates of the model parameters can be caused by their possible time varying nature which is missed by the policymakers[3]. In this case, ϕ_t^y and ϕ_t^π record general model uncertainty, that is, uncertainty that stems from the structure of the model and that is impossible to attribute to specific parameters in real-time.

While the coefficients of the AS and AD can be estimated at a fixed frequency, the information flow accessible to central bank and relevant for policy decisions is continuous and sometimes not apt for a direct use in the estimation process. Nevertheless, this information flow can be useful for policymakers to form an opinion on the variance of ϕ_t^y and ϕ_t^π. To model this idea we as-

[3] See Rubio-Ramirez and Villaverde (2007) for empirical evidence in favour of parameter drifting in DSGE models and the literature therein for empirical evidence on time varying parameters in dynamic models.

sume that policymakers use all the available information and their judgment to form an opinion on the exogenous volatility of the inflation and output gap processes. Then, they consider this volatility as proxy for the variance of ϕ_t^y and ϕ_t^π in the determination of the optimal monetary policy.

Within this framework, the central bank optimization problem consists of finding the interest rate path that maximizes its preferences subject to the AD and AS and to the opinion on the level of uncertainty proxied by the exogenous volatility of the inflation and output gap processes. Turning to central bank preferences, they are described by the following standard loss function

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\mu \pi_{t+\tau}^2 + \lambda y_{t+\tau}^2 + \nu \left(i_{t+\tau} - i_{t+\tau-1} \right)^2 \right], \tag{3}$$

where μ, λ and ν are weights that express the preferences of the central bank for the inflation and output gap stabilization targets, and the instrument smoothing target, respectively. Regarding the interest rate smoothing preference for a central bank that pursues inflation targeting see, for example, Svensson (2010), p. 2, Holmsen et al. (2008), and Woodford (2003).

2.1 Optimal monetary policy with Non-certainty Equivalence

In our analysis we let the central bank consider undistinguishable uncertainty via multiplicative shocks. On the one hand, the use of a multiplicative rather than additive shock is consistent with the multiplicative nature of the uncertainty surrounding the structural parameters and the properties of the exogenous disturbances. On the other hand, it provides a convenient way of investigating optimal monetary policy in presence of uncertainty. It is well known, in fact, that the linear-quadratic setup features certainty equivalence, which implies that optimal monetary policy does not depend on additive uncertainty. Instead, considering ϕ_t^y and ϕ_t^π as multiplicative shocks we can relax the certainty equivalence assumption and study how uncertainty affects the optimal policy. Relaxing the certainty equivalence assumption is also important in that allows us to introduce a key realistic aspect of monetary policy, namely the policymakers' focus on the distribution forecasts of the target variables rather than the mean forecasts.

We follow this route by assuming a discrete support for ϕ_t^y and ϕ_t^π and that in any period these shocks can take n_j different values corresponding to n_j exogenous modes drawn by nature and indexed by $j_t \in \{1, 2, ..., n_j\}$. Thus ϕ_t^y and ϕ_t^π correspond to $\phi_{j_t}^y$ and $\phi_{j_t}^\pi$, respectively. Then, the Markov Jump-Linear-

Quadratic approach developed by Svensson and Williams (2007a) and further discussed in Svensson and Williams (2007b) is adopted to solve the central bank optimization problem in presence of multiplicative shocks[4]. Thus, we let the mode j_t follow a Markov process with constant transition probabilities and start by assuming that each transition probabilities has the same value

$$P_{jk} \equiv \Pr\{j_{t+1} = k | j_t = j\} = \frac{1}{n_j}, \qquad \forall \;\; j,k \in \{1,2,...,n_j\}. \tag{4}$$

Next, let $P \equiv [P_{jk}]$ be the Markov transition matrix and $p \equiv \left(p_{1t},...,p_{n_j t}\right)'$ (with $p_{jt} \equiv \Pr\{j_t = j\}$) the central bank's subjective probability distribution over the modes in period t. We now assume that the central bank does not observe the modes and that does not update its subjective distribution of modes through the observation of the economy[5]. Thus its subjective distribution $p_t \equiv \left(p_{1t},...,p_{n_j t}\right)'$ evolves according to the exogenous transition probabilities, that is

$$p_{t+\tau} = (P')^{\tau} p_t, \qquad \tau \geq 0. \tag{5}$$

As to the central bank knowledge before choosing the instrument i_t at the beginning of period t, the information set consists of the transition matrix P, the central bank's subjective probability distribution over the modes in period t and subsequent periods via (5), the n_j different values that each of the matrices can take in any mode and, finally, the realizations of X_t.

Given (4), the unique stationary distribution of the modes associated with the Markov transition matrix P is a uniform distribution. Thus the transition probabilities described by (4) capture the case of generalized modes uncertainty in which modes are serially i.i.d. The motivation to consider this case lies in the interest of studying optimal monetary policy when the central bank has a minimal knowledge on the multiplicative shocks, specifically it only knows their bands and considers any realization as equally likely. In section 5, to check for the robustness of the results, we explore an alternative assumption where the modes exhibit some persistence. We now assume that there are $n_j = 5$ modes so that the stationary distribution for the shock ϕ^h_{jt},

[4]Our analysis follows the approach presented in Svensson and Williams (2007a).

[5]Svensson and Williams (2007a) base the no-learning perspective on the *forgetting the past* assumption according to which, when the central bank and the private sector make decisions in period t, they forget past observations of the economy so that they cannot use current observations to update their beliefs.

for $h = y, \pi$, is given by

ϕ^h_{jt}	$\left(1 - 2\delta_{\phi^h}\right)$	$\left(1 - \delta_{\phi^h}\right)$	1	$\left(1 + \delta_{\phi^h}\right)$	$\left(1 + 2\delta_{\phi^h}\right)$
$\Pr\left(\phi^h_{jt}\right)$	0.2	0.2	0.2	0.2	0.2

with the parameter δ_{ϕ^h} allowing to change the variance of the distribution. Moreover, we assume that the multiplicative shocks and the additive exogenous disturbances to the economy are independent so that modes j_t and innovations ε_t are independently distributed[6].

Summing up, the central bank cannot observe the modes yet find the optimal monetary policy

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\left(\rho_\pi \pi_{t+1|t} + \rho_y y_t\right) \tag{6}$$

taking into account the volatility of the multiplicative shocks through its information set[7].

3 Model estimation

To choose the coefficients of the AD and AS equations (1) and (2), we jointly estimate these and the monetary policy equation in (3) using US data for the period 1969Q4-2009Q2. We use the Effective Federal Funds rate as the nominal interest rate. Inflation is the annual proportional change in the GDP price index. For inflation, we (i) use real-time median forecasts of GDP price inflation obtained from the Survey of Professional Forecasters (SPF) database maintained by the Federal Reserve Bank of Philadelphia, and (ii) replace GDP price inflation forecasts with their actual values. The output gap is proxied by (i) real-time output detrended by a quadratic trend, (ii) real-time output detrended by a Hodrick-Presott (1997) trend, (iii) final output detrended by the Congressional Budget Office (CBO) measure of potential GDP, and (iv) final-time output detrended by a Hodrick-Presott (1997) trend. To construct the real-time output gap data, we estimate for each quarter both a quadratic and a Hodrick-Prescott trend using real-time output data available in that

[6] The driving forces in this economy are the the additive disturbances ε_t and the multiplicative shocks modeled through the modes j_t. Diferently from multiplicative shocks, the presence of additive disturbances does not have any effect on the optimal monetary policy due to certainty equivalence. Nonetheless, in the New-Keynesian model where variables denote log-deviations from steady state values, the role of additive disturbances is to allow the economy departing from the steady state equilibrium.

[7] Details of the application of the Svensson and Williams approach are reported in Appendix B.

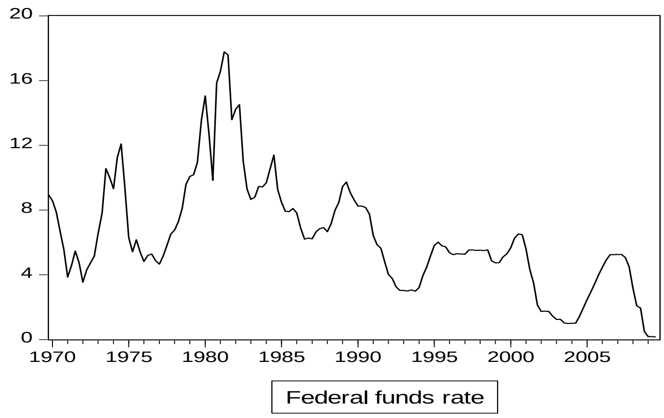
quarter. The output gap for the quarter is the end-of-sample residual from that quadratic trend and Hodrick-Prescott (HP) trend regressions, respectively. This means, for example, that in constructing the output gap data for the period 1969Q4 to 2009Q2 (159 quarters), we re-estimated 159 regressions[8]. To tackle the end-point problem in calculating the HP trend (see Mise et al, 2005a,b), we applied an autoregressive AR(n) model (with n set at 4 to eliminate serial correlation) to each of the real-time and final-time output measures. The AR model was used to forecast six additional quarters that were then added to each of the series before applying the HP filter. Figure 1 plots the federal funds rate and the different measures of inflation and the output gap. The federal funds rate is higher during the 1970s and early 1980s and reaches its lowest level following the 2007-2009 financial crisis. Inflation is higher during the 1970s and early 1980s; it also rises towards the end of our sample. Compared to the other output gap measures, real-time output detrended by a quadratic trend filter suggests a more severe downturn in the mid-1970s and following the 2007-2009 financial crisis. The two HP output gap measures move very much in line with each other (these have a correlation of 0.88). Empirical estimates using these two measures produced qualitatively similar results. In what follows, and in order to save space, we only report estimates using the real-time HP output gap measure (results based on the final-time HP output gap measure are available on request).

Equations (1), (2) and (3) are estimated jointly using the continuously updated Generalized Methods of Moments (GMM) estimator of Hansen et al (1996) with a constant, four lags of the interest rate, four lags of inflation and four lags of the output gap as instruments for all three equations. Column (i) of Table 1 reports real-time estimates with output detrended by a quadratic trend and with SPF inflation forecasts; column (ii) reports real-time estimates with output detrended by a Hodrick-Prescott trend and with SPF inflation forecasts, and column (iii) reports final estimates with output detrended by the CBO measure of potential output and with inflation forecasts replaced by actual inflation values. We also report the system's J statistic which tests the validity of the instruments used (Hansen, 1982). We estimate that the weight on inflation ranges from 1.53 to 2.12; the weight on the output gap ranges from 1.07 to 1.56 and the persistence parameter ranges from 0.83 to 0.90. Our estimates indicate a more aggressive response of policymakers to inflation and the output gap using real-time as opposed to final data.

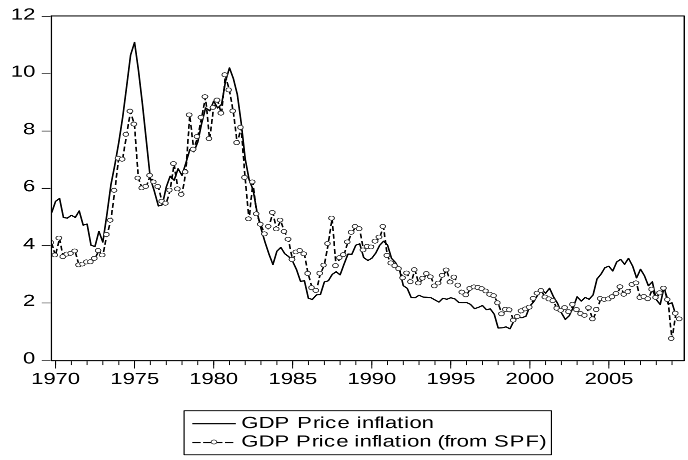
[8] Preliminary unit root analysis (results are available on request) indicated that the output gap is stationary whereas the order of integration of the interest rate and inflation is more ambiguous; we assume that all variables are stationary (see e.g. Clarida et al, 2000, for a discussion of similar issues).

Figure 1: Federal funds rate, inflation and output gap data

(a) Federal funds rate



(b) Inflation measures



(c) Output gap data

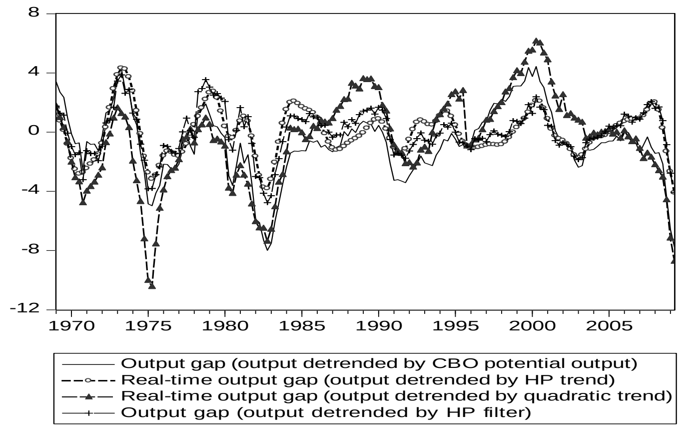


Table 1: Simple Taylor rule model estimates using GMM

Sample: 1969Q4-2009Q2

	(i)	(ii)	(iii)
Interest rate equation			
ρ_i	0.83 (0.03)	0.90 (0.03)	0.89 (0.02)
ρ_π	2.12 (0.31)	1.98 (0.50)	1.53 (0.30)
ρ_y	1.16 (0.26)	1.56 (0.85)	1.07 (0.47)
Adjusted R^2	0.92	0.90	0.93
Regression SE	0.94	0.95	0.89
Output gap equation			
α_y	0.53 (0.03)	0.51 (0.01)	0.54 (0.02)
α_r	0.05 (0.02)	0.05 (0.01)	0.06 (0.02)
Adjusted R^2	0.95	0.85	0.96
Regression SE	0.53	0.67	0.52
Inflation equation			
β_π	0.91 (0.04)	0.83 (0.03)	0.51 (0.02)
β_y	0.11 (0.03)	0.06 (0.01)	0.03 (0.01)
Adjusted R^2	0.97	0.95	0.98
Regression SE	0.42	0.38	0.20
J stat	[0.35]	[0.32]	[0.35]

Notes: All models include an intercept term; estimates of this are not reported.
(i): Real-time estimates. These use output detrended by a quadratic trend.
(ii): Real-time estimates. These use output detrended by a Hodrick-Prescott trend.
(iii) Final estimates. These use output detrended by the CBO measure of potential output.

Numbers in parentheses are the standard errors of the estimates. J stat is the p-value of a chi-square test of the system's overidentifying restrictions (Hansen, 1982). The instruments are a constant, four lags of the interest rate, inflation and the output gap.

The inflation estimates are in line with other results in the literature (e.g. Judd and Rudebusch, 1998, Clarida et al, 2000, Castelnuovo, 2003, Alcidi et al. 2009, and Martin and Milas, 2009) and satisfy the Taylor (1993) principle that excessive inflation should trigger increases in the real interest rate[9].

Our estimates suggest that the weight on past output in the aggregate demand equation ranges between 0.51 and 0.54 whereas the weight on past inflation in the aggregate supply equation ranges between 0.51 and 0.91. The estimates of the AS imply that backward looking inflation effects are more important than forward looking ones. This is in line with Rudd and Whelan (2005) and Linde (2005) but contradicts e.g. Gali and Gertler (1999), Gali et al (2005) and Kim and Kim (2008). The model with final data fits monetary policy and the AD and AS equations best as it delivers the lowest regression standard error and the highest adjusted R2. This finding is consistent with other evidence that policymakers do not respond to real-time output data, for example in the context of fiscal policy (see for example IMF, 2008, chapter 5). One interpretation of this finding is that estimates of policy rules based on final data may be misleading since they assume a policy response to data policymakers did not possess at the time (e.g. Orphanides and van Norden, 2005). An alternative interpretation is that policymakers do not in fact place that much weight on real-time data, which according to Adam and Cobham (2004), does not correspond "precisely to what researchers would like - the output gap as understood at the time by policymakers - which seems nearly impossible to identify".

Finally, it is worth making the following points. First, since the system is estimated jointly, the performance of the different variants of the policy rule depends on the fit of the other equations in the model. Second, and perhaps more importantly, the fit of the estimated equations is contingent on having allowed for the smoothing interest rate coefficient to vary between the different specifications in Table 1. Third, due to endpoint problems with the HP filter, specification (i) is much more trustworthy than specification (ii). However, as mentioned above, we have tried to hedge against this problem by extending the GDP series using the AR forecasts before applying the HP filter.

[9] Gerberding et al (2005) and Gerdesmeier and Roffia (2005) find that the use of real-time output data as opposed to final output data increases the output effect in the Taylor rule for the Bundesbank and the EU area, respectively. A possible explanation is that the magnitude of the response using revised data could suffer from downward bias owing to the errors-in-variables problem.

4 Optimal monetary policy response to output gap and inflation uncertainty

This section starts by presenting the theoretical predictions of the model considering the separate and joint impact of the exogenous level of inflation and output gap volatility on the optimal monetary policy. It then tests the model empirically using US data.

4.1 Theoretical predictions of the model

We choose the parameter values for the AD and AS equations based on the final-time estimates reported in column (iii) of Table 1 (which fit the data best). We then compute the optimal monetary policy response to changes in the volatility of the inflation and output gap processes under the assumption, common in the literature, that the central bank pursues flexible inflation targeting and wishes to smooth the interest rate path (that is, $\mu = 1$, $\lambda = 0.1$ and $\nu = 0.2$)[10].

In our model the vector of state variables is given by $X_t = \left(\pi_t,\ \pi_{t+1|t},\ y_t,\ i_{t-1}\right)'$ so that due to the Markov Jump-Linear-Quadratic setup, the optimal monetary policy, for a given value of multiplicative uncertainty, is a linear function of X_t[11]. Since the optimal coefficient for current inflation, π_t, is always zero, the optimal policy turns out to depend only on $\left(\pi_{t+1|t},\ y_t,\ i_{t-1}\right)$ as anticipated in (6). Furthermore, in order to allow comparisons of the coefficient for inflation and output-gap with much of the earlier literature, we present the results of the analysis in terms of the long-run optimal monetary policy

$$i_t = \rho_\pi \pi_{t+1|t} + \rho_y y_t,$$

and discuss separately at the end of section 4.1.1. the relation between uncertainty and the optimal policy response to the lagged interest rate[12].

Figure 2 reports the coefficients of the optimal monetary policy for increasing values of the standard deviations of $\phi^y_{\jmath t}$ and $\phi^\pi_{\jmath t}$, i.e. σ_y and σ_π. The range of the standard deviations has been chosen in order to obtain a realistic mea-

[10] We focus on flexible inflation targeting as, in practice, no inflation targeting central bank pursues strict inflation targeting.

[11] Although $\pi_{t+1|t}$ is an expectation, in the model with a two-period policy transmission lag to affect inflation this variable is predetermined and thus belongs to the vector of the state variables. This can be easily seen by lagging the aggregate supply one period and then taking the expectation at time t. In section 5, we relax the assumption of policy transmission lags and consequently $\pi_{t+1|t}$ is no longer a state variable.

[12] Given the short-run policy (6), the long-run policy turns out to be $i_t = \rho_\pi \pi_{t+1|t} + \rho_y y_t$.

sure of the volatility for the output and inflation one-period ahead forecast errors. As to the former, we assumed that the upper bound of the standard deviation of the shock to the output gap is 0.4. It follows that, in any period t, due to the presence of uncertainty captured by the factor $\phi^y_{j_t}$, actual output gap may differ from the one-period ahead forecast of, at the most, 56 percentage points of its value[13]. For example, if the expected value of the output gap based on the period t information set is 1%, then its actual value in period $t+1$ may shift at the most to 0.44% or 1.56%. This range is in line with the low reliability of output gap estimates in real-time discussed for example by Orphanides and van Norden (2002) who find that ex-post revisions of the real-time output gap estimates can be of the same order of magnitude as the output gap itself. Turning to inflation, we let $\sigma_\pi \in [0,\ 0.175]$ so that in any period t actual inflation may differ from the one-period ahead forecast of, at the most, 25 percentage points of its value. This level of the variability of inflation is consistent with the view that forecasting errors related to inflation estimates tend to be less than half the ones related to output gap estimates[14].

Describing our findings, first Figure 2 (a, b) shows that the optimal response to a state variable increases in the level of exogenous volatility surrounding that variable. Since the admissible range of output gap volatility is more than two times as large as the one of inflation volatility, this increase is remarkably large for the optimal response to the output gap in presence of output gap volatility.

To gain the intuition for this result it is useful to recall that relaxing the certainty equivalence assumption the central bank objective is to have the distribution forecasts of inflation and the output gap that "look good". In this model, the quality of the distribution forecasts, measured for instance by their volatility, is negatively affected by two factors: the perceived exogenous volatility in the inflation and output gap processes and the distance of the economy from its long-run equilibrium. While the role played by the former is evident, to see the role played by the latter consider that due to the multiplicative nature of the shock capturing the exogenous volatility in the inflation and output gap processes, the further away is the economy from the long-run equilibrium, the larger is the impact of the shock for a given level of its volatility. Now, when there is an exogenous increase in the volatility of the shocks, the length of the potential deviations of the economy from its steady state increases and may combine with the second factor in a self-enforcing cycle

[13]Given the uniform distribution for ϕ^y_j reported before, when $\sigma_y \in [0,\ 0.4]$ it follows that $\phi^y_j \in [0.44,\ 1.56]$.

[14]See Sims (2002) for the root mean square errors of several US inflation and output growth estimates including the Green Book ones.

that quickly deteriorates the distribution forecasts. To take this contingency into account monetary policy needs to move preemptively. It thus gets more reactive in presence of an increase in the volatility of the inflation and output gap.

Figure 2 also shows that the optimal response to inflation is inversely related to output gap volatility, whereas the optimal response to the output gap tends to be unrelated to inflation volatility; see panels (a) and (b) respectively. This symmetry breaking occurs as output gap volatility introduces a trade-off between the quality of the distribution forecasts for inflation and the output gap. In contrast, the presence of inflation volatility does not introduce a similar trade-off. To see the mechanism at work, let us consider the behavior of the central bank in two different scenarios. In the first one, inflation deviates from its long-run value, i.e. $\pi \neq 0$, in presence of output gap volatility; in the second one, the output gap deviates from its long-run value, i.e. $y \neq 0$, in presence of inflation volatility. In the first case, the policymakers' attempt to stabilize inflation requires a deviation of the output gap from its long-run value. This implies a loss for the central bank. However, the presence of output gap volatility now adds a further loss as it makes it potentially harder to take the output gap back to the equilibrium in the subsequent periods. Thus, policymakers will trade-off slower inflation stabilization for smaller deterioration of the output gap distribution forecast. This gets reflected in an attenuation of the policy response to inflation. Conversely, when the output gap deviates from its long-run value and the central bank faces inflation uncertainty, the attempt to stabilize the output gap will not require a deviation of inflation from its long-run value. On the contrary, by stabilizing the output gap, policymakers prevent output gap deviations from perturbing inflation via the Phillips curve. This explains why the policy response to the output gap is not affected by inflation uncertainty.

The result that accounting for uncertainty can lead to a more aggressive policy has precedents in the literature. Söderström (2002), with a backward-looking model found that the policy responsiveness to inflation increases with the uncertainty on the persistence of this variable. Extending the analysis with a microfounded forward-looking model where policymakers minimize a social welfare loss, Kimura and Kurozumi (2007) confirmed this result. The current paper, abstracting from specific sources of uncertainty considered in previous contributions, suggests that a more aggressive policy response to a state variable should occur when the exogenous *overall volatility* surrounding that variable increases. It is worth noting that, in real-time, information on changes in specific sources of uncertainty seems more difficult to gather than general information on variations in the uncertainty of the output gap and inflation processes. Thus, the previous result matters for policy design in that it unveils the utility of limited information on the general uncertainty in the output gap and inflation processes when central banks cannot rely on detailed information on the uncertainty sources.

We also note that a distinct implication of the paper's multiplicative set-up is that the forecasting precision is negatively correlated with the size of the output gap and the level of inflation as implied by the multiplicative specification of the IS equation and the New Keynesian Phillips Curve. In other words, the further the economy departs from the steady state, the more difficult it becomes to forecast the evolution of the economy. One practical way of assessing this is to compare, for instance, the Bank of England's fan chart for GDP growth in the pre-crisis Inflation Report of February 2007 with that at the height of the recession in the Inflation Report of August 2009 (both reports are available from the website of the Bank of England). The former forecasts GDP growth in the (0.75%, 4.75%) range whereas the latter forecasts a much wider GDP growth range of (-1%, 5.5%). This highlights the difficulty in forecasting the further away the economy moves from equilibrium.

4.1.1 Optimal policy responses with both inflation and output gap volatility

So far we have considered the case in which the exogenous volatility surrounding one process (either the output gap or inflation) increases as the volatility surrounding the other process is constant. While investigating this scenario has been instructive to see the specific contributions of σ_π and σ_y on the optimal policy, a more realistic one occurs when both processes are surrounded by exogenous uncertainty and, in real-time, policymakers perceive shifts in the volatility of one process relative to the volatility of the other process.

To study this case, we introduce an output gap relative volatility ratio and investigate how the optimal policy reacts to changes in this ratio. We define the output gap relative volatility ratio as $\sigma_y / (\sigma_y + \sigma_\pi)$ and impose the following inverse relation between σ_y and σ_π

$$\sigma_y = \overline{\sigma}_y - \frac{\overline{\sigma}_y}{\overline{\sigma}_\pi}\sigma_\pi,$$

where $\overline{\sigma}_y = 0.4$ and $\overline{\sigma}_\pi = 0.175$ refer to the upper bound for the volatility of the output gap and inflation processes, respectively. Thus

$$\frac{\sigma_y}{\sigma_y + \sigma_\pi} = \frac{1}{\left(1 - \frac{\overline{\sigma}_y - \overline{\sigma}_\pi}{\overline{\sigma}_y \overline{\sigma}_\pi}\sigma_\pi\right)} - \frac{1}{\overline{\sigma}_\pi}\frac{1}{\left(1 - \frac{\overline{\sigma}_y - \overline{\sigma}_\pi}{\overline{\sigma}_y \overline{\sigma}_\pi}\sigma_\pi\right)}\sigma_\pi.$$

Figure 2: Optimal policy response to y_t and $\pi_{t+1|t}$ in presence of uncertainty on the output gap and inflation processes. Central bank preferences: μ=1, λ=0.1, v=0.2.

(a) output gap uncertainty **(b) inflation uncertainty**

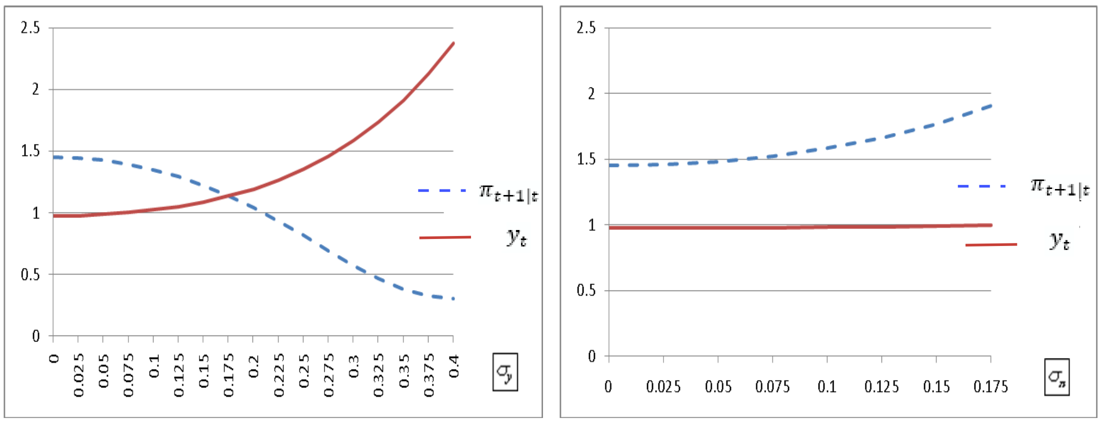
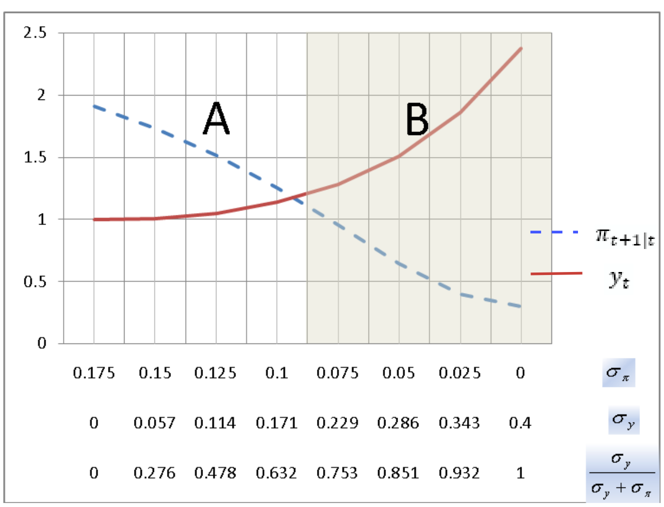


Figure 3: Optimal policy response to y_t and $\pi_{t+1|t}$ in presence of relative output gap uncertainty. Central bank preferences: μ=1, λ=0.1, v=0.2.



It is worth noting that the ratio is equal to zero if and only if there is maximum volatility in the inflation processes and no volatility in the output gap process; on the other hand, it is equal to one if and only if there is maximum volatility in the output gap process and no volatility in the inflation process, that is

$$\frac{\sigma_y}{\sigma_y + \sigma_\pi} = \begin{cases} 0 \ \text{ iff } \ \sigma_\pi = \overline{\sigma}_\pi, \quad \sigma_y = 0 \\ 1 \ \text{ iff } \sigma_\pi = 0, \quad \sigma_y = \overline{\sigma}_y \end{cases} .$$

We also note that the level of overrall exogenous volatility, $\sigma_y + \sigma_\pi$, associated with the relative output gap volatility is comparable to the cases analyzed in the previous section where either exogenous output gap volatility or inflation volatility was considered. In particular, the overall exogenous volatility is at least (at most) equal to the maximum volatility that we had with only exogenous inflation (output-gap) volatility, that is $\sigma_y + \sigma_\pi \in [0.175,\ 0.4]$.

Figure 3 plots the optimal policy coefficients for the output gap and inflation versus the output gap relative volatility ratio. Figure 3 shows that when the volatility in the output gap is inversely related to the volatility in inflation, movements in the volatilities of y and π should be associated with changes in the same direction of the policy response to these variables. Furthermore, by equally splitting the y and π volatility ranges we notice that with more volatility in the inflation process than the output gap process (region A), the optimal monetary policy tends to react more to inflation than to the output gap in a fashion similar to what predicted by the Taylor rule. Instead, in the opposite case (region B), the policy response to the output gap exceeds the one to inflation. Finally, in region B, the spread between the policy responses is remarkably larger than in region A and the response to the less volatile state variable tends to become negligible, while it remains important in region A. This latter response is in line with the breaking symmetry effect previously described, which is caused by the different impact of the y and π volatility on the optimal policy.

Summing up, these findings show that also in the more realistic case in which policymakers face uncertainty in both processes, there is (i) a positive relation between the volatility of a state variable and reactiveness of the associated policy response, and (ii) an asymmetric policy behavior associated with movements in the relative volatility.

It is important to note that the results presented in section 4.1. and 4.1.1 show the optimal long-run policy response to inflation and the output gap taking implicitly into consideration the role played by the optimal response to the lagged interest rate, i.e the degree of optimal policy inertia. When we focus on the short-run optimal policy, which reveals optimal coefficients for $\left(\pi_{t+1|t},\ y_t,\ i_{t-1}\right)$, we have the coefficient for i_{t-1} falling monotonically for increasing values of σ_y and the same qualitative behavior for the response to $\left(\pi_{t+1|t},\ y_t\right)$. Thus with only output-gap uncertainty, or more output gap uncertainty relative to inflation uncertainty, monetary policy becomes less inertial. Since in these two cases the optimal response to the output gap gets larger than the response to inflation, less inertia turns out to amplify more the response to the output gap than the response to inflation. This explains why the difference in these responses is larger in presence of output gap uncertainty[15].

4.2 Empirical application on US monetary policy

The theoretical predictions reported in Figure 3 are now tested on US monetary policy over the 1969Q4-2009Q2 period. We construct the volatility measures σ_{yt}, $\sigma_{\pi t}$, and the relative volatility ratio $\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t$, respectively, by taking the 8-quarter moving standard deviation of inflation and the output gap (results using a 16-quarter moving standard deviation are qualitatively similar). Our measures of volatility are reported in Figure 4. Inflation volatility is greatest in the 1970s, and towards the end of our sample. Output gap volatility declines throughout the 1980s with resurgences in the early 1990s, after the 9/11 terrorist attacks, and following the financial crisis at the end of our sample. We have also considered as measures of shock uncertainty the Root Mean Square Errors for output and inflation instead; empirical results based on these very measure were poor compared to the ones reported below.

To allow for asymmetric volatility effects, we express the monetary policy rule as:

$$
\begin{aligned}
i_t = {} & \rho_i i_{t-1} + (1-\rho_i)\, I_t^{\sigma_y/(\sigma_y+\sigma_\pi)} \left[\rho^+_{\pi,0} + \rho^+_{\pi,1}\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t\right]\pi_{t+1|t} \\
& + (1-\rho_i)\, I_t^{\sigma_y/(\sigma_y+\sigma_\pi)} \left[\rho^+_{y,0} + \rho^+_{y,1}\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t\right] y_t \\
& + (1-\rho_i)\left[1 - I_t^{\sigma_y/(\sigma_y+\sigma_\pi)}\right]\left[\rho^-_{\pi,0} + \rho^-_{\pi,1}\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t\right]\pi_{t+1|t} \\
& + (1-\rho_i)\left[1 - I_t^{\sigma_y/(\sigma_y+\sigma_\pi)}\right]\left[\rho^-_{y,0} + \rho^-_{y,1}\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t\right] y_t
\end{aligned} \tag{7}
$$

[15] Specifically, focusing on the optimal coefficient for the lagged interest rate, we find that with only increasing output gap uncertainty this coefficient falls over the range $[0.66,\ 0.48]$, with only increasing inflation uncertainty it falls over the range $[0.66,\ 0.65]$, and finally with increasing relative output-gap uncertainty it falls over the range $[0.65,\ 0.48]$. To save space we do not plot the response to the lagged interest rate but this is available on request.

where $I_t^{\sigma_y/(\sigma_y+\sigma_\pi)}$ is the indicator function:

$$I_t^{\sigma_y/(\sigma_y+\sigma_\pi)} = \begin{cases} 1, & \text{if } \left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t \geq \left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_{t-1} \\ 0, & \text{if } \left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t < \left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_{t-1} \end{cases} \tag{8}$$

Model (7) using the indicator function (8) differs from the linear Taylor rule model (6) in that it allows for a regime-switching relationship between the interest rate, inflation and the output gap depending on whether there is higher relative volatility in the output gap process (in which case increasing values of $\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t$ are observed) against higher relative volatility in the inflation process (in which case decreasing values of $\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t$ are observed). The response to inflation switches from $\rho^+_{\pi,0} + \rho^+_{\pi,1}\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t$ when there is higher relative volatility in the output gap process to $\rho^-_{\pi,0} + \rho^-_{\pi,1}\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t$ when there is higher relative volatility in the inflation process.

Similarly, the response to the output gap switches from $\rho^+_{y,0} + \rho^+_{y,1}\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t$ to $\rho^-_{y,0} + \rho^-_{y,1}\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t$. We expect higher relative volatility in the output gap process to raise the response of monetary policy to the output gap $\left(\rho^+_{y,1} > 0\right)$ and lower the response of monetary policy to inflation $\left(\rho^+_{\pi,1} < 0\right)$. On the other hand, higher relative volatility in the inflation process should lower the response of monetary policy to the output gap $\left(\rho^-_{y,1} < 0\right)$ and increase the response to inflation $\left(\rho^-_{\pi,1} > 0\right)$. The model in (7) simplifies to a model with symmetric volatility effects if $\rho^+_{y,0} = \rho^-_{y,0}$, $\rho^+_{y,1} = -\rho^-_{y,1}$, $\rho^+_{\pi,0} = \rho^-_{\pi,0}$, and $\rho^+_{\pi,1} = -\rho^-_{\pi,1}$.

To allow for multiplicative uncertainty effects, the model in (7) is estimated jointly with the AD-AS equations (1) and (2) where the crossproduct shocks ϕ^y and ϕ^π are proxied by σ^y and σ^π, respectively. To save space, we only report estimates of (7) in Table 2 (estimates of (1-2) are very similar to those reported in Table 1 and are available on request). We estimate that $\rho^+_{y,1} > 0$, $\rho^+_{\pi,1} < 0$, $\rho^-_{y,1} < 0$, and $\rho^-_{\pi,1} > 0$. These estimates are statistically significant, suggesting that higher (lower) relative volatility in the output gap process raises (lowers) the response of monetary policy to the output gap and lowers (raises) the response to inflation. All three models estimated in columns (i)-(iii) of Table 2 fit the data better than the corresponding models in Table 1. Therefore, both inflation and output gap volatility matter for US monetary policy. Amongst all estimated models, the model with final data (reported in column (iii) of Table 2) delivers the best fit. For this model, the average output gap response drops from 1.61 when there is higher relative volatility in the output gap to 1.48 when there is higher relative volatility in inflation. On the other hand, the average inflation response increases from 1.49 when there is higher relative volatility in the output gap to 1.56 when there is higher relative volatility in inflation. Consistent with Figure 3, the average output gap response is higher (lower) than the average inflation response when there is higher (lower) relative volatility in the output gap process. These average estimates provide some evidence of asymmetries in the response to inflation and the output gap when volatility is considered.

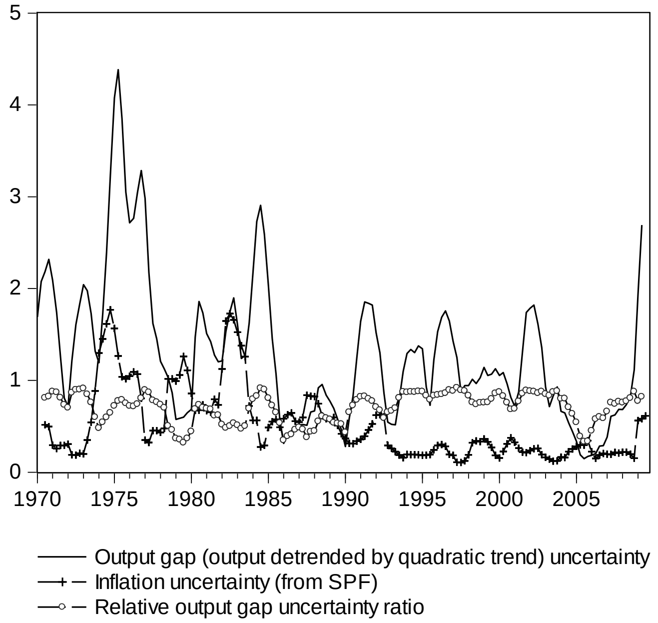
Estimates of the real-time models (in columns (i) and (ii) of Table 2) also suggest that the average inflation response increases when there is higher relative volatility in inflation. On the other hand, the output gap response increases when there is higher relative output gap volatility for the model in column (i) but not for the one in column (ii); the latter finding is arguably due to the $\rho_{y,0}^{+}$ coefficient being imprecisely estimated.

To fix ideas, we discuss the implications of equation (7) for optimal monetary policy in the U.S. using the estimates in column (i) of Table 2. First, consider the case where the U.S. economy experiences a Japanese style deflation with low inflation as well as low inflation volatility in one period followed by an even lower inflation volatility in the next period (such that the relative inflation volatility ratio drops). In this case, the average response of policymakers to inflation is equal to 1.88, whereas the average response to the output gap is equal to 0.90. Contrast the above responses with the case where the U.S. economy experiences a 1970s style increase in inflation volatility, with inflation volatility in one period followed by an even higher inflation volatility in the next period (such that the relative inflation volatility ratio rises). In this case, policymakers become more concerned with inflation and less concerned with output gap fluctuations, as the average response to inflation rises from 1.88 to 2.12, whereas the average response to the output gap drops from 0.90 to 0.85.

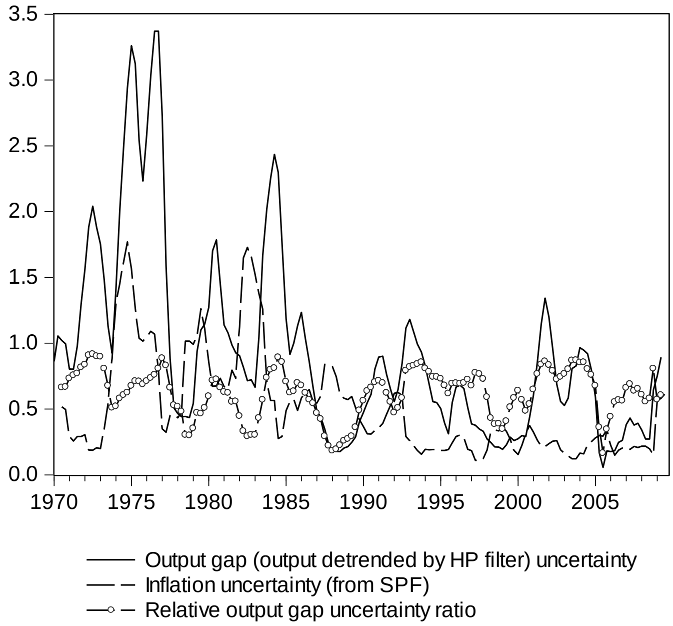
We have also estimated alternative volatility measures based on the structural shocks derived from a Vector Autoregressive (VAR) system with six lags (chosen by the Akaike Information Criterion) in output gap, inflation and the interest rate. Using the above ordering, we have identified the structural output gap, inflation and interest rate shocks using the Cholesky decomposition (for more details of the structural VAR approach see e.g. Amisano and Giannini, 1997). As an alternative, we have augmented the VAR with commodity price inflation (based on the spot price index of all commodities from the Commodity Research Bureau) as an additional variable.

Figure 4: Output gap uncertainty, inflation uncertainty and relative output gap uncertainty ratio

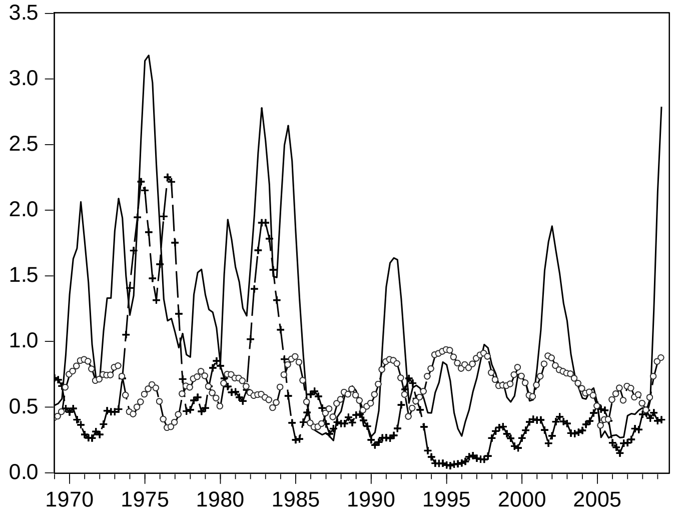
(a) Real-time data, output detrended by quadratic trend



(b) Real-time data, output detrended by Hodrick-Prescott filter

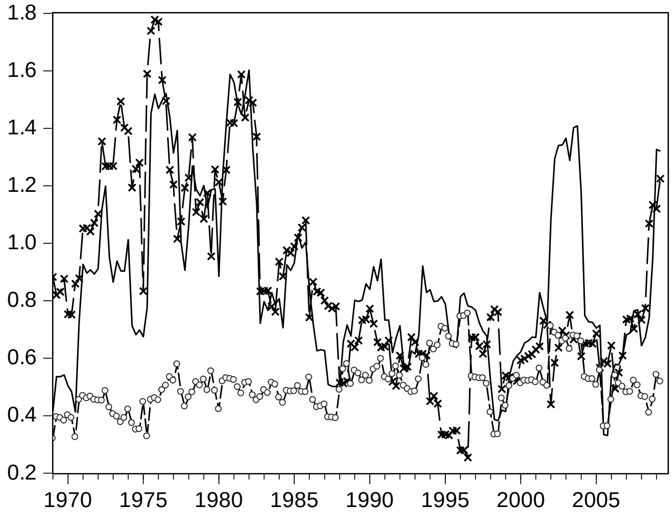


(c) Final data, output detrended by CBO measure of potential output



Output gap (output detrended by CBO potential output) uncertainty
Inflation uncertainty
Relative output gap uncertainty ratio

(d) Real-time data, output detrended by quadratic trend. Measures are based on structural shocks from a Vector Autoregressive (VAR) system



Output gap (output dtrended by quadratic trend) uncertainty
Inflation uncertainty
Relative output gap uncertainty ratio

Table 2: Taylor rule model estimates with uncertainty effects using GMM Sample: 1969Q4-2009Q2

	(i)	(ii)	(iii)	(iv)
Interest rate equation				
ρ_i	0.84 (0.03)	0.85 (0.02)	0.89 (0.03)	0.83 (0.03)
$\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t \geq \left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_{t-1}$ **regime**				
$\rho^{+}_{\pi,0}$	2.19 (0.30)	2.36 (0.40)	2.16 (0.45)	2.27 (0.32)
$\rho^{+}_{\pi,1}$	-0.83 (0.37)	-1.46 (0.45)	-1.85 (0.64)	-0.79 (0.35)
Average inflation effect	**1.88**	**1.84**	**1.49**	**2.05**
$\rho^{+}_{y,0}$	0.44 (0.20)	0.40 (0.24)	1.04 (0.40)	0.79 (0.26)
$\rho^{+}_{y,1}$	1.23 (0.50)	2.70 (1.01)	1.57 (0.60)	0.61 (0.42)
Average output effect	**0.90**	**1.39**	**1.61**	**0.94**
$\left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_t < \left(\frac{\sigma_y}{\sigma_y+\sigma_\pi}\right)_{t-1}$ **regime**				
$\rho^{-}_{\pi,0}$	1.84 (0.31)	1.76 (0.38)	1.00 (0.31)	1.92 (0.28)
$\rho^{-}_{\pi,1}$	0.84 (0.40)	1.75 (0.71)	1.88 (0.61)	0.82 (0.40)
Average inflation effect	**2.12**	**2.22**	**1.56**	**2.11**
$\rho^{-}_{y,0}$	1.19 (0.32)	2.98 (1.0	1.95 (0.65)	1.15 (0.29)
$\rho^{-}_{y,1}$	-0.99 (0.41)	-4.21 (1.6	-1.53 (0.70)	-1.12 (0.40)
Average output effect	**0.85**	**1.85**	**1.48**	**0.81**
Adjusted R^2	0.96	0.95	0.97	0.96
Regression SE	0.51	0.66	0.47	0.50
J stat	[0.29]	[0.30]	[0.32]	[0.33]

Notes: All models include an intercept term; estimates of this are not reported. Numbers in parentheses are the standard errors of the estimates. J stat is the p-values of a chi-square test of the system's overidentifying restrictions (Hansen, 1982). The instruments are a constant, four lags of the interest rate, inflation and the output gap and one lag of inflation uncertainty and output gap uncertainty.
(i): Real-time estimates. These use output detrended by a quadratic trend.
(ii): Real-time estimates. These use output detrended by a Hodrick-Prescott trend.
(iii) Final estimates. These use output detrended by the CBO measure of potential output.
(iv): Real-time estimates. These use output detrended by a quadratic trend. Inflation and output gap uncertainty measures are based on the structural shocks from a Vector Autoregressive (VAR) model in output gap, inflation and the interest rate.

This is routinely added to the VAR in order to limit the "price puzzle", that is, the positive response of prices to interest rate increases (see e.g. Boivin and Giannonni, 2006 and references therein). Doing so makes no qualitative difference to what follows. Figure 4d) plots our uncertainty measures based on the 8-quarter moving standard deviation of the VAR-derived structural shocks for inflation and the real-time output gap (the latter is detrended by quadratic trend; empirical results based on the CBO measure of potential output and a HP trend filter were poor and for this reason not reported). In line with our previous results, the estimates reported in column (iv) of Table 2 suggest that (i) the average inflation response increases when there is higher relative volatility in inflation and (ii) the output gap response increases when there is higher relative output gap volatility; however, the $\rho^{+}_{y,1}$ coefficient is imprecisely estimated and the model fits the data worse than the model which uses final data in column (iii) of Table 2. All in all, our estimates provide some evidence of asymmetries in the response to inflation and the output gap when uncertainty is considered.

5 Robustness analysis

At this point, some natural questions inspire a battery of experiments to check for the robustness of our results[16]. We first consider to what extent, if any, changes in the central bank preferences may affect these results. This question matters in that policymakers' preferences are not known with certainty and might also change due to special facts or contingencies. Thus we consider the case where output gap stabilization is as important as inflation stabilization[17]. The result of this experiment is reported in Figure 5 and shows that policymakers' preferences do not affect the monotonicity or the curvature type of the paths; they only affect the degree of concavity or convexity of these paths.

We then ask what happens to the case where the multiplicative shocks exhibit some persistence. Arguably, this could be due to some exogenous disturbance and/or change in the structure of the economy whose medium or long-lived nature is not known yet by the policymakers. To address this question, we introduce some inertia into the Markov chain. Thus, a new Markov matrix, P, is constructed such that: (i) the probability that the shock keeps the same value over two periods is equal to 0.5, (ii) the probability that

[16] The current analysis is conducted with the same parameters used before unless the experiments require otherwise, in which case the new values for the parameters (e.g. loss functions weights and transition probabilities) are reported.

[17] These alternative preferences are captured by setting $\mu = 1$, $\lambda = 1$, and $\nu = 0.2$ in the loss function.

the shock jumps to adjacent values is equal to 0.3, and (iii) by skipping the adjacent values, the probability that the shock jumps to the closer remaining values is equal to 0.1. Therefore, P takes the form

$$P = \begin{bmatrix} 0.5 & 0.3 & 0.1 & 0.1 & 0 \\ 0.15 & 0.5 & 0.15 & 0.1 & 0.1 \\ 0.1 & 0.15 & 0.5 & 0.15 & 0.1 \\ 0.1 & 0.1 & 0.15 & 0.5 & 0.15 \\ 0.1 & 0.1 & 0.15 & 0.15 & 0.5 \end{bmatrix}.$$

Given the Markov matrix above, its associated stationary distribution for the shock ϕ^h_{jt} for $h = y, \pi$ is given by

ϕ^h_{jt}	$(1-2\delta_{\phi^h})$	$(1-\delta_{\phi^h})$	1	$(1+\delta_{\phi^h})$	$(1+2\delta_{\phi^h})$
$\Pr(\phi^h_{jt})$	0.1598	0.2371	0.2371	0.2371	0.1598

.

Results in this case (available on request) are very similar to the previous ones based on serially i.i.d. modes.

Finally, we consider to what extent, if any, the realistic policy transmission lags embedded in the model affect the relation between inflation and output gap uncertainty and optimal policy. We then relax the assumption of a one-period lag between policy action and output gap response and of a further one-period lag between output gap and inflation changes. Accordingly, the AD and AS take the more conventional form

$$\begin{aligned} y_{t+1} &= \left[\alpha_y y_t + (1-\alpha_y)\, y_{t+2|t+1} - \alpha_r r_{t+1} + \varepsilon^y_{t+1}\right] \phi^y_{t+1} \\ \pi_{t+1} &= \left[\beta_\pi \pi_t + (1-\beta_\pi)\, \pi_{t+2|t+1} + \varepsilon^\pi_{t+1}\right] \phi^\pi_{t+1} \end{aligned}$$

and the vector of state variable is now given by $X_t = (\pi_t, y_t, i_{t-1})$. The results of the experiment are reported in Figures 6-7.Comparing these findings with the ones in Figures 2-3 shows that abstracting from transmission lags leads only to minor quantitative changes.

Figure 5: Optimal policy response to y_t and $\pi_{t+1|t}$ in presence of uncertainty on the output gap and inflation processes. Central bank preferences: μ =1, λ =1, ν =0.2.

(a) output gap uncertainty **(b) inflation uncertainty**

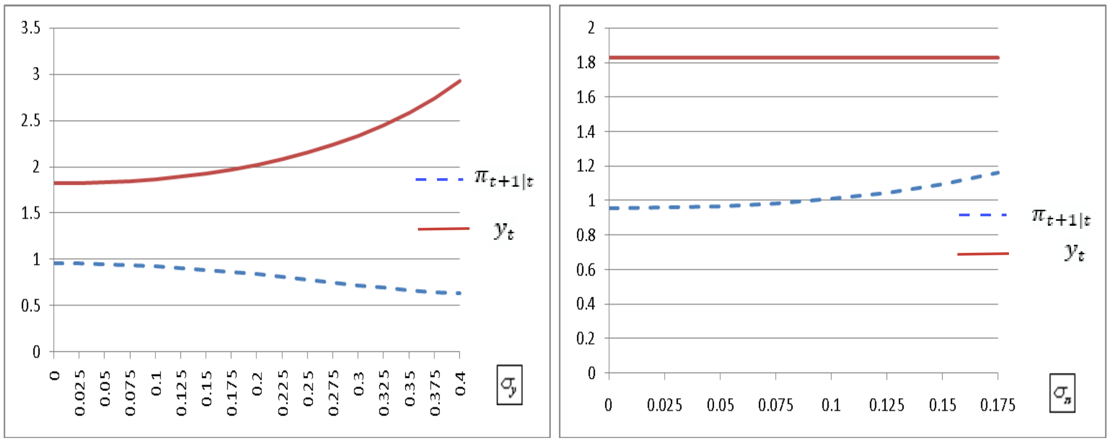


Figure 6: Optimal policy response to y_t and π_t with no policy transmission lags and in presence of uncertainty on the output gap and inflation processes. Central bank preferences: μ=1, λ=0.1, ν=0.2.

(a) output gap uncertainty **(b) inflation uncertainty**

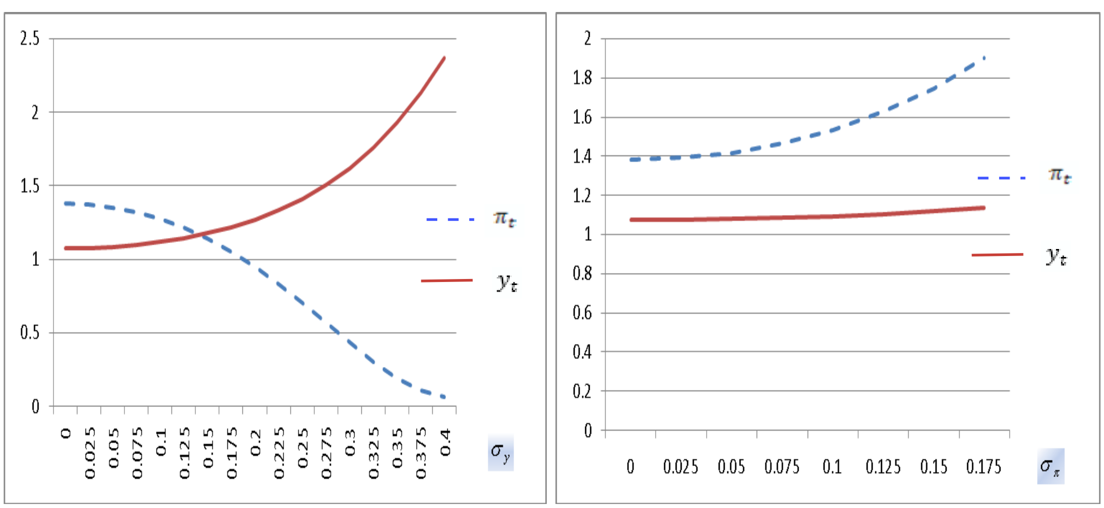
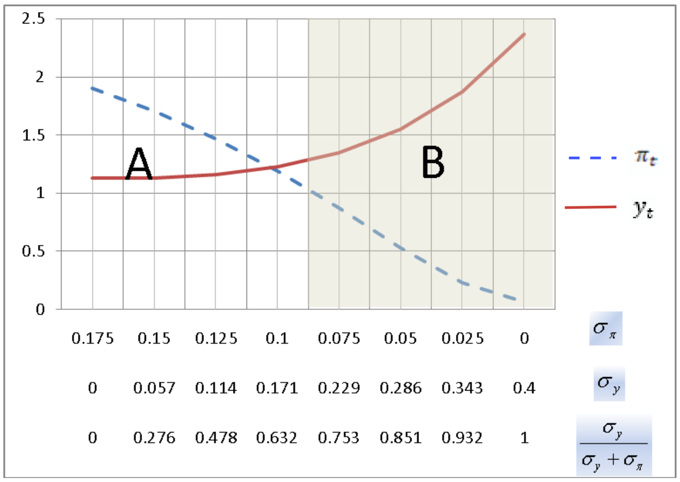


Figure 7: Optimal policy responses to y_t and π_t with no policy transmission lags and in presence of relative output gap uncertainty. Central bank preferences: μ=1, λ=0.1, ν=0.2.



6 Conclusions

This paper investigates optimal real-time monetary policy when policymakers consider the presence of undistinguishable uncertainty about the state and structure of the economy proxied by the exogenous volatility of the inflation and output gap processes.

First, the paper shows that in presence of undistinguishable uncertainty either on inflation or the output gap, considering this uncertainty in the policy decisions results in a more aggressive response to the uncertain state variable. Furthermore, the optimal response to inflation is inversely related to output gap uncertainty while the optimal response to the output gap tends to be unrelated to inflation uncertainty.

Second, in presence of exogenous uncertainty in both the inflation and output gap processes, when there is relatively more uncertainty on inflation than on the output gap, optimal monetary policy resembles the Taylor rule. On the other hand, when the uncertainty on the output gap exceeds the one on inflation, optimal monetary policy tends to respond more strongly to the output gap and to ignore inflation. Finally, in intermediate cases, the policy response to the state variables tends to be similar.

These results are based on a preemptive behavior of the central bank aiming to reduce the risk of large deviations of the economy from its long-run equilibrium, which would deteriorate the distribution forecasts for inflation and the output gap. In an empirical test carried out on the US economy, we find that the model predictions tend to be consistent with the data.

The model discussed in the current paper can be extended to allow for the effects of other types of uncertainty such as exchange rate uncertainty. We intend to address these issues in future research.

A Details on microfoundations

A.1 Preferences and technology

The economy features a continuum of unit mass of identical households, each producing a different variety of a consumption good indexed by j. For the sake of simplicity, but without loss of generality, we assume preferences only on consumption, see for example Svensson (2000). To obtain realistic inertia in the household behavior we introduce habits persistence following Abel (1990) so that the household's utility function is

$$E_t \sum_{\tau=0}^{\infty} \delta^\tau U\left(C_{t+\tau}, \overline{C}_{t+\tau}\right), \qquad U\left(C_{t+\tau}, \overline{C}_{t+\tau}\right) = \frac{\left(C_t/\widetilde{C}_{t-1}^{\kappa}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \qquad \sigma, \kappa \in (0,1),$$

where δ denotes the intertemporal discount factor, C_t consumption of consumer/producer j, which is a composite good given by the Dixit-Stiglitz aggregator of the continuum of varieties of the consumption good

$$C_t \equiv \left[\int_0^1 \left(C_t(j)\right)^{1-\frac{1}{\vartheta}} dj\right]^{\frac{1}{1-\vartheta}},$$

with $\vartheta > 1$ denoting the elasticity of substitution among any two differentiated varieties; finally, $\widetilde{C}_t$ is aggregate consumption, and κ and σ denote the degree of habits persistence in consumption and the intertemporal elasticity of substitution respectively.

As in Flamini (2007) we assume that the composite good used for consumption is also used for production and let $Y_t^P(j)$ be the quantity of the composite good used by firm j to produce its own differentiated variety $Y_t(j)$ with the production function

$$Y_t(j) = f\left[Y_t^P(j)\right],$$

where f is an increasing, concave, isoelastic function. It follows that the input requirement function for any firm j is given by

$$Y_t^P(j) = V\left[Y_t(j)\right], \tag{9}$$

where $V \equiv f^{-1}$, so that the firm's variable costs are given by

$$P_t V\left[Y_t(j)\right],$$

where

$$P_t \equiv \left[\int_0^1 p_t(j)^{1-\vartheta} dj\right]^{1/(1-\vartheta)}$$

is the price of the composite good, and the firm's marginal costs are

$$P_t V_y\left[Y_t(j)\right].$$

A.2 Steady state

Assuming flexible prices, the monopolistic competitive representative firm j sets the optimal price $p_t(j)$ in any period as a markup on marginal costs,

$$p_t(j) = \mu P_t V_y [Y_t(j)], \tag{10}$$

where $\mu > 1$ is the desired markup.

Considering the preferences of the household, the demand function for each variety j is

$$Y_t(j) = Y_t \left[\frac{p_t(j)}{P_t}\right]^{-\vartheta}, \tag{11}$$

where

$$Y_t = \left[\int_0^1 Y_t(j)^{1-\frac{1}{\vartheta}} dj\right]^{\frac{1}{1-\vartheta}}.$$

Next, rewriting (10) as

$$\frac{p_t(j)}{P_t} = \mu V_y [Y_t(j)]. \tag{12}$$

and substituting (11) into (12) yields

$$\left[\frac{Y_t(j)}{Y_t}\right]^{-\frac{1}{\vartheta}} = \mu V_y [Y_t(j)]. \tag{13}$$

Now, following Woodford (2003, ch. 3), we notice that the RHS of (13) is increasing in $Y_t(j)$ so that there is a unique $Y_t(j)$ that solves this equation given Y_t. Thus in equilibrium each firm will supply the same quantity which must equal Y_t. As a result the steady state level of output, Y, is the solution to

$$\mu^{-1} = V_y(Y),$$

that is

$$Y = V_y^{-1}\left(\mu^{-1}\right).$$

Since all the firms use the same technology, it also follows that $Y_t^P(j) = Y_t^P$ so that

$$Y_t^P = V(Y_t), \tag{14}$$

and using the market clearing condition

$$Y_t = C_t + Y_t^P \tag{15}$$

we finally obtain that in steady state

$$Y^p = V\left[V_y^{-1}\left(\mu^{-1}\right)\right]$$

$$C = V_y^{-1}\left(\mu^{-1}\right) - V\left[V_y^{-1}\left(\mu^{-1}\right)\right]$$

A.3 Aggregate demand

We can now log-linearize (15) around steady state values obtaining

$$y_t = \varphi c_t + (1-\varphi)\, y_t^p, \qquad \varphi \equiv \frac{C}{Y}, \tag{16}$$

where $y_t \equiv \log \frac{Y_t}{Y}$ and denotes the output gap, $c_t \equiv \log \frac{C_t}{C}$, and $y_t^P \equiv \log \frac{Y_t^P}{Y^p}$. Then, recalling that the function f is isoelastic, log-linearizing (14) and substituting y_t^p into (16), leads to

$$y_t = \varphi c_t + (1-\varphi)\, \xi y_t,$$

where the ξ is the output elasticity the input requirement function (14). Factorizing y yields

$$y_t = \frac{\varphi}{1-(1-\varphi)\,\xi} c_t,$$

and assuming for sake of simplicity but without loss of generality that a percentage increase of the quantity produced requires the same percentage increase of the input, that is $\xi = 1$, we obtain

$$y_t = c_t. \tag{17}$$

In order to reproduce a realistic one-period lag in the transmission of policy action to real activity, we assume as in Rotemberg and Woodford (1997) and Boivin and Giannoni (2006) that consumption decisions are predetermined one period in advance, specifically we let households choose the index C_t at date $t-1$. Thus, preferences maximization subject to a budget constraint and a no-Ponzi condition leads to the following log-linearised Euler equation

$$c_{t+1} = \alpha_y c_t + (1-\alpha_y)\, c_{t+2|t} - (1-\alpha_y)\, \sigma r_t + \varepsilon_{t+1}^y, \quad \alpha_y \equiv \frac{\kappa\,(1-\sigma)}{1+\kappa\,(1-\sigma)}, \tag{18}$$

where for sake of simplicity and without significant loss in generality we have approximated $r_{t+1|t}$ with r_t. Finally, considering (17), equation (18) can be rewritten in terms of the output gap as

$$y_{t+1} = \alpha_y y_t + (1-\alpha_y)\, y_{t+2|t} - \alpha_r r_t + \varepsilon^y_{t+1}, \qquad \alpha_r \equiv (1-\alpha_y)\,\sigma.$$

A.4 Derivation of the aggregate supply

In order to have a realistic two-period lag in the transmission of monetary policy to inflation we follow Rotemberg and Woodford (1997) and Boivin and Giannoni (2006) assuming that pricing decisions are made two periods in advance. We also assume the Calvo (1983) staggered price scheme and follow Christiano and al. (2005) in allowing the firms that cannot up-date the price optimally to index their price to previous inflation. Thus, firm j profit maximization problem is

$$\max_{\widetilde{P}_{t+2}} E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \lambda_{t+\tau+2} \left\{ \frac{\widetilde{P}_{t+2}\Psi_{t+\tau+1}}{P_{t+2+\tau}} Y_{t+\tau+2} \left[\frac{\widetilde{P}_{t+2}\Psi_{t+\tau+1}}{P_{t+2+\tau}} \right]^{-\vartheta} - V\left[Y_{t+\tau+2} \left(\frac{\widetilde{P}_{t+2}\Psi_{t+\tau+1}}{P_{t+2+\tau}} \right)^{-\vartheta} \right] \right\},$$

where $(1-\alpha)$ is the probability that in any period the producer choose an optimal price, $\Psi_{t+\tau+1} \equiv \left(\frac{P_{t+\tau+1}}{P_{t+1}} \right)^{\zeta}$, with ζ denoting the degree of indexation to previous period inflation when it is not possible to optimally up-date the price, and finally λ_t and $\widetilde{P}_{t+2}$ denote the marginal utility of consumption, and the new price chosen in period t for period $t+2$, respectively. The first-order condition is

$$E_t \left\{ \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \lambda_{t+\tau+2} \left[\frac{\widetilde{P}_{t+2}\Psi_{t+\tau+1}}{P_{t+2+\tau}} - \mu V' \left(Y_{t+\tau+2} \left(\frac{\widetilde{P}_{t+2}\Psi_{t+\tau+1}}{P_{t+2+\tau}} \right)^{-\vartheta} \right) \right] Y_{t+\tau+2} \left(\frac{\widetilde{P}_{t+2}\Psi_{t+\tau+1}}{P_{t+2+\tau}} \right)^{-\vartheta} \right\} = 0\ ,$$

where $\mu \equiv -\frac{\vartheta}{1-\vartheta}$, which can be rewritten as

$$E_t\left\{\sum_{\tau=0}^{\infty}\alpha^{\tau}\delta^{\tau}\lambda_{t+\tau+2}\left[\frac{X_{t+2}\prod_{s=1}^{\tau}\Pi_{t+1+s}^{\zeta}}{\prod_{s=1}^{\tau}\Pi_{t+2+s}}\right.\right. \tag{19}$$

$$\left.\left.-\mu V'\left(Y_{t+\tau+2}\left(\frac{X_{t+2}\prod_{s=1}^{\tau}\Pi_{t+1+s}^{\zeta}}{\prod_{s=1}^{\tau}\Pi_{t+2+s}}\right)^{-\vartheta}\right)\right]Y_{t+\tau+2}\left(\frac{X_{t+2}\prod_{s=1}^{\tau}\Pi_{t+1+s}^{\zeta}}{\prod_{s=1}^{\tau}\Pi_{t+2+s}}\right)^{-\vartheta}\right\}=0\ ,$$

where $X_{t+2}\equiv\frac{\widetilde{P}_{t+2}}{P_{t+2}}$ and $\Pi_{t+2}\equiv\frac{P_{t+2}}{P_{t+1}}$.

In equilibrium any firm that is free to choose the price in period t will choose the same price, $\widetilde{P}_t$, and the remaining firms that are not free to choose the price in period t will keep the previous period price updated to the previous period inflation. Thus, the aggregate price is given by

$$P_t=\left[\alpha\left(P_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\zeta}\right)^{1-\vartheta}+(1-\alpha)\,\widetilde{P}_t^{(1-\vartheta)}\right]^{\frac{1}{1-\vartheta}}$$

$$\Longleftrightarrow\Pi_t=\left[1-(1-\alpha)\,X_t^{(1-\vartheta)}\right]^{-\frac{1}{1-\vartheta}}\alpha^{\frac{1}{1-\vartheta}}\Pi_{t-1}^{\zeta}.$$

Then we log-linearize the first order condition around the steady state. Let us allow bounded fluctuations in $(\lambda_t,\ X_t,\ \Pi_t,\ Y_t)$ around the steady state $(\lambda,\ 1,\ 1,\ Y,)$ and let small letters be log-deviations from their steady state value. Then we obtain

$$v'=\omega y\,(j)\,, \tag{20}$$

$$\pi_t=\zeta\pi_{t-1}+\frac{1-\alpha}{\alpha}x_t, \tag{21}$$

where $\omega>0$ is the elasticity of V' with respect to $Y_t\,(j)$. Then, log-linearizing (19) yields

$$E_t\left\{\sum_{\tau=0}^{\infty}\alpha^{\tau}\delta^{\tau}\left[x_{t+2}-\sum_{s=1}^{\tau}(\pi_{t+2+s}-\zeta\pi_{t+1+s})\right.\right.$$

$$\left.\left.-\omega\left(y_{t+\tau+2}-\vartheta\left(x_{t+2}-\sum_{s=1}^{\tau}(\pi_{t+2+s}-\zeta\pi_{t+1+s})\right)\right)\right]\right\}=0$$

which can be rewritten as

$$E_t \left\{ \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \left[(1+\omega\vartheta) \left(x_{t+2} - \sum_{s=1}^{\tau} (\pi_{t+2+s} - \zeta \pi_{t+1+s}) \right) - \omega y_{t+\tau+2} \right] \right\} = 0 \ . \tag{22}$$

Now note that

$$\begin{aligned} \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \sum_{s=1}^{\tau} (\pi_{t+2+s} - \zeta \pi_{t+2+s-1}) &= \sum_{s=1}^{\infty} (\pi_{t+2+s} - \zeta \pi_{t+1+s}) \sum_{\tau=s}^{\infty} \alpha^\tau \delta^\tau \\ &= \sum_{s=1}^{\infty} (\pi_{t+2+s} - \zeta \pi_{t+1+s}) \frac{\alpha^s \delta^s}{1-\alpha\delta} \\ &= \frac{1}{1-\alpha\delta} \sum_{\tau=1}^{\infty} \alpha^\tau \delta^\tau (\pi_{t+2+\tau} - \zeta \pi_{t+1+\tau}) \ . \end{aligned}$$

Then, equation (22) can be rewritten as

$$E_t \left\{ \frac{1+\omega\vartheta}{1-\alpha\delta} x_{t+2} - \frac{1+\omega\vartheta}{1-\alpha\delta} \sum_{\tau=1}^{\infty} \alpha^\tau \delta^\tau (\pi_{t+2+\tau} - \zeta \pi_{t+2+\tau-1}) - \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau y_{t+\tau+2} \right\} = 0$$

$$\Longleftrightarrow$$

$$\begin{aligned} E_t x_{t+2} &= E_t \left\{ \sum_{\tau=1}^{\infty} \alpha^\tau \delta^\tau (\pi_{t+2+\tau} - \zeta \pi_{t+2+\tau-1}) + \frac{1-\alpha\delta}{1+\omega\vartheta} \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \omega y_{t+\tau+2} \right\} \\ &= E_t \left\{ \alpha\delta (\pi_{t+2+1} - \zeta \pi_{t+2}) + \frac{1-\alpha\delta}{1+\omega\vartheta} \omega y_{t+2} \right\} + \alpha\delta E_t x_{t+2+1}. \end{aligned}$$

Next, approximating δ by unit in order to ensure the Natural-rate hypothesis and combining the previous equation with (21) we obtain

$$\pi_{t+2|t} = \frac{\zeta}{(1+\zeta)} \pi_{t+1|t} + \frac{1}{(1+\zeta)} \pi_{t+3|t} + \frac{(1-\alpha)^2}{(1+\zeta)\,\alpha\,(1+\omega\vartheta)} \omega y_{t+2|t}.$$

Finally, adding the error term $\varepsilon^\pi_{t+2} + \frac{\zeta}{1+\zeta} \varepsilon^\pi_{t+1}$ to both sides of the previous equation yields

$$\begin{aligned} \pi_{t+2} &= \frac{\zeta}{(1+\zeta)} \pi_{t+1} + \frac{1}{(1+\zeta)} \pi_{t+3|t} + \frac{1}{(1+\zeta)} \frac{(1-\alpha)^2}{\alpha\,(1+\omega\vartheta)} \omega y_{t+2|t} + \varepsilon^\pi_{t+2} \\ &= \beta_\pi \pi_{t+1} + (1-\beta_\pi)\, \pi_{t+3|t} + \beta_y y_{t+2|t} + \varepsilon^\pi_{t+2}. \end{aligned}$$

B Key elements of the Svensson and Williams approach

We first expand the product of the multiplicative shocks with the terms in the square brackets of the RHS in (1-2) and then rewrite the aggregate demand and supply in State-space form as

$$X_{t+1} = A_{11j_{t+1}} X_t + A_{12j_{t+1}} x_t + B_{1j_{t+1}} i_t + \varepsilon_{t+1} \tag{23}$$
$$E_t x_{t+1} = A_{21j_t} X_t + A_{22j_t} x_t + B_{2j_t} i_t \tag{24}$$

where X_t and x_t are, respectively, the vectors of predetermined and forward looking variables

$$X_t = \left(\pi_t,\ \pi_{t+1|t},\ y_t,\ i_{t-1}\right)',$$
$$x_t = \left(y_{t+1|t}, \pi_{t+2|t}\right)',$$

ε_t is the vector of additive exogenous disturbances, and finally the matrices

$$A_{11j_{t+1}},\ A_{12j_{t+1}},\ B_{1j_{t+1}},\ A_{21j_t},\ A_{22j_t},\ B_{2j_t}, \tag{25}$$

are random, each free to take n_j different values in period t corresponding to the n_j modes indexed by $j_t \in \{1, 2, ..., n\}$.

Turning to the central bank loss function (3), it can be rewritten as[18]

$$E_t \sum_{\tau=0}^{\infty} \delta^{\tau} L\left(X_{t+\tau}, i_{t+t}\right),$$

where the period loss function can be expressed in matrix form as

$$L\left(X_{t+\tau}, i_{t+t}\right) \equiv \begin{bmatrix} X_t \\ i_t \end{bmatrix}' W \begin{bmatrix} X_t \\ i_t \end{bmatrix}, \tag{26}$$

and W is a positive semidefinite matrix depending on the central bank preferences. Thus the central bank problem consists of minimizing (26) subject to (23-24) and (5). Following the approach described in Svensson and Williams

[18] Notice two differences with the loss function considered in the general case by Svensson and Williams (2007) in section 2.1. First, in our model the loss depends only on the stabilization of inflation, the output-gap and the first difference of the interest rate which implies that the forward looking variables x_t do not appear in the loss function. Second, there is no uncertainty on the weight to the target variables in the loss function so that the loss does not depend directly on the modes.

(2007a), this problem, which is not recursive due to the presence of forward looking variables, needs to be converted to a recursive one. The approach consists in expanding appropriately the state and control space to use the recursive saddlepoint method developed by Marcet and Marimon (1998). Specifically, the vector of lagged Lagrange multipliers, which will be denoted by Ξ_{t-1}, corresponding to the forward-looking equations is added as a vector of additional state variables. Furthermore, the vector of current values of these multipliers, which will be denoted by γ_t, as well as the expected values of the forward looking variables themselves, which will be denoted by z_t, are added as vectors of additional control variables. Accordingly, equation (24) is replaced by two equivalent equations

$$\begin{aligned} E_t x_{t+1} &= z_t, \\ 0 &= A_{21j_t} X_t + A_{22j_t} x_t - z_t + B_{2j_t} i_t, \end{aligned}$$

where z_t is a vector of additional forward-looking variables. Then solving the second equation for x_t we obtain

$$x_t = \widetilde{x}\left(X_t, z_t, i_t, j_t\right),$$

which is a convenient representation of the forward-looking variables in the application of the Marcet and Marimon method. This method requires to introduce the dual period loss function

$$\widetilde{L}\left(\widetilde{X}_t,\ z_t,\ i_t,\ \gamma_t,\ j_t\right) \equiv L\left(X_t, i_t\right) - \gamma_t' z_t + \Xi_{t-1}' \frac{1}{\delta} \widetilde{x}\left(X_t, z_t, i_t, j_t\right) \tag{27}$$

and find a saddlepoint for the dual intertemporal loss function

$$E_t \sum_{\tau=0}^{\infty} \delta^{\tau} \widetilde{L}\left(\widetilde{X}_t,\ z_t,\ i_t,\ \gamma_t,\ j_t\right)$$

such that the dual intertemporal loss function is maximized over $\{\gamma_{t+\tau}\}_{\tau\geq 0}$ and minimized over $\{z_{t+\tau},\ i_{t+\tau}\}_{\tau\geq 0}$ subject to (23-24) and (5).

To solve this problem, we let $s_t \equiv \left(\widetilde{X}_t',\ p_t'\right)'$ denote the perceived state of the economy where

$$s_{t+1} \equiv \begin{bmatrix} X_{t+1} \\ \Xi_t \\ p_{t+1} \end{bmatrix} = g\left(s_t, z_t, i_t, \gamma_t, j_t, j_{t+1}, \varepsilon_{t+1}\right)$$

$$\equiv \begin{bmatrix} A_{11j_{t+1}} X_t + A_{12j_{t+1}} \widetilde{x}\left(X_t, z_t, i_t, j_t\right) + B_{1j_{t+1}} i_t + \varepsilon_{t+1} \\ \gamma_t \\ P^{'} p_t \end{bmatrix}$$

and introduce the conditional dual value function which gives the dual intertemporal loss conditional on the true state of the economy $(s_t,\ j_t)$

$$\widehat{V}\left(s_t,\ j\right) \equiv \int \widetilde{L}\left(\widetilde{X}_t,\ z\left(s_t\right),\ i\left(s_t\right),\ \gamma\left(s_t\right),\ j\right) + \delta \sum\nolimits_k P_{jk} \widehat{V}\left[\overline{g}\left(s_t, j, k, \varepsilon_{t+1}\right), k\right]$$
$$\varphi\left(\varepsilon_{t+1}\right) d\varepsilon_{t+1},$$

$\forall j \in n_j$, where $\varphi(.)$ denotes a generic probability density function and $\overline{g}\left(s_t, j, k, \varepsilon_{t+1}\right)$ is the perceived state of the economy in $t+1$ considering that in equilibrium the control variable i_t and the control vectors γ_t and z_t depend only on s_t. This allows presenting the true dual value function as the average of the conditional value functions according to the perceived distributions of modes:

$$\widetilde{V}\left(s_t\right) = E_t \widehat{V}\left(s_t, j_t\right) = \sum\nolimits_j p_{jt} \widehat{V}\left(s_t, j_t\right), \tag{28}$$

and the true dual value function, in turns, using the saddlepoint method solves

$$\widetilde{V}\left(s_t\right) = \max_{\gamma_t} \min_{(z_t, i_t)} E_t \left\{ \widetilde{L}\left(\widetilde{X}_t,\ z_t,\ i_t,\ \gamma_t,\ j_t\right) \right. \tag{29}$$

$$\left. + \delta \widehat{V}\left[g\left(s_t, z_t, i_t, \gamma_t, j_t, j_{t+1}, \varepsilon_{t+1}\right), j_{t+1}\right] \right\} \tag{30}$$

subject to the law of motion of the perceived state of the economy

$$\begin{aligned} X_{t+1} &= A_{11j_{t+1}} X_t + A_{12j_{t+1}} \widetilde{x}\left(X_t, z_t, i_t, j_t\right) + B_{1j_{t+1}} i_t + \varepsilon_{t+1} \\ \Xi_t &= \gamma_t \\ p_{t+\tau} &= \left(P^{'}\right)^{\tau} p_t. \end{aligned}$$

Next notice that if modes were fixed we would have a standard Linear Quadratic problem whose the familiar solution is a quadratic value function and a linear policy. In the current model, since the belief evolution is exogenous, the value function is then quadratic in $\widetilde{X}_t$ for a given belief, that is for a given perceived probability distribution p_t. In other words the value function is belief-dependent quadratic which implies that the optimal policy is belief-dependent linear, specifically a linear function of the expanded state $\widetilde{X}_t$ for a given p_t.

Now the fact that the value function is quadratic in $\widetilde{X}_t$ for a given p_t, matters in that it can be expressed in a matrix form. Furthermore, accounting for (26), the dual period loss function (27) can be also written in matrix form in terms of the expanded vector of predetermined variables $\widetilde{X}_t$ and the expanded vector of control variables, i.e. $\widetilde{i}_t \equiv (z_t', i_t, \gamma_t')'$. This allows writing the value function for the dual problem (28) and the Bellman equation (29) in matrix form. Then it is possible to obtain a first order condition with respect to $\widetilde{i}_t$ which allows determining the following form of the policy function

$$\widetilde{i}_t = \widetilde{F}(p_t)\widetilde{X}_t, \tag{31}$$

where $\widetilde{F}(p_t)$ is a matrix depending on the perceived distribution of the modes in period t. It is worth noting that the matrix $\widetilde{F}(p_t)$ is unknown as it depends on the value function matrix which is unknown too at this stage. Then, equation (31) is embedded in the Bellman equation in matrix form leading to the so called Riccati equation. The latter is finally used in the algorithm developed by Svensson and Williams to determine the unknown value function matrix and the matrix $\widetilde{F}(p_t)$ through numerical iterations.

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