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Abstract

Strategic price interaction on networks with rival and interchangeable services are well-known to produce damaging externalities with which the number of agents acting independently can interact in non-linear ways. We examine how varying the number of independent agents can impact the relative performance of platform models on a transport network whose design can mitigate some of the damaging externalities in the 2-agent setting. We show that increasing the number of agents can preserve or enhance some of the benefits of the platform models under some circumstances but the platform structure, that abates damaging externalities with 2-agents, can constrain beneficial competitive forces with more agents, damaging relative performance.

Keywords: Platform; Strategic Interaction; Multi-operator; Transport Network; Pricing; Welfare

JEL #s: D43, L13, L91, R40

1 Introduction

Mobility-as-a-Service (MaaS) is an emerging concept in the field of transport which places emphasis on matching the mobility needs of users combining transport services across operators and modes. Benefits of having a single service satisfying a user's origin-destination requirements by integrating transport components across operators are recognised to include improved access and inclusion alongside broader advantages such as reduced dependence on private car use and pollution (e.g., see Jittrapirom et al., 2017). Such innovations also have a potential role to play in helping drive the net zero and levelling-up agendas in countries where regional inequalities in transport infrastructure in second-tier cities holds back economic growth and the transition away from private vehicle use.

One aspect of MaaS, that until recently has not received much attention (e.g., see Polydoropoulou et al., 2020), concerns what business structures supporting MaaS provision might look like.[1] Whilst König et al. (2016) set out and analyse the relative attractiveness and merits of alternative organisations of MaaS structures, van den Berg et al. (2022) provide insights into the implications of the different pricing structures associated with different business models and their impacts on relative profit and welfare outcomes. Both studies recognise the importance of potential differences across MaaS provision in terms of the organisation of the supply chain of mobility services including which agents are involved in setting prices.

In common with wider practice in transport studies in recent times, van den Berg et al. (2022) embed their alternative MaaS regimes in an Economides and Salop (1992)-type network model, essentially extending the model to allow a new agent which provides a user platform for purchasing combined complementary multi-operator services on the network.[2] This network setting envisages transport operators competing in the provision of horizontally differentiated services that have rival and interchangeable parts, capturing important countervailing strategic interactions often evident in real-world transport networks. One of the key results is to demonstrate that one platform, denoted the Integrator, offers to improve the welfare outcomes on the network relative to a Free-Market case based upon its pricing configuration, and if services are not too highly substitutable also improves on profit too. As such they demonstrate an important potential benefit arising from some of the business models in terms of solving a strategic interaction problem that can be present in multi-operator transport settings under the Free-Market.

However, as is typically the case in models using this framework in a transport setting, the number of agents is assumed as fixed at $n = 2$. McHardy (2022) demonstrates that the countervailing strategic forces in the network make the $n = 2$ model susceptible to predictions that do not generalise well to larger networks with more operators, as increasing n can

[1] Until quite recently, the main contributions around MaaS have been focused on categorisation and analysis of existing and pilot MaaS projects (e.g., see for instance König et al., 2016; Ebrahimigharehbaghi et al., 2018), and the MaaS supply chain and wider operating environment, its agents and their functions (e.g., see Kamargianni and Matyas, 2017)

[2] This modelling framework has been widely employed in the transport literature with many examples of work explicitly using it or nested within it (e.g., see Shy, 1996; Lin, 2004; McHardy et al., 2012; Bataille and Steinmetz, 2013; Silva and Verhoef, 2013; Socorro and Viecens, 2013; van den Berg, 2013; Clark et al., 2014; D'Alfonso et al., 2016; Kuang et al., 2020; van den Berg et al., 2022).

change the balance of the countervailing forces and in different ways under different regimes, potentially reversing the orderings of these regimes in profit and welfare terms. Indeed, they also show that allowing a larger number of services in the modelling framework can facilitate the investigation of real-world transport network characteristics which are not possible in the $n = 2$ setting but which can have further implications for the relative performance of different regimes. For instance, the $n = 2$ model assumes rival operators each provide only a single service i.e., one route with unit frequency. In practice, route numbers and/or service frequencies are choice variables for operators which, if included in the analysis, can result in different regimes supporting different network sizes, again with the potential to reorder the performance rankings of regimes.

In this paper we extend the analysis of van den Berg et al. (2022) to the n-operator setting and examine how increasing n beyond 2 can change the relative performance of the three main MaaS platform business models outlined therein.[3] The earlier observation about the potential non-monotonic impacts of raising n on the relative performance of different regimes alongside the point of the MaaS being to integrate services across *multiple* providers, suggests this exercise is both relevant and has the potential for important new insights. We also consider a number of other extensions to the network model including (i) allowing demands for single (domestic) as well as round-trip travel, and (ii) making service frequency a choice variable for operators. In each case, we show that some of the key conclusions drawn in van den Berg et al. (2022) can be sensitive to these generalisations. However, in common with van den Berg et al. (2022), wider potential benefits of the MaaS platform models are, for the main part, assumed away. Given the expected benefits of MaaS include improving the user experience via additional services such as journey planning and simplified ticketing, we would expect that adding such benefits into the modelling framework would unambiguously favour platform-based regimes over non-platform ones given the additional surplus available under those regimes. We briefly consider this issue and show that, whilst this can be the case, it need not be so.

Our main methodological contribution is to extend the analysis of the business models introduced under a 2-operator setting to the n-operator case as well as introducing other common transport network characteristics such as frequency choice, single (domestic) demands and generalised cost benefits of platform regimes. Given the analysis produces new insights into the relative performance of different business regimes there are also policy implications of our work. In addition, we also introduce and analyse a Multi-operator Ticketing Card (MTC) model of pricing, currently permitted under a block exemption in the UK which is under statutory review and due to expire in 2026. We consider a stylised representation of this regime and compare its performance alongside the other regimes offering further timely policy insights.

The following Section introduces the base model, focusing solely on network price effects, and sets out the network strategic interaction problem identified above which the platform can help address and which underlies its potential performance advantages over the Free-Market, as indicated in van den Berg et al. (2022). Section 3 then derives the equilibria in

[3]These are the Intermediary, Platform and Integrator models, which we introduce more formally later. They also consider two other regimes, independent operators and a one-sided regulation case which relate directly to scenarios outlined in Economides and Salop (1992) but are not central to their findings.

the three platform regimes, along with the Free-Market and MTC cases. Section 4 analyses the regimes and sets out the key findings under the base model. Section 5 then considers how a number of extensions to the base model (allowing frequency as a choice variable, single and return demands and platform generalised cost benefits) can impact the relative performance of the regimes. Section 6 concludes.

2 The Base Model

Consider a transport network where all users combine a component x and y journey in their Origin-Destination (OD) travel and there are n operators, with operator i offering a component pair $\{x_i, y_i\}$, which can be combined in the in-service journey J_{ii}. Operator j ($j \neq i = 1, ..., n$) similarly offers a horizontally differentiated component pair $\{x_j, y_j\}$. Users can combine operator i's x component with operator j's y component to form the cross-service journey J_{ij}. Hence passengers can choose to undertake one of n differentiated OD journeys with a single operator (an in-service journey). By the nature of the set-up these services are imperfect substitutes. Let P_{ii} be price for the OD journey J_{ii} with operator i. By the above assumptions, the relationship between two rival operators' prices is strategic complements. Operators' prices are increasing in the OD price of a rival operator, i.e., best response functions in prices will be upward sloping: $\frac{\partial P_{jj}}{\partial P_{ii}} > 0$. Therefore, amongst rival operators we have a situation of differentiated Bertrand competition - strategic interaction across the in-service OD prices places downward pressure on those prices, and increasing the number of operators n will tend to intensify price competition resulting in lower prices.

If we allow passengers to undertake J_{ij} OD journeys combining operator i's x_i and operator j's y_j components, then we have an additional $n(n-1)$ differentiated cross-service journey options. Let p_i be the price set by operator i for travel on either one of its component journeys x_i or y_i.[4] Hence, a cross-service OD journey J_{ij} has price $P_{ij} = p_i + p_j$. On the one hand, since these cross-service journeys are substitutes amongst themselves (i.e., J_{12} is a substitute for J_{31}) but also substitutes with respect to the n in-service journeys (i.e., J_{12} is a substitute for J_{33}), their OD prices P_{ij} are strategic complements with respect to all other OD prices: the cross-service journeys add to the downward pressure on OD prices. However, on the other hand, the x and y component prices have a complementary relationship which is characterised by strategic substitutes. Operators' component prices are decreasing in the price of a rival's component price, i.e., component price best response functions are downward sloping: $\frac{\partial p_j}{\partial p_i} < 0$. Therefore, amongst rival operators we have a situation where strategic interaction places upward pressure across component prices, and increasing the number of operators will tend to intensify these upward price pressures. Since the cross-service journeys are substitutes for in-service journeys, the upward pressure on cross-service prices through the strategic substitutes channel extends to lifting the downward (strategic complement) pressure on in-service prices.

[4]Whilst we can readily incorporate different prices for different components, in the current paper symmetry conditions will result in operators setting the same price on each component.

The Network Problem

Hence, in transport networks where services have substitute and interchangeable aspects, independent price setting can involve two countervailing strategic interactions acting on prices in opposing directions. As a result, Free-Market price setting in a network can yield in-service prices below the monopoly level, but cross-network prices above the monopoly level, which can result in welfare outcomes worse than monopoly (e.g., see McHardy, 2022). Essentially, in the presence of monopoly, the socially harmful strategic substitute externalities and socially beneficial strategic complement externalities are both internalised. It can be shown that where the services are not close substitutes, internalising the harmful strategic substitute externalities can have the dominant effect with monopoly improving welfare relative to a Free-Market outcome with n independent operators.

Let operator i provide a single $\{x, y\}$ component pair, differentiated from each of the other $\{x, y\}$ pairs according to the demand system:

$$Q_{ii} = a - bP_i \; + d\left[\sum_{k\neq i}^{n} P_k + 2\sum_{k\neq i}^{n}(p_i + p_k) + \sum_{k\neq m,i}^{n}\sum_{m\neq i}(p_m + p_k)\right] \tag{1}$$

$$Q_{ij} = a - b(p_i + p_j) + d\left[P_i + \sum_{k\neq i}^{n} P_k + \sum_{k\neq i,j}^{n}(p_i + p_k) + \sum_{k\neq i}^{n}(p_i + p_k) + \sum_{k\neq m,i}^{n}\sum_{m\neq i}(p_m + p_k)\right]$$

We underpin this demand system with a quasi-linear utility function employed in many transport and industrial economics studies (e.g., see Hackner, 2000; Silva and Verhoef, 2013), which incorporates network service density effects and imposes:[5]

$$a \equiv \frac{\alpha}{(1 + \gamma(N-1))}, \quad b \equiv \frac{1 + \gamma(N-2)}{(1-\gamma)(1 + \gamma(N-1))}, \quad d \equiv \frac{\gamma}{(1-\gamma)(1+\gamma(N-1))} \tag{2}$$

Service density effects arise, as at given prices, additional $\{x, y\}$ pairs bring additional consumer surplus rather than result in a pure redistribution of a fixed surplus. This means that adding new operators with new services has two potential channels to impact surplus: density effects and added competition.

Regarding costs, marginal costs in transport settings are often treated as constant (e.g., see Clark et al., 2014), for which there is empirical justification (e.g., see Jørgensen and Preston, 2003). For much of the analysis constant marginal cost has no impact on the relative size of key equilibria variables across regimes, and hence zero constant marginal cost can be assumed without further loss of generality. After the main analysis we do introduce non-linear marginal costs and a fixed cost per $\{x, y\}$ pair, which we will formally present later.

3 Regimes

In this Section we present the three main platform regimes in van den Berg et al. (2022) alongside a Free-Market case and a stylised MTC as introduced earlier.

[5]The function can be expressed as $U(\boldsymbol{Q}, M_0) = \alpha \sum_t^N Q_t - \frac{1}{2}\left[\sum_t^N Q_t^2 + 2\gamma \sum_t^N \sum_{r>t} Q_t Q_r\right] + M_0 \quad (r \neq t = 1, ..., N)$, where M_0 is a composite good, and is classed as a Spence (1976)-Shubik and Levitan (1980)-type (see Choné and Linnemer, 2020).

3.1 Free-Market

In the Free-Market regime, operators set their in-service price and cross-service price components simultaneously and independently. Given the symmetry of the model we assume here, and henceforth, that each operator receives half of the revenue on each of its cross-service operations. Hence, each operator i solves the problem:

$$\max_{P_{ii},p_i} \pi_i = P_{ii}Q_{ii} + p_i \sum_{j\neq i}^{n}(Q_{ij} + Q_{ji}) \quad (i \neq j = 1, ..., n)$$

which results in the equilibrium in-service and cross-service prices (where convenient, we exploit symmetry and denote in- and cross-service prices as P and P_x, respectively, with quantities treated accordingly):

$$P^{FM} = \frac{3\alpha(1-\gamma)}{6 + (2n^2 - 3n - 5)\gamma}, \quad P_x^{FM} = \frac{4\alpha(1-\gamma)}{6 + (2n^2 - 3n - 5)\gamma}$$

As outlined above, the Free-Market regime experiences both externalities such that whilst the in-service price lies below the monopoly level, the cross-service price can exceed the monopoly level for sufficiently low n and γ (see McHardy, 2022).

3.2 Integrator Platform

In the Integrator Platform, the operators set their in-service prices and the Platform sets the common (given symmetry) cross-service price, P_x, taking a share, ϕ of the associated revenues.[6]

Each operator i solves the following problem:

$$\max_{P_{ii}} \pi_i = P_{ii}Q_{ii} + \frac{1}{2}(1-\phi)P_x \sum_{j\neq i}^{n}(Q_{ij} + Q_{ji})$$

The Integrator, who supplies the platform, simultaneously solves the problem:

$$\max_{P_x} \pi^{int} = \phi P_x \sum_{j\neq i}^{n}(Q_{ij} + Q_{ij}) \tag{3}$$

which results in the equilibrium in-service and cross-service prices:

$$P^I = \frac{\alpha(1-\gamma)[2 + \gamma(n-1)(3+n-\phi)]}{\Delta}, \quad P_x^I = \frac{\alpha(1-\gamma)(2\gamma n^2 - 3\gamma + 2)}{\Delta} \tag{4}$$

where $\Delta \equiv 4 + \gamma^2(n-1)\{[3n^2 - 6 - n(3-\phi)]\} + 2\gamma(2n^2 + n - 5)$. Analysis of the equilibrium prices yields the following Proposition.

Proposition 1. [7] *Under the Integrator, (i) single- and cross-service prices are equal under $\gamma = 0$, and, (ii) single-service (cross-service) prices are strictly (weakly) falling in the Integrator's share of profit, ϕ: $P_\phi^I < 0, P_{x\,\phi}^I \leq 0$*

[6] Anecdotal evidence supports the convention of assuming ϕ to be low and around 2% (e.g., van den Berg et al., 2022).

[7] Where relevant, proofs are reported in Appendix A.

The intuition behind this can best be understood with respect to the Network Problem outlined above. We know that in the Free-Market outcome, cross-service prices can exceed monopoly levels reducing downward pressure on in-service prices and yielding welfare outcomes worse than monopoly. Here, though, the Integrator is internalising the strategic substitute externality which raises cross-service prices. It is also taking a share of the associated profit which alters the balance of the channels through which operators earn their rewards, increasing the relative importance of in-service revenue as ϕ increases. As we have seen above, in setting its in-service price operator i knows that, taking other prices as given, a reduction in P_{ii} will yield a market share gain relative to other in-service and cross-service operations gaining it profit through its in-service revenues but damaging its profit through lost revenues on its cross-service revenues. The latter effect therefore injects a resistance to incentives for in-service price cutting. As the Integrator takes a larger share of the cross-service profit so it insulates the operator from this resistance that would otherwise result in lower in-service prices and lower prices more generally given services are substitutes. The following result encapsulates this in consumer surplus and welfare terms.

Corollary 1. *Under the Integrator, Consumer surplus and Welfare are weakly increasing in the Integrator's share of profit, ϕ: $W^I_\phi, CS^I_\phi \geq 0$.*

The following Proposition confirms that the lower prices under the Integrator with higher levels of ϕ result in lower aggregate profit, which is not as straightforward as one might expect.

Proposition 2. *Under the Integrator, aggregate profit is weakly decreasing in the Integrator's share of profit, ϕ: $\pi^I_\phi \leq 0$*

To understand why the above result is not necessarily obvious, note that under the Free-Market for sufficiently low levels of n and γ, cross-service prices exceed the Monopoly level (see McHardy, 2022), and hence reducing cross-network prices can increase profit. However, here the Integrator prevents cross-service prices exceeding monopoly levels, even with $\phi = 0$, and any further reductions in prices are moving strictly away from monopoly levels reducing aggregate profit.

Note that the Integrator regime is likely in practice to have benefits beyond the pricing ones captured here, e.g., reducing the generalised cost of cross-service travel. For instance in the absence of the platform, cross-service travel might come at an increased generalised cost relative to in-service travel, associated with buying multiple tickets. We might think of this in terms of the vertical intercept for cross-service demand under the Free-Market having a lower intercept (α_x) than in-service demand (α): $\alpha_x < \alpha$. The platform then reduces the generalised cost of cross-service travel, bringing the cross-service α_x in line with the single-service level, α. Though we abstract away from this potential platform benefit in the analysis in the next Section, we explore the potential impact of such considerations on prices, profit and welfare in Section 5.3.

3.3 Passive Platform

In this platform regime, the operators set prices as in the Free-Market case but pay a share ϕ of cross-service revenue to the platform. The platform combines the cross-service component prices to form P_x^P but does not engage in strategic price setting.[8]

Each operator i solves the problem:

$$\max_{P_{ii},p_i} \pi_i = P_{ii}Q_{ii} + \frac{1}{2}(1-\phi)P_x \sum_{j\neq i}(Q_{ij}+Q_{ji}) \quad (i \neq j = 1, ..., n)$$

which results in the equilibrium in-service and cross-service prices:

$$P^P = \frac{3\alpha(1-\phi)(1-\gamma)\left[1+\gamma(n-1)\left(n+1-\frac{2\phi}{3}\right)\right]}{\Delta_1},$$
$$P_x^P = \frac{4\alpha(1-\gamma)\left[1-\phi+\gamma\left((1-\phi)n^2+\frac{3\phi}{2}-1\right)\right]}{\Delta_1}$$

where:

$$\Delta_1 \equiv 6(1-\phi)+\gamma(1-\phi)(n+1)(8n-11)$$
$$+2(n-1)\gamma^2\left[n^3(1-\phi)+\frac{n^2}{2}(1-\phi)+n(2-\phi)^2+\frac{5}{2}(1-\phi)\right] \geq 0$$

It is straightforward to show that prices here are not monotonic in ϕ. However, note that under this regime the second-order condition is not met across the full parameter set (even for $n = 2$), specifically for higher levels of ϕ. Noting that, given anecdotal evidence, interest is likely to be around very low levels of ϕ, we limit the profit-share parameter under the Passive regime to ensure interior solutions and that the second-order condition are satisfied:

Assumption 1. *To ensure internal solutions and that the second-order condition for the Passive platform is satisfied, let* $\phi^P \in \left[0, \frac{1}{3}\right)$

All future results including the Passive platform will be subject to this condition. Like the Free-Market case the Passive platform, therefore, is subject to both externalities, whereas the Integrator internalises the strategic substitute externality. However, as a platform regime it potentially benefits from reducing the generalised cost of cross-service travel relative to the Free-Market, as discussed above and again we will analyse the potential for this to materialise in price, profit and welfare in Section 5.3.

3.4 Intermediary Platform

In this model the operators set their prices as in the Free-Market case but the cross-service component prices that they set are not the final prices, rather they are the prices they charge the Platform, and the Platform takes these input prices and sets the cross-service price to maximise its profit across all cross-service sales.

Hence, we have a two-stage process with operators setting all their prices in the first stage and the Platform setting the cross-service prices, based on the operators' first-stage

[8] van den Berg et al. (2022) denote this the "Platform" regime. Since we apply the term platform to distinguish MaaS from non-MaaS regimes, we rename this case the Passive platform, reflecting the passive role that the provider has, taking a share of revenue but not engaging in setting prices.

choices. By backward induction, the Intermediary platform solves the following problem in stage two:

$$\max_{\boldsymbol{P_{ij}}} \pi^{im} = \sum_{j\neq i}^{n} \sum_{i} (P_{ij} - p_i - p_j) Q_{ij}$$

taking all P_{ii} and p_i as given, where $\boldsymbol{P_{ij}}$ is the $n(n-1)$-vector of cross-service prices. This results in the first-order condition:

$$\frac{\partial \pi^{im}}{\partial P_{ij}} = Q_{ij} - b[P_{ij} - p_i - p_j] + d \sum_{k\neq i,j}^{n} [P_{ik} - p_i - p_k] + d \sum_{k\neq i}^{n} [P_{ki} - p_i - p_k] + d \sum_{m\neq k,i}^{n} \sum_{k\neq i} [P_{mk} - p_m - p_k] = 0$$

Taking the total differential across all $n(n-1)$ first-order conditions, we have the following optimising Intermediary stage-two responses to a change in stage one prices:[9]

$$\frac{\partial P_{ij}}{\partial P_{ii}} = \frac{\partial P_{mk}}{\partial P_{ii}} = \frac{d}{2[b - d(n^2 - n - 1)]}, \quad \frac{\partial P_{ij}}{\partial p_i} = \frac{1}{2}$$

Taking these as given, in the first stage operator i solves the following problem:

$$\max_{P_{ii}, p_i} \pi_i = P_{ii} Q_{ii} + p_i \sum_{j\neq i}^{n} (Q_{ij} + Q_{ji})$$

yielding equilibrium input component price, and in-service and cross-service prices, respectively:

$$p^{IM} = \frac{4\alpha(1 + \gamma(n-1))(1-\gamma)}{\Delta_2}, \quad P^{IM} = \frac{\alpha[6 + \gamma(n^2 + 3n - 4)](1-\gamma)}{\Delta_2},$$
$$P_x^{IM} = \frac{2\alpha\left[20 + (4n^2 + 7n - 16)(n-1)\gamma^2 + (4n^2 + 28n - 38)\gamma\right](1-\gamma)}{\Delta_2(1 + \gamma(n-1))}$$

where $\Delta_2 \equiv 12 + (3n^3 - 8n^2 - 3n + 8)\gamma^2 + (4n^2 + 6n - 22)\gamma$.

3.5 Multi-operator Ticket Card

We now briefly introduce an alternative pricing system that currently operates in the UK allowing firms to jointly set cross-service prices under rules permitted by the Public Transport Ticketing Scheme Block Exemption (Competition Commission, 2001), henceforth Block Exemption. The recommended pricing rule set out by the Competition Commission (see Department for Transport, 2013, pp.22) for cross-service tickets is:

$$P_x = \text{“Average or median single fares x Estimated [typical] ticket usage x Passenger discount for purchasing a multi-journey ticket”} \tag{5}$$

and hence the cross-service price is set as a discount on the weighted average of the prevailing in-service prices on the network. We employ the simplified characterisation of this pricing

[9] This yields the two-stage game's closed-loop solution (e.g., see Fudenberg and Tirole, 1991, pp. 132).

structure in McHardy (2022) whereby we assume: (i) a zero discount (ensuring in the symmetric model that the cross- and in-service prices are the same), (ii) an average journey of one (the Block Exemption requires three or more) and, for now (iii) no potential transaction costs benefits of the scheme (e.g., gains from the need to buy only a single ticket rather than two individual tickets).

Operator i, therefore, solves the problem:

$$\max_{\{P_{ii}\}} \pi_i = P_{ii}Q_{ii} + (n-1)P_x Q_x - F$$

which from McHardy (2022), with symmetry and a zero discount, yields the common equilibrium price under the MTC:

$$P^M = \frac{(2n-1)(1-\gamma)\alpha}{(n^3 - n^2 - 4n + 2)\gamma + 4n - 2} \tag{6}$$

We also envisage a scenario where the MTC regime as set out in Eq. (5) is coordinated and delivered by a (passive) platform charging a percentage ϕ of cross-service revenue resulting in the uniform equilibrium price:

$$P^M(\phi) = P_x^M(\phi) = \frac{\alpha(n(2-\phi) + \phi - 1)(1-\gamma)}{(\gamma n^3 - \gamma n^2 + 2\gamma n\phi - 4\gamma n - 2\gamma\phi - 2n + 2\gamma + 4n + 2\phi - 2)} \tag{7}$$

Proposition 3. *The uniform price under the MTC is weakly decreasing in ϕ.*

4 Analysis

4.1 Free-Market versus Integrator

In this subsection we consider aspects of the relative performance of the Integrator regime compared with the Free-Market. In van den Berg et al. (2022) Proposition 1 under the setting of $n = 2$ it is found that the Integrator model is superior to the Free-Market case on consumer surplus and welfare terms. In essence, the Integrator, by internalising the externality that forces cross-service prices up under the Free-Market, benefits consumers and welfare. The following Proposition considers how the price story underlying this result generalises.

Proposition 4. *Whilst under the Integrator with $n \in \{2, 3\}$, in-service (cross-service) prices are weakly (strictly) lower than their equivalents under the Free-Market, the relative size of prices is ambiguous for larger n, and there exist open-intervals of parameter combinations (n, γ, ϕ), with sufficiently high γ (low ϕ), supporting a reversal in the ordering of in- and cross service prices.*

Corollary 2. *Whilst the Integrator strictly dominates the Free-Market on consumer surplus and welfare terms under $n \in \{2, 3\}$, the reverse ranking arises for $n \geq 4$ on some selection of γ and ϕ.*

Thus the consumer surplus and welfare superiority of the Integrator model over the Free-Market is not robust to a larger network: the policy prescription that moving from the Free-Market to an Integrator model is welfare enhancing is not generally true.

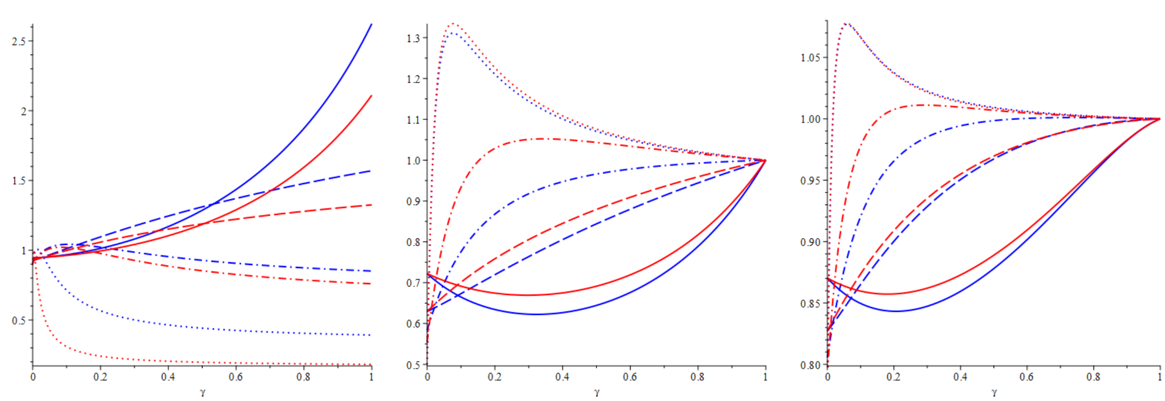
In terms of the intuition behind this result, whilst the Integrator internalises the strategic substitutes externality across component prices (preventing the cross-service prices exceeding the monopoly level) the negative effects of this externality are most strong when n is small and there is a low level of substitutability between the differentiated services. In addition, with the Integrator acting as a single agent setting all cross-service prices, it is able to stifle the intra-cross-service pro-competitive price substitutability effects. Under the Free-Market, for higher n and higher levels of substitutability, these interdependencies amongst the cross-service prices naturally add to the downward competitive price pressure across the network. Effectively, the Integrator, which is useful at countering the damaging cross-service effects with low n and γ becomes relatively harmful by also curtailing the strategic complementarity effects amongst the cross-service OD prices when n and/or γ are not low.

Regarding aggregate profit, it is straightforward to show that neither regime dominates in the case of $n = 2$ or more generally. However, the following Proposition indicates that, regardless of n, if the degree of substitutability is sufficiently low then the Integrator regime has a particular attraction.

Proposition 5. *For sufficiently low levels of substitutability the Integrator regime strictly dominates the Free-Market in aggregate profit, consumer surplus and welfare terms.*

Hence, even before we factor in non-price potential benefits of the platform, we can see the Integrator model can yield a Pareto improvement over the Free-Market under certain conditions, which is not tied to a low level of n.

Figure 1: Profit, Consumer Surplus and Welfare under the Free-Market relative to the Integrator with $\phi \in \{0, 1\}$



(a) Profit (b) Consumer Surplus (c) Welfare

Red $\phi = 0$, Blue $\phi = 1$; —— $n = 2$, - - - $n = 3$, -·-·- $n = 5$, ······ $n = 10$

Of course, as we will see later, it is not just aggregate profit across the regimes that might matter. In the platform regimes, profit is shared across the operators and the platform provider and differences in the relative size of these components across regimes can, depending on n, impact on incentives, for instance, in relation to potential to invest in services, or around the attractiveness of alternative regimes and ultimate choice of regime to support. In the two-operator case, van den Berg et al. (2022) observe that operator profit under the Integrator lies below the Free-Market level for sufficiently high levels of substitutability. However, the following Proposition shows that the reverse is true for a larger network.

Proposition 6. *Whilst for $n \in \{2, 3\}$, operator profit is strictly higher under the Free-Market than the Integrator for sufficiently high γ, this ordering is reversed for $n \geq 4$ and $\phi \in [0, \bar{\phi})$, where $\bar{\phi} \in (0, 1)$.*

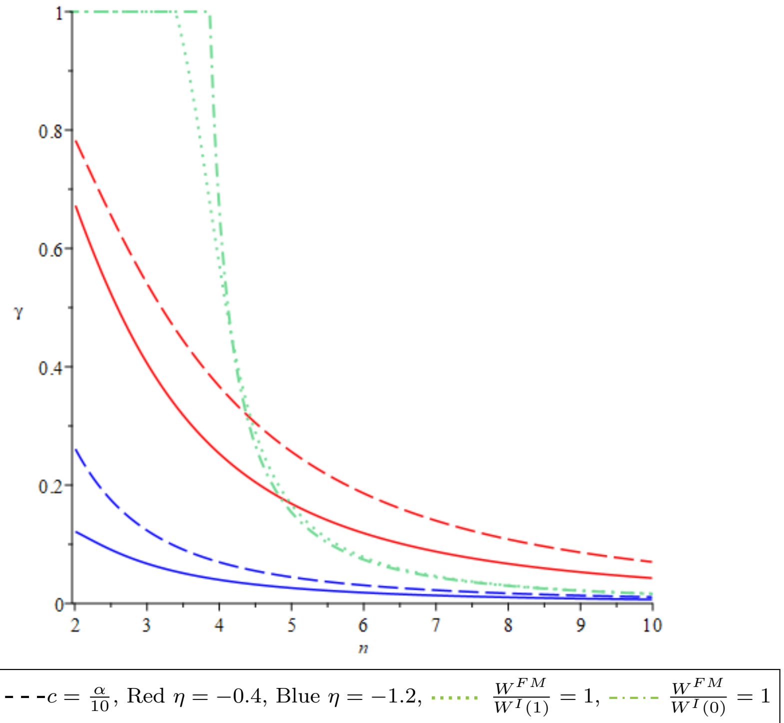
The intuition underlying this is that when n and γ are small, the Integrator is helping reduce prices relative to the Free-Market and in particular the profit damaging cross-service price, which under the Free-Market, exceeds the monopoly level in this region. However, for high levels of substitutability and low operator numbers the cross-service price under the Free-Market is not above monopoly levels and the price reducing effect of the Integrator damages profit. At higher levels of n the single price setter on cross-service prices under the Integrator better insulates the market from the intensity of independent prices across all n^2 services protecting profit except for very low γ where there is very little price competition under any regime. That under higher levels of n, Integrator profit can be higher than under the Free-Market when the services are strong substitutes, show van den Berg et al. (2022) Proposition 2 does not generalise.

Figure 1 reports ratios of profit, consumer surplus and welfare for the Free-Market relative to the Integrator regimes under $n \in \{2, 3, 5, 10\}$ and $\phi \in \{0, 1\}$. supporting Corollaries 1 and 2 and Propositions 5 and 6.

Figure 2 reports four calibration contours of a real Free-Market equilibrium pairing of n and γ, under zero marginal cost, $c = 0$, and constant marginal cost per passenger of $\frac{\alpha}{10}$, and two 'extremal' estimates of the average market price elasticity of demand.[10] Hence, if the market operates in line with the Free-Market model and the elasticity of demand is -0.4 and marginal cost is zero, then the market will be at an (n, γ) combination on the solid red line. The green lines in the Figure represent (n, γ) contours for which welfare under the Free-Market and the Integrator are equal under $\phi = 1$ (dotted) and $\phi = 0$ (dash-dot). Points to the right (left) of these contours have $\frac{W^{FM}}{W^{I}(\phi)} > 1$ (<1). Suppose we move from a situation where the market initially operates in line with the Free-Market model and the elasticity is -1.2, to the platform with the Integrator and the number of operators remains the same. Then, since the blue lines lie everywhere below the green lines in the Figure, in all cases considered here welfare will improve. On the other hand, the same change in regimes under the elasticity of -0.4 would result in a reduction in welfare for sufficient operator numbers, as the red lines lie above the green lines once n extends beyond 4 or 5.

[10] For a derivation of the calibrated market equilibria see McHardy (2022). The basis for the selection of elasticities in this exercise is long-run price estimates from a variety of studies including (i) Goodwin (1992) with elasticities for bus -0.6 and rail -1.1 based on an average of numerous other studies, (ii) Small and Winston (1999) U.S. urban (intercity) elasticities for rail -0.6 (-0.7), bus -0.9 (-1.2) and air -0.4

Figure 2: Calibrated Free-Market Equilibria (n, γ) Combinations for $\eta \in \{-0.4, -1.2\}$ and Marginal Cost $c \in \{0, \frac{\alpha}{10}\}$ and $\frac{W^{FM}}{W^{I}(\phi)} = 1$ contours for $\phi \in \{0, 1\}$



4.2 Passive Platform v Integrator and Free-Market

Analysis of the equilibrium prices across the Free-Market, Integrator and Passive models yields the following Proposition.

Proposition 7. *Under the Passive platform, (i) the in- and cross-service prices coincide with their Free-Market equivalents at either $\gamma = 0$ or $\phi = 0$, otherwise (ii) the in-service (cross-service) price is strictly lower (higher) than under the Free-Market.*

Proposition 8. *Whilst (i) aggregate profit, consumer surplus and welfare are identical under the Passive platform and Free-Market for $\gamma = 0$ and/or $\phi = 0$, (ii) for $\gamma \in (0, 1]$ and $\phi \in (0, \frac{1}{3})$, comparisons of welfare and consumer surplus across the Passive platform and Free-Market are ambiguous and aggregate profit is strictly greater under the Free-Market for $n \in \{2, 3, 4\}$ but otherwise ambiguous.*

It is important to note that in cases where welfare, consumer surplus and aggregate profit outcomes vary across the regimes, the differences are very small. In terms of intuition, first note that the Passive platform and the Free-Market regimes are identical under $\phi = 0$ in the absence of non-price setting aspects of the platform as we have assumed thus far. Further, note that under the Passive platform with $\phi > 0$ the operators are deterred from keeping cross-service prices down as they lose a share of the associated profit and price reductions on the cross-service services feeds into greater competition for the in-service prices where they have the full share of profit. Ultimately, the upward pressure on cross-service prices relative to under the Free-Market exaggerate the damage of high cross-service prices under the latter having a damaging impact on relative welfare.

Figure 3 reports the ratios of Free-Market and Integrator profit, consumer surplus and welfare relative to the Passive platform for $\phi \in [0, \frac{1}{3})$ (in line with Assumption 1). It is notable that the green lines, presenting the ratio of outcomes under the Free-Market relative to the Passive platform, are very close to unity and are not very sensitive to changes in n suggesting that the difference between the two regimes in profit, consumer surplus and welfare terms is not substantial.

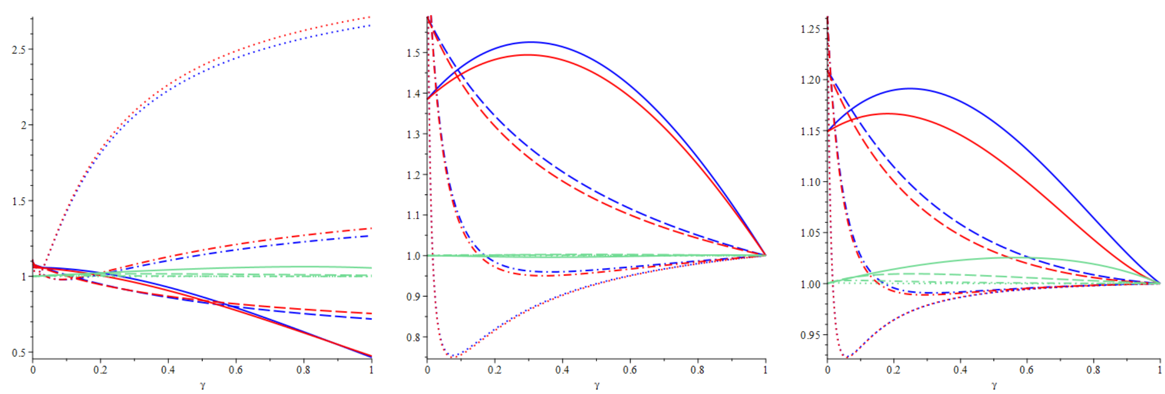
The following Proposition shows that the situation is a little more complex in relation to the comparison of the Passive platform against the Integrator.

Proposition 9. *Under the Passive platform, (i) the in-service (cross-service) price is weakly (strictly) higher than under the Integrator for $n \in \{2, 3\}$, but (ii) this does not generalise unambiguously for $n \geq 4$.*

Corollary 3. *Consumer surplus and welfare are strictly greater under the Integrator relative to the Passive platform for $n \in \{2, 3\}$, but this does not generalise to $n \geq 4$.*

Figure 3 reflects the reversal of the consumer surplus and welfare rankings in favour of the Integrator at lower levels of substitutability for higher levels of n. It is also clear from the Figure that the relative size of aggregate profit is ambiguous across the two regimes.

Figure 3: Profit, Consumer Surplus and Welfare under the Free-Market and Integrator relative to the Passive Platform with $\phi \in \{0, \frac{1}{3}\}$



(a) Profit (b) Consumer Surplus (c) Welfare

Red $\frac{R^I(0)}{R^P(0)}$; Blue $\frac{R^I(\frac{1}{3})}{R^P(\frac{1}{3})}$; Green $\frac{R^{FM}}{R^P(\frac{1}{3})}$;$R \in \{\pi, CS, W\}$; —— $n = 2$, - - - $n = 3$, -·-·- $n = 5$, ······ $n = 10$

4.3 Intermediary versus Integrator and the Free-Market

Comparison of the equilibrium prices across the Free-Market, Integrator and Intermediary regimes leads to the following results.

Proposition 10. *Under the Intermediary, the (i) in-service price is weakly greater than under the Free-Market and Integrator (with equality for both at $\gamma = 0$, and additionally for the latter at $n = 2$), (ii) cross-service price is strictly greater than under the Free-Market and Integrator.*

Corollary 4. *Consumer surplus and welfare are strictly greater under the Free-Market and Integrator than the Intermediary.*

The intuition here is straightforward. Prices under the Intermediary (weakly) exceed those under the Free-Market and the Integrator regimes resulting in worse consumer surplus and welfare outcomes in line with the double marginalisation that comes through the Intermediary setting cross-service prices based on operators intermediate component service prices. This places upward pressure on the $n(n-1)$ cross-service prices which have a strategic complement relationship with the n single-service prices.

Proposition 11. *Aggregate profit is strictly greater under the Free-Market and Integrator regimes than the Intermediary for sufficiently low n and γ.*

Corollary 5. *For sufficiently low γ and n, the Free-Market and Integrator regimes strictly dominate the Intermediary in profit, consumer surplus and welfare terms.*

4.4 Integrator versus MTC

We now turn to the *MTC* model of pricing and note at the outset that this regime has a strong attraction since McHardy (2022) shows it to weakly dominate the Free-Market regime in consumer surplus and welfare terms regardless of n.

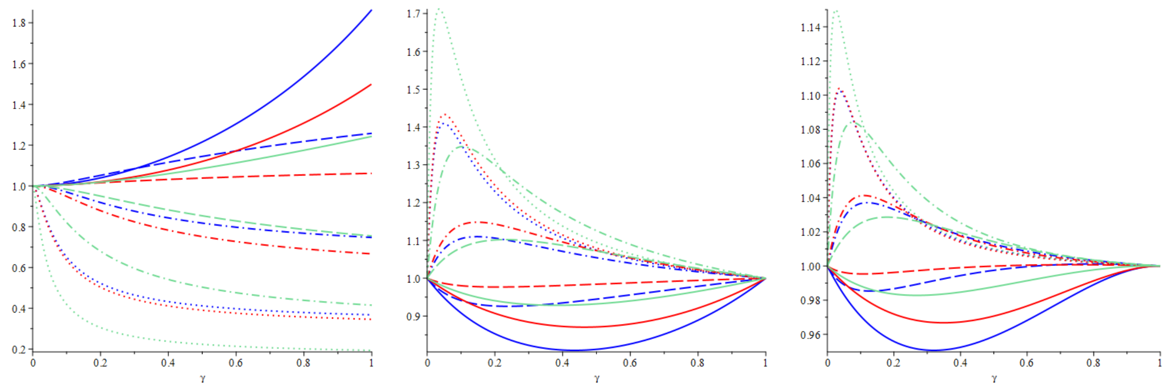
Proposition 12. *In the case of $\phi = 0$, the Integrator (i) prices are strictly lower than under the MTC for $n = 2$ and $\gamma \in (0, 1]$, with equality at $\gamma = 0$, and (ii) cross-service (in-service) price is strictly greater than under the MTC for $n > 3$ ($n > 4$) and $\gamma \in (0, 1]$ with equality at $\gamma = 0$.*

Corollary 6. *For $\phi = 0$ and $\gamma \in (0, 1]$, consumer surplus and welfare under the Integrator (i) are strictly greater than under the MTC for $n = 2$, (ii) are strictly lower than under the MTC for $n > 4$, (iii) are equal to that under the MTC with $\gamma = 0$.*

Hence, the Integrator regime has the potential to improve on the *MTC* on welfare terms over some selection of n and γ, emphasising the potential attractiveness of this platform. The following proposition shows that, although the above welfare gains are underpinned by consumer surplus gains, it is still able to maintain higher profits than the *MTC* at the same time.

Proposition 13. *(i) In the special case of $n = 2$ industry profit is weakly greater under the MTC than the Integrator with equality at $\gamma = 0$, (ii) In the case of $\phi = 0$ industry profit is also weakly greater under the MTC than the Integrator for $n \in \{2, 3\}$, but the inequality is reversed for $n \geq 5$, (iii) these results do not generalise across the domain of ϕ.*

Figure 4: Profit, Consumer Surplus and Welfare under $MTC(\phi)$ relative to the Integrator $I(\phi)$ with $\phi \in \{0, 1\}$



(a) Profit (b) Consumer Surplus (c) Welfare

Red $MTC(0), I(0)$; Blue $MTC(0), I(1)$; Green $MTC(1), I(1)$; —— $n = 2$, - - - $n = 3$, -·-·- $n = 5$, ······ $n = 10$

4.5 Profit Incentives

Having established that different platforms are attractive from a welfare or aggregate profit perspective at different levels of n, γ and ϕ, we now turn our attention to which ones would be preferred by operators and platform providers. Hence, we might be able to shed light on which regimes might be selected into by the operators and determine whether, for instance, there is a profit incentive at a given parameter setting for operators and platform providers to opt for a platform that performs badly from a welfare perspective at that parameter selection.

Beginning with the case of the platform providers, Proposition 14 reports the key findings in relation to the three platforms introduced earlier which involve the platform charging a fee equal to a share ϕ of cross-service revenue, earning profit, π_{plat}.

Proposition 14. *For $\phi \in [0, 0.1]$, the platform provider's profit, π_{plat}, is (i) strictly higher under the Integrator than the MTC for $n \in \{2, 3\}$ (with equality at $\gamma = 0$) and otherwise broadly greater if γ is not too low, (ii) strictly higher under for the Integrator relative to the Passive platform for $n = 2$, and broadly strictly greater elsewhere, (iii) strictly greater under the MTC than the Passive platform for $n = 2$, and otherwise if γ is sufficiently low.*

Hence, in the special case of $n = 2$ the platform provider would prefer the Integrator model over the MTC and Passive platforms, and this is broadly true for higher n, for sufficiently high levels of substitutability. The following Proposition now considers the case of the non-fee based platform, the Intermediary, whereupon we constrain $\phi \leq 0.1$ to allow comparisons of the fee-regimes in and around the anecdotal 0.02 level.

Proposition 15. *The Intermediary is strictly more profitable for the platform provider than the other platform regimes for $\phi \in [0, 0.1]$.*

Turning to the relative attractiveness of each regime to the operators, π_{op}:

Proposition 16. *For $\phi \in [0, 0.1]$, operator profit, π_{op}, is (i) broadly greater under the Integrator than the other regimes for $n \geq 5$ and, for $n \in \{2, 3, 4\}$, if γ is not too high, (ii) strictly greater under the Intermediary than the other regimes for $n = 4$ if γ is not too low, (iii) broadly greater under the Free-Market than the other regimes for $n \in \{2, 3\}$ if γ is not too low.*

Corollary 7. *Whilst there will generally be disagreement between operators and platform providers about the optimal choice of platform, in the case of $n = 4$ and γ not too low, there is a uniform preference for the Intermediary model which is strictly worse in welfare and consumer surplus terms than the Integrator and Free-Market regimes.*

One of the implications of Corollary 7 is that if there is a risk that n should be such that operators and platform providers are united in their preference for the Intermediary model, then encouraging a wider selection of operators will result in shifting operator preferences to the higher-performing Integrator model.

5 Network Extensions

We now explore the impact on the regime rankings of including a number of additional factors which feature in transport networks and their modelling but are currently outside the analysis of our Base Model.

5.1 Frequency selection with two operators

Until now we have been concerned with a model in which additional $\{x, y\}$ service pairs are offered by new 'rival' operators. However, in practice incumbent operators can generally offer a variety of services on a given route at different times and/or offer multiple alternative routes. We now seek to explore this multi-frequency/route possibility in an optimising framework. For brevity, henceforth, we refer to multi-frequency, whereby multi-route would be analytically equivalent. Hence, we move from a uni-frequency situation, in which there are n rival operators, each offering one $\{x, y\}$ service pair, to a multi-frequency one, where operator i offers n_i service pairs.

Before proceeding to a numerical exercise, it is important to understand how moving from a uni- to multi-frequency setting impacts on the strategic forces in the market under our different regimes. Recall that in the case of uni-frequency operators, an additional $\{x, y\}$ service pair brings one new in-service journey and $2(n-1)$ new cross-service journey combinations. However, in the multi-frequency setting an operator's new x and y components can additionally be combined as further in-service journeys with their existing x and y components. Hence, moving from a uni- to multi-frequency setting will change the balance of in- and cross-service journeys, increasing the former and reducing the latter. For a given network size moving to the multi-frequency case reduces the weight of cross-network pricing which is the source of the potentially damaging strategic substitute price effects discussed earlier: we would expect prices on services converted from cross- to single-service to be lower reducing potentially harmful effects associated with the strategic substitutability across cross-service pricing. In addition, for a given number of alternative OD journeys across the network there are more prices set independently in the uni-frequency case than under multi-frequency. This is because an operator with multiple frequencies will maximise profit setting the in-service and cross-service component prices simultaneously across its n_i $\{x, y\}$ service pairs. With fewer prices being set independently by independent operators, we would expect this to have an anti-competitive upward effect on prices. Given different regimes have different exposure to each of these two effects, the move to multi-frequency has the potential to alter the relative performance of different pricing regimes.

We now consider the optimising frequency choices of operators under the Free-Market and Integrator regimes assuming two rival operators ($n = 2$), and as indicated above we need to introduce costs. Let F be the fixed cost for providing a unit of frequency (an $\{x, y\}$ pair), so total fixed cost for operator i with service frequency n_i is n_iF. In order to advantage or disadvantage a lower price regime we also specify a marginal passenger cost for an operator i, which takes value $\bar{c}_i$ in the case the operator has zero passengers and changes at a rate ϵ with each in-service OD journey, and on a pro rata basis for each cross-service journey. Hence, the total cost for operator i is:[11]

$$C_i = \left\{\bar{c}_i + \epsilon\left[n_i^2 Q_{ii} + n_i n_j Q_x\right]\right\}\left(n_i^2 Q_{ii} + n_i n_j Q_x\right) + n_i F \tag{8}$$

where $\{.\}$ is marginal cost per full in-service OD passenger journey. Marginal cost is constant under $\epsilon = 0$, whilst $\epsilon < 0$ ($\epsilon > 0$) represents economies (diseconomies) in marginal costs per passenger.

It is also important to note some differences arising in the multi-frequency setting around demand and operator revenue. Given demand symmetry, operators will charge the same price across all their in-service journeys, P_{ii}, and their component to any cross-service price, p_i, will also be common. In- and cross-service demands are then, respectively:

$$\begin{aligned} Q_{ii} &= a - bP_{ii} + d\left[(n_i^2 - 1)P_{ii} + n_j^2 P_{jj} + 2n_i n_j P_x\right] \\ Q_x &= a - bP_x + d\left[n_i^2 P_{ii} + n_j^2 P_{jj} + (2n_i n_j - 1)P_x\right] \end{aligned}$$

where $P_x = p_i + p_j$ in the case of the Free-Market and P_x is selected by the platform in the case of the Integrator. Operator i under the Integrator now solves:

$$\max_{P_{ii}} \pi_i = n_i^2 P_{ii} Q_{ii} + (1-\phi) n_i n_j P_x Q_x - \left\{\bar{c}_i + \epsilon\left[n_i^2 Q_{ii} + n_i n_j Q_x\right]\right\}\left(n_i^2 Q_{ii} + n_i n_j Q_x\right) - n_i F$$

with $\phi = 0$ producing the objective for the operator in the Free-Market case. The Integrator now solves the problem:

$$\max_{P_x} \pi^{int} = \phi n_i n_j P_x Q_x$$

Table 1 reports the payoffs to operators 1 and 2 in the case of the Free-Market and Integrator where each operator has a choice of frequency, n_i. Two different sets of parametrisations are specified to investigate optimum strategic frequency selections with operator passenger cost economies and diseconomies ($\epsilon = -0.02$ and $\epsilon = 0.15$, respectively). The value of γ is selected to be in the region where profit under the Integrator is strictly greater than

[11] For example, see van den Berg et al. (2022), who use this cost specification in the case of two uni-frequency operators: $n = 2$ and $n_i = 1$.

under the Free-Market assuming two operators offering a single frequency and zero costs $\gamma \in \{0.10, 0.15\}$. The other parameterisations are then specified without loss of generality ($\alpha = 1$), based on anecdotal evidence ($\phi = 0.02$), or to ensure that there is a small set of viable frequencies under each regime with Nash equilibria supporting interior solutions for both operators ($F = 0.2$ and $\bar{c}_i \in \{0.05, 0.10\}$).[12]

Beginning with the case of cost economies ($\epsilon = -0.02$) combined with $\gamma = 0.15$ and $\bar{c}_i = 0.10$, Table 1 reports that, whilst there is a single Pure Strategy Nash Equilibrium (*PSNE*) under the Free-Market with each operator selecting a frequency of $n_i = 2$, it achieves a lower (expected) welfare than either the *PSNE* or Mixed Strategy Nash Equilibrium (*MSNE*) under the Integrator with a smaller (expected) aggregate frequency of $n_i + n_j = 3.0$.[13] In the relevant range the passenger economies of marginal cost aid the Integrator regime in welfare terms since it achieves higher total quantities relative to the Free-Market with the same frequency configuration to the extent that despite offering a lower number of frequencies, the smaller network under the Integrator with lower fixed cost exposure, offers higher welfare outcomes than the Free-Market.

Table 1: Multi-Frequency Profit Matrices with $\alpha = 1$, $\phi = 0.02$, $F = 0.2$, $\bar{c}_i \in \{0.05, 0.10\}$, $\epsilon \in \{-0.02, 0.15\}$, $\gamma \in \{0.10, 0.15\}$

$(\epsilon, \bar{c}_i, \gamma)$	Regime	Solution Characteristics	Expected n	Expected W
$(-0.02, 0.10, 0.15)$	Free-Market	PSNE $(n_i, n_j) = (2, 2)$	4.0	0.92
	Integrator	MSNE	3.0	0.94
	Integrator	PSNE $(n_i, n_j) = (1, 2)$	3.0	0.98
$(0.15, 0.05, 0.10)$	Free-Market	PSNE $(n_i, n_j) = (2, 2)$	4.0	0.51
	Integrator	PSNE $(n_i, n_j) = (1, 1)$	2.0	0.48

On the other hand, in the case of cost diseconomies ($\epsilon = 0.15$) with $\bar{c}_i = 0.05$ and $\gamma = 0.10$, each regime has a single *PSNE* where the Free-Market provides two frequencies per operator relative to one each for the Integrator and achieves a higher level of welfare.[14] We know (i) from Proposition 4 that with 2 or 3 uni-frequency operators and zero costs, the Integrator regime yields lower prices and higher quantities than the Free-Market regime, and (ii) under $\gamma \in \{0.10, 0.15\}$ with two uni-frequency operators the former is also strictly more profitable than the latter. Cost diseconomies are working against the Integrator reversing its profit superiority over the Free-Market resulting in it being unable to profitably sustain the same size network as the latter such that the latter both provides greater frequency but also covers the larger associated fixed costs to offer a welfare premium over the smaller Integrator network. On the other hand, with cost economies the Integrator is still unable to support as large a network as the Free-Market due to downward pressure on prices in the former, yet here the lower prices, with cost economies translate into higher welfare.

[12] We allow some part of each operator's fixed cost for their first unit of frequency to be sunk ruling out the same Nash equilibria under both regimes with a large Monopoly - i.e., one operator preferring zero profit from operating no services relative to a small loss in an interior solution.

[13] In the case of the Intermediary, there is no interior Nash equilibrium: the *PSNE* $(1, 3)$ involves a loss-making operator and the *MSNE* involves negative expected profits and associated welfare is below 0.90 in each case.

[14] In the case of the Intermediary, the Nash equilibrium is $n_i = 2$ but welfare is below that for the other regimes at 0.46.

5.2 Single- and Round-trip Demand

The analysis so far has been based on the assumption that all journeys involve the two component parts and hence rule out single-trip or domestic travel. This is potentially important as the existence of the latter demand types will have differential impacts on different regimes depending whether operators are able to separate single-component prices from dual-component prices. Hence, we now introduce a new traveller type where demand is for operator i's single-x component is X_i:

$$X_i = A - Bp_i + D\left(\sum_{j\neq i}^{n} p_j\right) \tag{9}$$

To keep matters simple we assume that the demand for X and Y components are identical allowing us to continue with a common x or y component price for operator i, p_i. Constrained optimisation of a suitably augmented utility function yields the above linear single-component travel demands where:[15]

$$A = \frac{\mu\alpha}{1+\gamma(n-1)}, \quad B = \frac{1+\gamma(n-2)}{[1+\gamma(n-1)](1-\gamma)}, \quad D = \frac{\gamma}{[1+\gamma(n-1)](1-\gamma)}$$

where the parameter $\mu \in [0,1]$ determines the relative strength of demand for the single- relative to dual-component journeys.[16] Higher values of μ increase the market size of single- relative to dual-component travel.

In the Free-Market setting, operators are not able to separate the single-component prices from their cross-service prices: $P_{ij} = p_i + p_j$. Hence, operator i solves:

$$\max_{P_{ii},p^i} \pi_i = P_{ii}Q_{ii} + p_i(X_i + Y_i) + \sum_{k\neq i}^{n} p_i(Q_{ik} + Q_{ki}) \tag{10}$$

Under the Integrator however, the operator's prices on the single-component journeys are distinct from the prices of cross-service travel. Under this regime, operator i solves the problem:

$$\max_{P_{ii}} \pi_i = P_{ii}Q_{ii} + p_i(X_i + Y_i) + (1-\phi)\sum_{j\neq i}^{n} P_{ij}Q_{ij} \tag{11}$$

whilst the Integrator solves the same problem as before in Eq. (3), with the constraint that $P_{ij} \leq p_i + p_j$ to avoid travelers buying two single-component tickets in place of the Integrator's equivalent cross-service ticket. Of course, they do not have the constraint that when they raise or lower single-component prices, their component of the cross-service price is raised or lowered accordingly. We would, therefore, expect the absence of this constraint

[15] We augment our earlier utility function according to: $U(\boldsymbol{Q},\boldsymbol{X},\boldsymbol{Y},M_0) = \alpha\sum_t^N Q_t + \mu\alpha\sum_k^n (X_k + Y_k) - \frac{1}{2}\left[\sum_t^N Q_t^2 + 2\gamma\sum_t^N\sum_{r>t} Q_tQ_r\right] - \frac{1}{2}\left[\sum_k^n (X_k^2 + Y_k^2) + 2\gamma\sum_k^n\sum_{m>k}(X_kX_m + Y_kY_m)\right] + M_0$.

[16] Lin (2004), for instance introduces single-component demands in the $n = 2$ case where $A = a$, $B = b = 1$ and $D = d$, effectively setting $\mu = 1$.

in the Integrator case to impact differentially on the strategic forces at play relative to the Free-Market regime and focus our attention on these two cases.[17]

Figure 5 illustrates the aggregate profit and welfare ratios for the Integrator relative to Free-Market under two specifications of the relative size of single versus dual demand: $\delta \in \{0.5, 1.0\}$.[18] The green lines represent the situation in the absence of single demand replicating the earlier analysis for comparative purposes. The complex interplay of the impact of the introduction of single demands on strategic pricing incentives is clear from the Figure with evidence of non-monotonicities in both welfare and profit terms. However, with the green lines typically sitting above their red and blue counterparts in both the welfare and profit cases, it is clear that the introduction of the single demands is acting to benefit the performance of the Free-Market relative to the Integrator on both measures. In terms of intuition, the single demands are having a constraining effect on the damaging externality that pushes cross-service prices above the monopoly level under the Free-Market and consequently improves it profit and welfare performance. This serves to suggest that an analysis of the gains of the platform without single demands will tend to overstate the associated gains relative to the Free-Market since the latter's performance problems are eased by the existence of single demand. Clearly, however, the blue lines do not everywhere sit above or below the red equivalents, meaning we cannot infer that a stronger single demand always results monotonically in a better result for the Free-Market.

5.3 Wider Platform Benefits

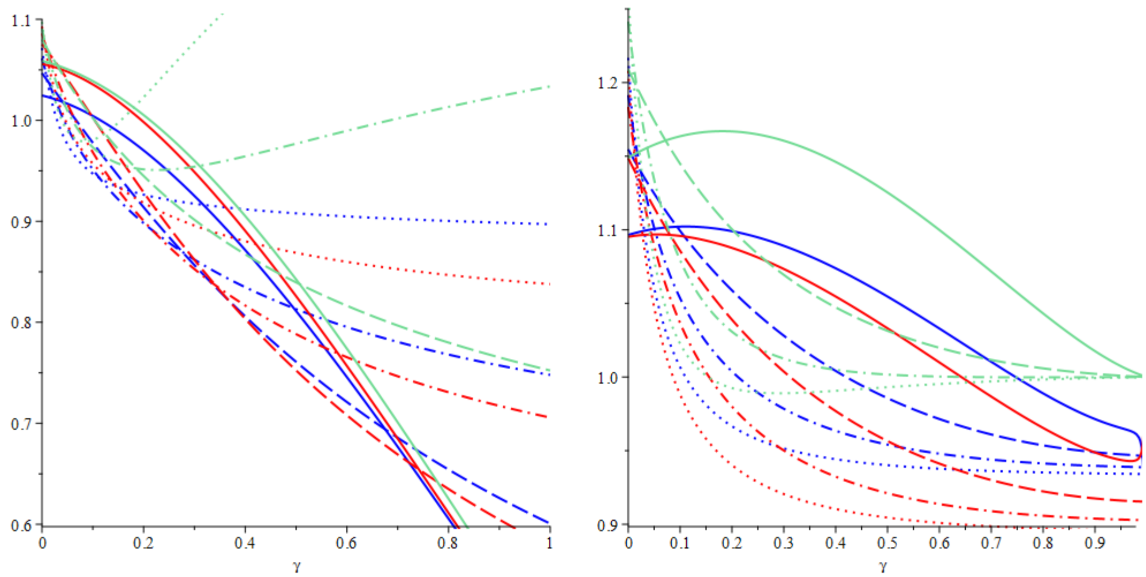
In this Section we briefly explore the implications of the platform offering value-added in the cross-service journey. We do this by allowing the Free-Market provision of cross-service travel to come at an elevated generalised cost, for instance associated with having to engage with two separate operators involving higher transactions costs. Let the term α in utility be reduced to $\alpha_x = (1-\rho)\alpha$ for cross-service journeys under the Free-Market, where $\rho \in [0, 1]$ is increasing in the size of the cross-service penalty for having to use two operators. In the case of the platform providers, we allow $\rho = 0$, since the platform eliminates the need to deal with more than one organisation in arranging cross-service travel. Hence, we need only update the non-platform case of the Free-Market. Implementing the cross-service penalty, ϕ yields equilibrium prices under the Free-Market:

$$
\begin{aligned}
P^{FM}(\rho) &= \frac{\alpha(1-\gamma)\left[3+\gamma(n^2-1)(3-2\rho)\right]}{(6+\gamma(2n^2-3n-5))(1+\gamma(n^2-1)} \\
P_x^{FM}(\rho) &= \frac{2\alpha(1-\gamma)\left\{\gamma(n+1)[2n(1-\rho)+3\rho-2]+2(1-\rho)\right\}}{(6+\gamma(2n^2-3n-5))(1+\gamma(n^2-1)}
\end{aligned} \tag{12}
$$

[17] We do not report the cases of the Passive and Intermediary Platform regimes as, the former produces extremely similar outcomes to the Free-Market case in the presence of single- and round-trip demand under the same parameterisations indicating that it remains very similar to the Free-Market regime with welfare ratios across the regimes very close to unity. On the other hand analysis of the Intermediary does not have interior solutions for large parts of the parameter set studied here.

[18] It is important to note that the analysis here involves interior solutions in all cases except $\delta = 0.5$ with $n \in \{4, 5\}$ where upon as γ increases $p^I < 2P_x^I$, whereupon passengers could undertake cross-service journeys more cheaply combining two single tickets.

Figure 5: Profit and Welfare with Single- and Round-trip Demand under the Integrator relative to the Free-Market with $\delta \in \{0.5, 1.0\}$ and $n \in \{2, 3, 4, 5\}$



(a) Profit (b) Welfare

Blue $\delta = 0.5$, Red $\delta = 1.0$, Green - No Single Demand; —— $n = 2$, - - - $n = 3$, -·-·- $n = 4$, ······ $n = 5$

A priori, it might be supposed that imposing a hit to surplus on cross-service travel under the Free-Market would unambiguously damage the performance of this regime in profit, consumer surplus and welfare terms relative to the platform regimes which do not experience the surplus penalty.

However, McHardy (2022) shows that the Free-Market in- and cross-service prices are weakly falling in ρ such that, despite the hit to cross-service surplus due to the additional generalised cost, for sufficiently low levels of γ and n, there is an interval of ρ over which profit and welfare are increasing in ρ. This has the following implications for comparisons with the platform regimes.

Proposition 17. *The introduction of a generalised cost penalty on cross-service travel in the Free-Market case (i) may improve or worsen its performance in profit and welfare terms relative to the platform regimes which do not incur the penalty, and, (ii) in particular can reverse the consumer surplus and welfare ranking in favour of the Intermediary for sufficiently high γ.*

Hence, whilst the introduction of cross-service generalised cost penalties under the Free-Market regime can change the size of profit and welfare either up or down relative to the various platform regimes, and in some cases change the rankings on these performance criteria, for sufficiently high γ it can invalidate Corollary 4, with the Intermediary now being able to dominate the Free-Market in consumer surplus and welfare terms.

6 Conclusions

Recent interest in the potential for platform systems to offer integrated passenger transport services across a range of operators and modes has resulted in various insights into associated gains in terms of passenger mobility, accessibility and pollution. Beyond this, however, the platform structure has the potential to partially resolve a strategic interaction externality that can result in the Free-Market performing badly in profit and welfare terms. Indeed, van den Berg et al. (2022) in their analysis of pricing outcomes under MaaS business platform models, conclude, amongst other things, that the Integrator platform offers welfare gains over the Free-Market outcome, with the result driven by the platform's ability to mitigate this externality.

However, the analysis here is undertaken in a 2-agent setting and McHardy (2022) demonstrates that results in this type of model with $n = 2$ do not necessarily generalise to more agents. This paper re-examines the relative performance of the platform models in the n-agent setting. Regarding the Integrator model, which for $n = 2$ was everywhere superior in consumer surplus and welfare terms to the Free-Market, this does not generalise beyond a low level of operators with a reverse ordering of the regimes arising for $n \geq 4$ in consumer surplus and welfare terms. Indeed, a calibration exercise based on real-world elasticities shows there are circumstances where these elasticities are consistent with the Free-Market dominating the Integrator in welfare performance. However, we also show that there are circumstances under which the Integrator dominates the Free-Market in aggregate profit, consumer surplus and welfare terms for $n > 2$ more generally, and this is before taking into account any potential non-price platform benefits. A reversal in the performance ranking is also demonstrated in relation to other regimes.

We also consider a pricing scheme (Multi-Operator Ticketing Card) currently permitted in the UK in which firms are allowed to collude on the setting of a cross-service price. This is shown in McHardy (2022) to strictly dominate the Free-Market in consumer surplus and welfare terms. However, for low levels of n the Integrator dominates the MTC in consumer surplus and welfare, whilst also offering potential for higher profit.

We also consider two extensions to the modelling framework, allowing single and return demands and allowing each operator to provide more than one service with passenger cost economies. We show that the addition of real-world transport network characteristics can alter the welfare rankings of the Free-Market and Integrator regimes, hence indicating the importance of including these in modelling scenarios.

Turning to the incentives of operators and platform providers to adopt different platform regimes, we find that the Intermediary is the most profitable model for the platform providers. Whilst the Integrator regime is broadly the most profitable choice for operators if n in not too low and substitutability is not too high, there are parameter settings where the operators and platform suppliers are unanimous in opting for the Intermediary. This presents a potential problem as this platform performs worse than the Free-Market and Integrator platform in consumer surplus and welfare terms.

Finally, the analysis thus far was done assuming no additional effects or benefits arising from the platform provision. When we introduce a generalised cost penalty on Free-Market cross-service travel that is resolved by the platform, the welfare ordering was be reversed in favour of the Intermediary relative to the Free-Market for sufficiently high γ and n. Whilst it might be thought that the introduction of a cross-service generalised cost penalty under the Free-Market would necessarily worsen relative profit and welfare outcomes for the regime, we show that this need not be the case. Indeed, it is possible that the penalty can also help resolve a strategic externality under the Free-Market improving its relative performance.

Appendix

A Proof to Propositions

A.A Proof to Proposition 1

(i) This follows directly from observation, setting $\gamma = 0$ in Eq. (4). (ii) First, note that for interior solutions $\Delta > 0$. Then, we can write the partial derivative $\frac{\partial P^I}{\partial \phi} = -\frac{(n-1)}{\Delta}[\alpha(1-\gamma) + n\gamma^2 P^I]$, which is strictly negative for interior solutions in the relevant range. Similarly, $\frac{\partial P_x^I}{\partial \phi} = -\frac{n\gamma^2(n-1)P_x^I}{\Delta}$, which is weakly negative for interior solutions in the relevant range.

A.B Proof to Proposition 2

It is straightforward to see, using a maths program, that $\frac{\partial \pi^I}{\partial \phi} \leq 0$ has solutions on the full parameter set with equality for $\gamma \in \{0, 1\}$, and no solutions in the relevant range for the reverse inequality, completing the proof.

A.C Proof to Proposition 3

The partial derivative of the uniform MTC price with respect to ϕ is:

$$\frac{\partial P^M}{\partial \phi} = -\frac{(1-\gamma)\gamma n^2(n-1)^2}{[\gamma n^3 - \gamma n^2 + 2(2-\phi)(1-\gamma)n - 2(1-\phi)(1-\gamma)]^2} \leq 0$$

where the denominator is non-negative and the numerator is zero for $\gamma \in \{0, 1\}$.

A.D Proof to Proposition 4

Let $H^R \equiv \frac{R^{FM}}{R^I(\phi=0)}$, where $R \in \{P, P_x\}$, which are continuous in (n, γ). Under $n \in \{2, 3\}$, $H^P = 1$ has a single solution at $\gamma = 0$, there are no solutions for $H^{P_x} = 1$, and there exist a parameter combinations, e.g., $\phi = 0$, $n \geq 4$ and $\gamma > \frac{2}{2n^2 - 8n + 3}$, where $H^R < 1$ which, given continuity of H^R and R^I which are non-increasing in ϕ (by Proposition 1), also means there are non-empty intervals for $\phi > 0$ where $H^R < 1$, completing the proof.

A.E Proof to Proposition 5

Let $H^{CS} \equiv \frac{CS^{FM}}{CS^I}$, $H^W \equiv \frac{W^{FM}}{W^I}$, and $H^\pi \equiv \frac{\pi^{FM}}{\pi^I}$. Since $\lim_{\gamma \to 0} H^{CS} = \frac{5+4n}{9n}$, $\lim_{\gamma \to 0} H^W \approx \frac{(20n+7)}{27n}$ and $\lim_{\gamma \to 0} H^\pi = \frac{8n+1}{9n}$, there is a half-open set on γ: $\gamma \in [0, \bar{\gamma})$ where H^{CS}, H^W and H^π are all strictly below unity for all selections of n and ϕ in the relevant range, completing the proof.

A.F Proof to Proposition 6

(i) follows directly from observing Figure 1(a). (ii) Let $H = \frac{\pi^{FM}}{\pi^I(\phi=0)}$. Note that H is continuous in the relevant range. For $n \geq 4$, solving for $H = 1$ yields two contours, $\gamma_1 \equiv$

$\frac{2}{2n^2-8n+3}$ and $\gamma_2 \equiv RootOf(-8+(10*n^5-28*n^4+7*n^3+10*n^2-11*n+24)*_Z^3+(8*n^4-2*n^3+14*n-52)*_Z^2+(-8*n+36)*_Z$. Solving for $\gamma_1-\gamma_2>0$ we find this is strictly satisfied for all $n \geq 4$, and $\lim_{n\to}\gamma_u = 0$ $(u \in \{1,2\})$. Finally, between the two contours $H>1$, whilst outside each contour $H<1$. Hence, at $\phi=0$, yielding the maximal operator profit under the Integrator, dominates the Free-Market on operator profit for $n \geq 4$ and $\gamma > \gamma_1$, and given continuity of H, ensures $H<1$ is also satisfied on these terms in an open interval of $\phi>0$, completing the proof.

A.G Proof to Proposition 7

Using a solve command in a maths program shows the full parameter set satisfies $P_x^P \geq P_x^{FM}$ $(P^P \leq P^{FM})$, with equality at either $\gamma=0$ or $\phi=0$ and strict inequality otherwise.

A.H Proof to Proposition 8

First, trivially, the two models coincide at $\phi=0$ and additionally yield the same prices under $\gamma=0$. Otherwise, simple animated plots over the domains of ϕ and γ and over a selection of n clearly show ambiguity in the relative performance of the regimes. Using a solve command in a maths program reveals aggregate profit comparisons to unambiguously favour the Free-Market for $n \in \{2,3,4\}$, which does not generalise to locally higher levels of n.

A.I Proof to Proposition 9

Performing a 3D plot of $H \equiv \frac{P^P}{P^I}$ and $H_x \equiv \frac{P_x^P}{P_x^I}$ for $\gamma \in [0,1]$ and $\phi \in (0,\frac{1}{3})$, reveals H_x everywhere strictly greater than one and the same for H except at $\gamma=0$ where $H=1$. Noting that the ratio relative to unity is otherwise ambiguous completes the proof.

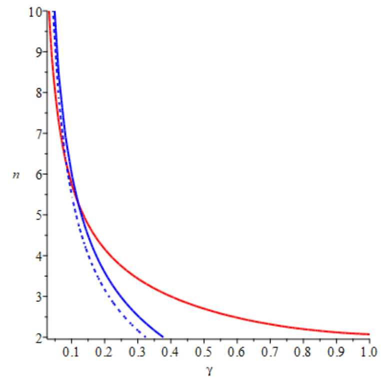
A.J Proof to Proposition 10

Let $H^R \equiv \frac{P^{IM}}{P^R}$ and $H_x^R \equiv \frac{P_x^{IM}}{P_x^R}$, where $R \in \{FM, I\}$. Solutions for $H^R \geq 1$ and $H_x^R > 1$ are across the entire parameter set with no solutions in the parameter set for the reverse inequalities. $H^R=1$ under $\gamma=0$, and also $H^{FM}=1$ for $n=2$ completing the proof.

A.K Proof to Proposition 11

The proof is easiest to communicated with reference to Figure 6 which plots three contours in (γ,n)-space where ratios of aggregate profit across regimes are equal to unity. The red contour represents the full set of (γ,n) combinations for which aggregate profit under the Intermediary is equal to that under the Free-Market. The blue lines capture the equivalent for the case of the Intermediary and Integrator with the solid (dashed) line in the case that the Integrator regime involves zero (full) payment of revenues to the Integrator, $\phi=0$ $(\phi=0)$. (γ,n) combinations below (above) the contours represents cases where aggregate profit is lower (higher) for the Intermediary relative to the other regime.

Figure 6: Equality Contours for Aggregate Profit under the Intermediary relative to the Free-Market and Integrator for $\phi \in \{0, 1\}$



Blue $\frac{\pi^M}{\pi^I(\phi)=1}$, Red $\frac{\pi^M}{\pi^{FM}} = 1$; —— $\phi = 0$, - - - $\phi = 1$

A.L Proof to Proposition 12

The proofs follow directly from examination of solve results for price ratios equal to one and strictly greater than one in a maths program.

A.M Proof to Proposition 13

The proofs follow directly from examination of solve results for price ratios equal to one and strictly greater than one in a maths program.

A.N Proof to Proposition 14

(i) Let $H^{MI} \equiv \frac{\pi^M}{\pi^I}$, which is continuous. For $n \in \{2, 3\}$, the former has a single solution for unity at $\gamma = 0$ and solutions everywhere else on the parameter set for $H < 1$. A 3D plot with animation reveals $H < 1$ is broadly true across the parameter set for $n > 3$. (ii) Let $H^{PI} \equiv \frac{\pi^P}{\pi^I}$, which is continuous. Solutions across the entire parameter set support $H^{PI} < 1$ for $n = 2$, and a 3D plot with animation reveals $H < 1$ is broadly true across the parameter set for $n > 3$. (iii) Let $H^{MP} \equiv \frac{\pi^M_{plat}}{\pi^P_{plat}}$, which is continuous. Given there are no solutions for $H^{MP} = 1$ for $n = 2$ and $H^{MP} > 1$ everywhere for $\gamma = 0$, given continuity and for $n/geq3$, $H^{MP} > 1$ for $\gamma = 1$, completes the proof.

A.O Proof to Proposition 15

Let $H^R \equiv \frac{\pi^{IM}_{plat}}{\pi^R_{plat}}$, where $R \in \{I, M, P\}$. Given the continuity of all H^R in the relevant interval, with no solutions for $H^R = 1$ and the existence of co-ordinates for $H^R < 1$ in the relevant range, completes the proof.

A.P Proof to Proposition 16

Let $H^{ef} \equiv \frac{\pi_{op}^{e}}{\pi_{op}^{f}}$, $(e \neq f = I, FM, IM, P, M)$. (i) First, studying 3D animated plots for H^{IM} and H^{IFM} reveals that H^{IM} is broadly greater than 1 across the parameter set, whilst this is broadly the case for H^{IFM} for $n \geq 4$. Second, note there are no solutions for $H^{IP} < 1$ and $H^{IIM} < 1$ for $n \geq 5$. Third, note H^{IIM} and H^{IP} are continuous and both are strictly greater (less) than 1 at $\gamma = 0$ $(\gamma = 1)$ for $n \in [2, 4]$. Noting, from above that the Integrator, which is inferior (superior) to the Intermediary for γ not too low (not too high), broadly everywhere dominates the Free-Market and MTC for $n = 4$. Fourth, note H^{IFM} is continuous and strictly greater (less) than 1 at $\gamma = 0$ $(\gamma = 1)$ for $n \in \{2, 3\}$, completing the proof. (ii) In addition to the above, note that H^{FMIM} is continuous and strictly greater (less) than 1 for $n \geq 4$ and $\gamma = 0$ $(\gamma = 1)$, completing the proof. (iii) In addition to the above, note that $H^{FMIM} > 1$ for $n \in \{2, 3\}$ and that $H^{FMP} \geq 1$ for $n \in \{2, 3\}$ with equality at $\gamma = 0$, completing the proof.

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