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The Error Correction Model as a Test for Cointegration

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Abstract

In this paper we generate critical values for a test for cointegration based on the joint significance of the levels terms in an error correction equation. We show that the appropriate critical values are higher than those derived from the standard F-distribution. We compare the power properties of this test with those of the Engle-Granger test and Kremers et al's t-test based on the t-statistic from an error correction equation. The F-test has higher power than the Engle-Granger test but lower power than the t-form of the error correction test. However, the F-form of the test has the advantage that its distribution is independent of the parameters of the problem being considered. Finally, we consider a test for cointegration between UK and US interest rates. We show that the F-test rejects the null of no cointegration between these variables although the Engle-Granger test fails to do so.

Keywords: Cointegration, error correction.

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I. INTRODUCTION

The use of the error correction model in applied econometrics goes back to Sargan (1964). However, its integration into modern time series econometrics began with the publication of two important papers in the mid 1970s. These were the analysis of the UK consumption function by Davidson, Hendry, Srba and Yeo – DHSY - (1978) and that of the UK demand for broad money by Hendry and Mizon (1978). These papers were important because they emphasised the potential importance of levels terms within a time series regression framework as a means of capturing the equilibrium interactions between variables.

Consider the following model set out by Hendry and Mizon which relates a variable y_t to its own lagged value and the current and lagged values of another variable x_t:

$$y_t = \beta_1 y_{t-1} + \gamma_0 x_t + \gamma_1 x_{t-1} + v_t \tag{1}$$

where $|\beta_1| < 1$ and v_t has zero mean, constant variance σ^2 and is serially independent. It is straightforward to see that equation (1) can be reformulated to give:

$$\Delta y_t = \gamma_0 \Delta x_t + (\beta_1 - 1) y_{t-1} + (\gamma_0 + \gamma_1) x_{t-1} + v_t \tag{2}$$

Given this formulation, a test of the joint hypothesis that the coefficients on y_{t-1} and x_{t-1} are zero is effectively a test of the hypothesis that the *y* and *x* processes have a common root equal to one i.e. it is appropriate to estimate the equation in first differences. Hendry and Mizon applied a model of this type to the UK demand for money function while

DHSY applied a similar model to the UK consumption function and in both cases rejected the common unit root assumption.

At the time this approach to time series model building was criticised by Williams (1978) on the grounds that "*A recent paper by Hendry and Mizon (1978) has suggested that it is possible to test whether a particular relationship should be estimated in levels or first differences by re-arranging the levels formulation into a first-difference formulation and a 'remainder' and then testing whether or not the coefficients in the 'remainder' are statistically significant. The fallacy in this approach is that in order for the estimation technique to be valid, it must be assumed that the error structure in the levels formulation is stationary*". In retrospect this statement is a neat summary of the cointegration problem which went on to become the major topic of research in time series econometrics for the next two decades.

Since the publication of the above articles our understanding of the theoretical and empirical properties of cointegrating relationships has increased enormously. Testing procedures have been developed by Engle and Granger (1987) and Kremers et al (1992) for single equation models and Johansen (1988) for multiple equation systems. The Engle-Granger procedure is to apply the Augmented Dickey-Fuller test to the residuals from a least squares regression between the levels of the variables. Appropriate critical values for this test have been computed by MacKinnon (1991). Kremers et al estimate an error correction model and use the t-ratio for the error correction term as their test statistic. Unfortunately the distribution of the test statistic depends on unobservable parameters of the specific problem and thus it may not be possible to implement the test in practice. This test has been investigated further by Zivot (1996) and Ericsson and MacKinnon (2002). Implementation of this test is considerably easier when the cointegrating vector is know prior to estimation. However, in many circumstances this is not a reasonable starting assumption. Finally, Banerjee et al.(1998) estimate an error correction model test, based on the null hypothesis of non-cointegration, but with the t-ratio version of the test suffering in finite samples when one tries to impose potentially invalid common-factor restrictions.

The focus of this paper is on testing for a single cointegrating vector. In section II we derive critical values for an alternative test based on the joint significance of the levels terms in an error correction model. We demonstrate that these critical values differ from those derived from the standard F-distribution in that they are consistently larger. In section III we compare the performance of our test statistic with the Engle-Granger test and the Kremers et al test. We show that our test has more power in rejecting a false null hypothesis when compared with the Engle-Granger test. We also show that our testing procedure has an advantage over the Kremers et al test in that it generates critical values which are not sensitive to the parameters of the particular error correction model we estimate. Section IV presents an example using monthly data for UK and US interest rates and Section V gives our conclusions.

II. CRITICAL VALUES FOR AN F-TEST FOR COINTEGRATION

We begin by assuming a general bivariate data generation process for y_t and x_t, $t = 1, \ldots T$. This is set out in equations (3)-(5).

$$\Delta y_t = \alpha_1 \Delta x_t + \alpha_2 y_{t-1} + \alpha_3 x_{t-1} + \varepsilon_{1,t} \tag{3}$$

$$\Delta x_t = \varepsilon_{2,t} \tag{4}$$

$$\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \sim IN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \right) \tag{5}$$

Next, we consider the simplest possible case in which y and x are independent random walks. This means we set $\alpha_i = 0; i = 1,2,3$. Based on these parameters we generate 10,000 replications using seeded pseudo random values for ε_1 and ε_2 generated by the EViews random number generator. We then estimate equation (3) by OLS and perform an F-test for the null hypothesis $H_0 : \alpha_2 = \alpha_3 = 0$ against the alternative $H_1 : \alpha_2 \neq 0$ or $\alpha_3 \neq 0$ for various sample sizes. In each case we discard the first 50 observations to avoid the danger of the start-up values biasing the results. The results are then tabulated and used to calculate empirical 10%, 5% and 1% critical values for the test statistic. The results are presented in Table One along with the critical values from the standard F-distribution. The values calculated are all considerable higher than those given by the critical values from the conventional F-distribution reflecting the fact that under the null hypothesis the series are not stationary and therefore classical statistical distribution results do not apply.

[Insert Table One here]

The critical values given in Table One are calculated on the basis of particular parameter values for the DGP. We also investigated the sensitivity of our results to changes in these parameter values in two ways. First, we allowed the relative variances of the y and x processes to change. We varied σ_1^2 / σ_2^2 within the range 10^{-8} and 10^{8} and found that the Monte Carlo critical values were completely insensitive to this ratio. Second, we experimented with values for α_1 within the range $-1 \leq \alpha_1 \leq 1$ and again found that our critical values were insensitive to the value chosen. All results in Table One are for a 'large' sample of 500 observations. Our conclusion is that the critical values we derived are not sensitive to the particular experimental design we have adopted.

Kremers et al adopt an alternative approach to testing for cointegration within an error correction framework. They begin by assuming that there is a 'natural' cointegrating vector for which we may wish to test. This often arises when there is a unit elasticity restriction which defines a constant ratio between the variables which make up the cointegrating relationship. For example, the error correction term for the DHSY consumption function consists of the lagged difference of the logarithms of consumption and disposable income. Given a natural cointegrating vector of this type we can impose the unit elasticity and then base a cointegration test on the t-ratio for the error correction term. In terms of our DGP we impose the restriction $\alpha_2 = -\alpha_3$ estimate an equation of the form given in equation (6) and test $H_0 : \alpha_2 = 0$ against the alternative $H_1 : \alpha_2 < 0$.

$$\Delta y_t = \alpha_1 \Delta x_t + \alpha_2 \left(y_{t-1} - x_{t-1} \right) + u_t \tag{6}$$

Kremers et al derive the distribution of the test statistic under the null hypothesis and note that it depends on the ‘signal to noise’ ratio where this is defined as $q = -(\alpha_1 - 1)\sigma_2 / \sigma_1$. The problem is that the critical values for their test (based on the t ratio for α_2) depend on this ratio and therefore on the specific parameters of the problem in question. Thus the Kremer et al test has the unattractive feature that critical values for the test depend on the specific values of the parameters of the problem being examined. This marks a distinct advantage of the F-test approach for which this problem does not apply.

Table Two gives Monte Carlo 5% critical values for the F-form and the t-form of the ECM test for different values of α_1 and the ratio σ_2 / σ_1. Whereas the F-form critical values are independent of these parameters, there is considerable variation in the t-form values. One interesting feature is that (provided $\alpha_1 < 1$) the critical value for the t-test always converges on the standard normal value as $\sigma_2 / \sigma_1 \to \infty$. This confirms the result derived by Kremers et al who show that as the signal to noise ratio increases then the distribution of the test statistic converges on the standard normal. Similarly the distribution of the test statistic gets closer to the standard normal for small values of the α_1 parameter.

[Insert Table Two here]

III. THE POWER OF ALTERNATIVE COINTEGRATION TESTS

The low power of the Engle-Granger test has proved to be a major drawback in applied work. This provides the motivating force behind Kremers et al's development of the ECM t-test. In this section we investigate the relative power properties of the three tests using a DGP in which the *y* and *x* processes are cointegrated. The DGP we assume takes the form:

$$y_t = 0.9\, y_{t-1} + 0.1\, x_t + \varepsilon_{1,t} \tag{7}$$

$$x_t = x_{t-1} + \varepsilon_{2,t} \tag{8}$$

As before we assume $\operatorname{var}(\varepsilon_1) = \sigma_1^2$, $\operatorname{var}(\varepsilon_2) = \sigma_2^2$ with $\operatorname{cov}(\varepsilon_1, \varepsilon_2) = 0$. Based on this experimental design we carry out 10,000 replications for a variety of different sample sizes and compute the percentage of rejections of the false null hypothesis that *y* and *x* are not cointegrated. The critical values used were the MacKinnon critical values for the Engle-Granger test and the Monte Carlo critical values from Table One for the F-form of the ECM test. For the t-form of the ECM test, we made the additional assumption that $\sigma_1^2 = \sigma_2^2$ and used Monte Carlo methods to calculate a set of critical values for the particular sample design given above. These are given for reference in Table Four.

The results of our experiments are reported in Table Three. These show a consistent ranking in terms of the power of the test. The Engle-Granger test consistently has the lowest power and performs very badly in small samples. The F-form of the ECM test has higher power but the t-form of the ECM test has consistently higher power than the other two tests. As the sample size increases then the power of all three tests increases and eventually converges on 100%.

Our results therefore indicate that the error correction approach to testing for cointegration is consistently more powerful than the Engle-Granger approach. Within the class of error correction tests, we find that the t-form of the test is more powerful than the F-form of the test when we know the correct set of critical values to apply. However, the critical values for the t-test can vary considerably depending on the particular nature of the problem being considered. Even if these parameters are known, then we need to generate appropriate critical values using Monte Carlo simulations and in practice these parameters may not be observable. Although the F-form of the test has lower power it does have the advantage that the critical values do not vary with the sample design and therefore this test is considerably easier to apply in practice.

[Insert Tables Three and Four here]

IV. EXAMPLE: UK AND US INTEREST RATES

In this section we present an example which illustrates the relative performance of the tests we have discussed in the previous sections. Our aim is to test for the existence of a cointegrating vector linking UK and US nominal interest rates. The rationale for the existence of a cointegrating vector between these two variables derives from the uncovered interest parity (UIP) condition. This states that the interest rate differential between similar assets in different countries should equal the expected rate of change of the exchange rate. Now it is a well established empirical fact that, in the vast majority of cases, exchange rates are integrated of order one. It follows therefore that, to be consistent with UIP, interest rates in the two countries should be cointegrated and that the cointegrating slope coefficient should be unity.

[Insert Figure One here]

Our data is illustrated in Figure One which shows the Treasury Bill rate for the UK and the US over the period February 1977 to December 2002. The data are taken from the International Monetary Fund *International Financial Statistics* database. Although the two series clearly exhibit some common features it is not clear whether they are cointegrated. Preliminary investigation of the data indicates that both the interest rate series and the sterling dollar exchange rate contain single unit roots. Therefore we proceed to the next stage of the analysis and perform cointegration tests for the two interest rates.

As a first step, we apply the two step Engle-Granger procedure. In the first stage we obtain the results reported in equation (9) for a regression of the UK rate on a constant and the US rate:

$$\begin{aligned} & i_{UK,t} = \underset{(0.623)}{3.3162} + \underset{(0.093)}{0.8448}\, i_{US,t} + \hat{u}_{1,t} \\ & T = 311 \qquad \hat{\sigma} = 2.20 \qquad DW = 0.10 \end{aligned} \tag{9}$$

$\hat{u}_{1,t} : t = 1, \ldots 311$ is the vector of OLS residuals. $\hat{\sigma}$ is the standard error of the regression and DW is the Durbin-Watson test statistic for first order autocorrelation. The standard errors reported in parentheses below coefficients are the Newey-West heteroscedasticity and autocorrelation adjusted standard errors. Equation (9) can be interpreted sensibly in terms of economic theory in that the slope coefficient is within two standard errors of one. However, the intercept term is apparently significant which indicates the possibility of a positive risk premium on UK assets. In the second stage of the analysis we apply an

Augmented Dickey Fuller (ADF) test to the residuals $\hat{u}_1$. Using the Schwarz criterion we determined the optimal number of lagged differenced terms in the auxiliary regression to be two and obtained a test statistic of -3.10. From the MacKinnon response surfaces we obtain critical values of –3.93, -3.36 and –3.06 at the 1%, 5% and 10% significance levels respectively. Thus the Engle-Granger test indicates that we cannot reject the null hypothesis that the two interest rates are not cointegrated at the 5% level, though it is possible to reject at the 10% level.

Next we consider the F-form of the error correction test. Estimation of an error correction model for the UK interest rate yields the results given in equation (10):

$$\begin{aligned}&\Delta i_{UK,t} = \underset{(0.086)}{0.0309} + \underset{(0.049)}{0.0998}\,\Delta i_{US,t} - \underset{(0.013)}{0.0499}\, i_{UK,t-1} + \underset{(0.015)}{0.0586}\, i_{US,t-1} + \hat{u}_{2,t} \\ &T = 311 \qquad \hat{\sigma} = 0.52 \qquad DW = 1.34\end{aligned} \tag{10}$$

The F-test for the joint significance of the two interest rate levels in equation (10) yields a value of 8.22. This compares with a 5% critical value of about 5.84 obtained from the Monte Carlo simulations reported in Table One. Indeed, the critical values reported in Table One indicate that in this case the F-test would also reject the null at the 1% significance level. Therefore, in this example, our results indicate that the F-test is more powerful in detecting a cointegrating vector than the Engle-Granger method.

Finally, we consider the t-form of the error correction test. First, we reparameterise equation (10) and obtain the results reported in equation (11):

$$\begin{aligned}&\Delta i_{UK,t} = \underset{(0.086)}{0.0309} + \underset{(0.049)}{0.0998}\,\Delta i_{US,t} - \underset{(0.013)}{0.0499}\left(i_{UK,t-1} - i_{US,t-1}\right) + \underset{(0.010)}{0.0087}\, i_{US,t-1} + \hat{u}_{2,t} \\ &T = 311 \qquad \hat{\sigma} = 0.52 \qquad DW = 1.34\end{aligned} \tag{11}$$

Since, the coefficient on the lagged US interest rate in (11) is statistically insignificant, it appears that the model accepts the restriction of a cointegrating slope coefficient equal to unity. Therefore we impose this restriction and obtain the results reported in equation (12):

$$\begin{aligned} \Delta i_{UK,t} &= \underset{(0.043)}{0.0933} + \underset{(0.049)}{0.0970}\,\Delta i_{US,t} - \underset{(0.013)}{0.0522}\left(i_{UK,t-1} - i_{US,t-1}\right) + \hat{u}_{3,t} \\ T &= 311 \qquad \hat{\sigma} = 0.52 \qquad DW = 1.34 \end{aligned} \tag{12}$$

The t-ratio for the error correction term in equation (12) is –3.97. We need to determine what are the appropriate critical values since we have seen that these are affected by the nuisance parameters which determine the ‘signal to noise’ ratio for this problem. The Monte Carlo critical values reported in Table One are for a sample size of 500. However, further simulations show that the 5% critical value of –2.89 for the most conservative case ($\sigma_2/\sigma_1 = 10^{-4}$, $\alpha = 0.9$) does not change when we reduce the sample size to 311 as in our empirical problem.. Therefore, even using the most conservative critical values, we can safely state that in this case the t-form of the error correction test will also reject the null of no cointegration.

V. CONCLUSIONS

In this paper we have used Monte Carlo methods to investigate the empirical distribution of the levels terms in the error correction relationship between a set of $I(1)$ variables. We generate critical values for the conventional F-test for the joint significance of the levels terms in such a regression and show that these are generally higher than the critical values from the F-distribution. Investigation of the power properties of this test indicate that it has higher power than the Engle-Granger test but lower power than a t-test based on the error correction model. However, the F-form of the test has the advantage that its distribution does not depend on the specific parameters of the problem being considered. Finally, we illustrate the value of our approach by considering the relationship between UK and US interest rates over the period 1977.02 to 2002.12. We show that it is not possible to reject the null of no cointegration between these variables at the 5% level using the Engle-Granger test. However, our alternative F-test rejects the null convincingly as does the t-form of the error correction test

References:

Banerjee, A., Dolado, J.J., and Mestre, Ricardo (1998) 'Error-Correction Mechanism Tests for Cointegration in a Single-Equation Framework', *Journal of Time Series Analysis*, Vol. 19, No.3, pp. 267-283.

Davidson, J.E.H., Hendry, D.F., Srba, F. and Yeo, J.S. (1978) 'Econometric Modelling of the Aggregate Time-Series Relationship between Consumer's Expenditure and Income in the United Kingdom', *Economic Journal*, Vol 88, No 352, pp. 661-692.

Ericsson, N. R. and MacKinnon, J. G. (2002) 'Distributions of Error Correction Tests for Cointegration', *Econometrics Journal*, Vol 5, pp. 285-318.

Engle, R.F. and Granger, C.W.J. (1987) 'Cointegration and Error Correction: Representation, Estimation and Testing', *Econometrica*, Vol 55, pp 251-276.

Hendry, D.F. and Mizon, G.E. (1978) 'Serial Correlation as a Convenient Simplification, not a Nuisance: A Comment on a Study of the Demand for Money by the Bank of England', *Economic Journal*, Vol 88, No 351, pp. 549-563.

Johansen, S. (1988) 'Statistical Analysis of Cointegration Vectors', *Journal of Economic Dynamics and Control*, Vol 12, pp 231-254.

Kremers, J.J.M, Ericsson, N. and Dolado, J.J. (1992) 'The Power of Cointegration Tests', *Oxford Bulletin of Economics and Statistics*, Vol 54, No 3, pp. 325-348.

MacKinnon, J.G. (1991) 'Critical Values for Cointegration Tests', in Engle, R.F. and Granger, C.W.J. (eds) *Long-Run Economic Relationships*, Oxford: Oxford University Press.

Sargan, J.D (1964) 'Wages and Prices in the United Kingdom: A Study in Econometric Methodology (with discussion)' in P.E. Hart, G. Mills and J.K. Whitaker (eds), *Econometric Analysis for National Economic Planning*, Colston Papers, vol. 16. pp. 25-63. London: Butterworth and Co.

Williams, D. (1978) 'Estimating in Levels or First Differences: A Defence of the Method Used for Certain Demand for Money Equations', *Economic Journal*, Vol 88, No 351, pp 564-568.

Zivot, E. (1996) 'The Power of Single Equation Tests for Cointegrating when the Cointegrating Vector is Prespecified', (Mimeo)

Table One: Critical Values for F-Form of Cointegration Test

k=2

	Monte Carlo Critical Values			Standard F-distribution Critical Values		
Sample Size	1%	5%	10%	1%	5%	10%
50	8.86	6.07	4.95	5.08	3.19	2.42
100	8.12	5.94	4.86	4.83	3.09	2.36
200	7.97	5.84	4.79	4.71	3.04	2.33
500	7.86	5.83	4.85	4.65	3.01	2.31

k=3

	Monte Carlo Critical Values			Standard F-distribution Critical Values		
Sample Size	1%	5%	10%	1%	5%	10%
50	7.25	5.26	4.35	4.23	2.80	2.20
100	6.61	5.00	4.22	3.99	2.70	2.14
200	6.39	4.89	4.19	3.88	2.65	2.11
500	6.39	4.86	4.15	3.82	2.62	2.09

k=4

	Monte Carlo Critical Values			Standard F-distribution Critical Values		
Sample Size	1%	5%	10%	1%	5%	10%
50	6.65	4.76	4.01	3.76	2.57	2.07
100	6.05	4.51	3.86	3.52	2.47	2.00
200	5.62	4.39	3.79	3.42	2.42	1.97
500	5.49	4.39	3.77	3.36	2.39	1.96

Table Two: 5% Critical Values for Alternative Error Correction Tests

		F-Test			T-Test		
		α_1			α_1		
		0	0.5	0.9	0	0.5	0.9
σ_2/σ_1	10^{-4}	5.83	5.83	5.83	-2.89	-2.89	-2.89
	1	5.83	5.83	5.83	-2.60	-2.79	-2.89
	10^{4}	5.83	5.83	5.83	-1.61	-1.61	-1.61

Table Three: Comparison of Power of the ECM test with the Engle-Granger Test

Rejection Frequency 10% Level

Sample Size	Engle-Granger Test	ECM (T-test)	ECM (F-Test)
50	11.8	51.2	28.4
100	25.5	89.4	65.2
200	71.5	99.9	98.3
500	100.0	100.0	100.0

Rejection Frequency 5% Level

Sample Size	Engle-Granger Test	ECM (T-test)	ECM (F-Test)
50	6.1	35.2	16.3
100	13.4	77.7	47.6
200	55.6	98.2	95.1
500	99.9	100.0	100.0

Rejection Frequency 1% Level

Sample Size	Engle-Granger Test	ECM (T-test)	ECM (F-Test)
50	1.3	11.2	4.1
100	3.2	41.3	20.4
200	18.7	95.8	79.7
500	99.2	100.0	100.0

The rejection frequencies reported above are calculated using the critical values from MacKinnon for the Engle-Granger test and the authors' Monte Carlo estimates for the error correction tests.

Table Four: Critical Values for the t-form of the ECM test

Sample Size	10%	5%	1%
50	-2.32	-2.65	-3.37
100	-2.28	-2.64	-3.36
200	-2.26	-2.60	-3.26
500	-2.25	-2.60	-3.26

These critical values are based on the particular sample design given in equations (7) and (8) and the assumption that $\sigma_1^2 = \sigma_2^2$.

Figure One: UK and US Treasury Bill Rates 1977.02-2002.12