Identity, Quantification, and Number

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abstract E. J. Lowe and others argue that there can be ‘uncountable’ things admitting of no numerical description. This implies that there can be something without there being at least one such thing, and that things can be identical without being one or nonidentical without being two. The clearest putative example of uncountable things is portions of homogeneous stuff or ‘gunk’. The paper argues that there is a number of portions of gunk if there is any gunk at all, and that the possibility of uncountable things is inadequately supported.

1. The quantification and identity principles

When I was a student I was taught that there were intimate connections between identity, quantification, and number.

First quantification and number. I was taught that for there to be something--anything at all--is for there to be at least one such thing. For there to be an $F$, or for there to be $Fs$, or for something to be $F$, is for there to be at least one thing or entity that is $F$. (I don’t mean anything special by ‘thing’. Everything is a thing--‘thing’ is for me just a completely general count noun.) We might put this by saying that for any kind to be instantiated is for there to be at least one thing that instantiates it. But the claim is not meant to require the existence of kinds or other universals. Nor is it meant to allow the possibility of nonkinds that could be instantiated without being instantiated by at least one thing. (*Being red* might be a nonkind.) So it might be better to appeal to a schema:

Something is $F$ if and only if at least one thing is $F$.

The claim is that every possible instance of this schema is true. Call this the *quantification principle*. 
As for identity and number, I was taught that for this and that to be identical is for them to be one, and that for them to be distinct or nonidentical is for them to be two:

\[ x = y \text{ if and only if } x \text{ and } y \text{ are one.} \]
\[ x \neq y \text{ if and only if } x \text{ and } y \text{ are two.} \]

It is because of this connection between identity and number, my teachers said, that we call the relation here expressed by the ‘=’ sign ‘numerical identity’. And I appeal to these principles in explaining to my own students the difference between numerical and qualitative identity. I don't know of any other way of explaining it. Call them the *identity principles*.

Exactly how these three principles relate is a nice question, but they show definite affinities. The two identity principles look inseparable. If being identical is being one, it's hard to see how being nonidentical could be anything other than being two. And if being nonidentical is being two, how could being identical not amount to being one? If identity implies anything at all about number, both principles must be true.

There are also connections between the quantification and identity principles, though they are less neat. Suppose this thing and that thing were identical and yet not one, falsifying the first identity principle left to right. Then presumably this thing would not be one on its own. So there would be something identical to that thing without there being one such thing. Nor would there be more than one of them. It follows that there would be something identical to that thing, yet not at least one such thing, contrary to the quantification principle. So the quantification principle appears to entail the first identity principle left to right.

We can almost derive the left-to-right component of the second identity principle from the quantification principle. Suppose something is *F* here and something is *F* there. By the quantification principle it follows that we have at least one *F* thing
here and at least one there. Now suppose further that one of the $F$ things here is distinct from one of the $F$ things there. So we have at least one $F$ thing and we have at least one other $F$ thing nonidentical to it. Does it not follow that there are at least two $F$ things? To put it the other way round: suppose this and that were nonidentical yet not two, so that the second identity principle were false left to right. Then it could not be that there is at least one such thing as this and at least one such thing as that. Yet there would be such a thing as this and such a thing as that. Hence, there would be something without there being at least one such thing.

Or again, suppose there is something, $x$, that is $F$. Given that everything is self-identical, $x=x$. Then by the first identity principle $x$ and $x$ are one. Does it not follow that at least one thing is $F$? So the first identity principle provides an argument for the left-to-right component of the quantification principle. (And its right-to-left component is, to my knowledge, wholly uncontroversial: if at least one thing is $F$, then something is $F$.)

2. The uncountability thesis

Because I was taught these principles at an impressionable age, perhaps, they seem true to me. Indeed, they sound like platitudes. But some philosophers think I was taught wrongly. Geach and Dummett, for example, say there are things that we cannot even begin to count. And if we cannot even begin to count them, we cannot say that there is at least one, contrary to the quantification principle (Geach 1980: 63; Dummett 1981: 547). Henry Laycock says things that at least appear inconsistent with both the quantification principle and the second identity principle.\footnote{He denies that ‘if and when we speak of this $F$, that $F$ or the other $F$, we speak in each and every case of just one $F$’ (1975: 416), which seems to amount to rejecting the quantification principle. And this passage sounds like a rejection of the second identity principle: ‘There is no natural way of thinking of the water in two distinct bottles as two distinct things and thus no way of thinking of the water in one bottle as one distinct thing’ (1972, 31).} But their most outspoken opponent is E. J. Lowe, who explicitly denies all three.
To say that there is something of a certain sort, Lowe claims, is not by itself to say or imply anything about how many there are—not even that there is at least one; so the quantification principle is false. Further, things can be identical without being one, and distinct without being two. Identity and nonidentity, by themselves, imply nothing about number. Lowe puts this by saying that things might have ‘determinate identity but not determinate countability’ (1998: 72).

In fact Lowe claims that the identity principles hold neither left to right nor right to left: this and that might be one without being identical (or at least without being definitely identical), or two without being (definitely) non-identical. Number need not imply anything (or at least anything definite) about identity or nonidentity: in Lowe’s terms, things might have ‘determinate countability but not determinate identity’ (1998: 62). I won’t discuss this claim here, and will consider only the left-to-right components of the principles.

How could there be something without there being at least one such thing? And how could things be identical without being one, or distinct without being two? It looks as if the answer can only be that some things simply do not admit of number. The concept of number, or of how many, does not apply to them. They are, in the strongest possible sense, uncountable.

The claim is not that some things are uncountable in the sense that there are too many to count. If we try to count the real numbers, for instance, by assigning one of the counting numbers 1, 2, 3,... to each of them, we find that when all the counting numbers have been used up there are still real numbers left over. But even though we cannot count them all, we can count any finite subset of them. We can at least begin counting them. In fact we can say exactly how many real numbers there are in total.

Nor is the claim that some things are so many as to be beyond the reach even of transfinite cardinals. Most mathematical logicians agree that for every number there is a set with more members that that number. One way of showing it is to
suppose that numbers are sets: specifically, each number is the set of sets with that number of members. Thus, 0 is the empty set, 1 is the set of singletons, 2 is the set of pairs, and so on. Then for every number there is at least one set with that number of members. But every set has a power set—the set of all its subsets—and the power set of any set has more members than the original set has. It follows that for any number there is a set with more than that number of members. And so for any number there are more things than that number. Hence there cannot be a number of things. But this does not prevent us from saying how many things there are: there are more than any number can capture. In fact this would seem to be a completely precise answer to the question of how many there are.

The claim is not merely that some things are not countable under a certain concept or predicate. Someone might argue that there’s no saying how many red things there are, insofar as there is no principled way of counting red things as such—the concept red thing provides no way of distinguishing one red thing from another. Still, the uncountability of red things qua red things would not entail their uncountability simpliciter. We can count roses, pillar boxes, and copies of The Thoughts of Chairman Mao, among other things. And it may be that every red thing is countable under some concept or other. The claim in question is that some things are not countable under any concept. They are uncountable simpliciter: there is no saying how many of anything they are.

The claim is not that certain things have no number because identity is always relative to a sortal concept. In that case something x and something y might be the same F and not the same G, but there would be no question of whether they are identical or distinct without qualification. Assuming a sortal-relative analogue of the identity principles, x and y would then be one F and two Gs; but there would be no question of whether they are one or two simpliciter. So we could ask how many Fs there are or how many Gs, but not how many things. There would be no such thing

\[2\] In this paragraph and the next I follow van Inwagen 2002.
as number *simpliciter*, but only number under a sortal. But this is not what Lowe has in mind. In fact he rejects the relativity of identity.

Nor, finally, is the claim that certain things have no *precise* number owing to vagueness. It may be that identity can be vague: there can be things that are neither definitely identical nor definitely not identical. Consider a simple case: there is something $x$ and something $y$ such that it is indeterminate whether $x$ is $y$, and nothing is determinately distinct from $x$ or from $y$. Then it will be indeterminate whether the number of things is one or two, and every other number will determinately not be the number of things. And that will be all the answer there is to the question of how many things there are. But it is still a perfectly meaningful answer--supposing, anyway, that the notion of vague identity is intelligible.

Lowe’s view is that some things simply do not admit of number or of numerical description. ‘It makes no sense even to inquire how many there are’ (1998: 74; see also 33, 50). Such things ‘cannot be assigned numbers--neither the number one nor any greater number’ (2009: 50). Dummett seems to mean the same when he says, ‘it simply makes no sense to speak of the number’ of some things. ‘There are some questions “How many?”’, he continues, ‘which can only be rejected, not answered’ (1981: 547). Such things exist, but we cannot say that there is at least one of them. Nor can we say that there is more than one. We cannot say that their number is between $m$ and $n$, or that there are more of them than any number can capture. We cannot say anything at all about how many there are. So it is an understatement to say that such things merely lack *determinate* countability. They have no countability at all, determinate or otherwise. The concept of number has no application to them. Call such things *strongly uncountable*. And call the claim that there could be strongly uncountable things the *uncountability thesis*.

Lowe concedes that the thesis sounds paradoxical. The word ‘things’ is plural, and plurality implies more than one. That seems to make it impossible for there to be *things* without there being more than one of them; and if we can say that there is
more than one thing of a certain sort, such things are not strongly uncountable. The grammar of the English language (and of every other language I am familiar with) makes the claim ‘there are uncountable things’ hard to state. This may show that the first speakers of Indo-European languages at least tacitly accepted the quantification and identity principles. Perhaps they too were taught wrongly. I hope this grammatical obstacle will not render my discussion unintelligible.

The precise relation between the uncountability thesis and the quantification and identity principles is not easy to establish. The falsity of any of the three principles would appear to entail the uncountability thesis. Suppose the thesis were false, so that for any things whatever, it were always possible to say how many there are, even if it were something like ‘an indenumerable infinity’ or ‘more than zero but not more than two’. Then there being something that is $F$ would entail there being at least one $F$ thing: any numerical description other than ‘zero’, be it vague or precise, entails ‘at least one’. The quantification principle would be true. And if there were any numerical description of $x$ and $y$, surely $x$'s being identical or nonidentical to $y$ would entail that $x$ and $y$ are one or two, respectively, as the identity principles say: the quantification and identity principles would seem to be true of any entities that admit of number at all. If everything admitted of number--if the uncountability thesis were false--the quantification and identity principles would be true.

But it is less clear whether the converse entailment holds. Geach and Dummett suggest not only that red things are strongly uncountable and do not admit of number, but that they do not admit of identity or diversity either. In that case their existence would be consistent with the identity principles. Some things might be strongly uncountable for reasons other than the falsity of the identity principles. If so, establishing the identity principles does not suffice to refute the uncountability thesis. But the quantification principle would appear to be true if and only if the uncountability thesis is false.
3. Portions of stuff

As examples of strongly uncountable things Lowe mentions tropes and facts: they exist, but we can say nothing about how many there are—not even that there is at least one. Geach and Dummett, following Frege, speak of red things (though to my knowledge Frege himself never endorsed the uncountability thesis). I will set aside tropes and facts and consider instead a case that both Lowe and Laycock appeal to: portions of stuff. (I will return to red things later.) Suppose there is water in the glass. Then, says Lowe, we can ask how much water there is, but not how many (1998: 74; see also 2009: 49-51.). The point is not merely that grammar forbids the question ‘How many water are there?’. It likewise forbids the question ‘How many Aristotle are there?’; yet Aristotle is not uncountable: there is exactly one thing that is Aristotle (Chappell [1970: 64] makes a similar point). As far as that goes, grammar forbids the statement ‘it makes no sense even to inquire how many water there are’. In order to state Lowe’s view about water and the like, we need a count noun of which the water in the glass, as well as the water in this part of the glass and the water in that part of it, are instances. Let us call such things portions. (*Part*, *parcel*, *mass*, and *quantity* are sometimes pressed into use for the same purpose.) Then the view is that portions of stuff--of water, or gold, or matter generally--are strongly uncountable. Although there are such things, it makes no sense even to inquire how many there are. Nor is there any answer to the question--not even an imprecise answer such as ‘at least one’. They are beyond numerical description.

For something to be a portion of stuff is different from its being what Lowe calls a *piece* of stuff. Drops of water, chunks of ice, and grains of sand are pieces. They contrast with their surroundings and have nonarbitrary boundaries. (*More precisely, the entire boundary of a piece is nonarbitrary: the matter in the upper half of a certain grain of sand would be a portion but not a piece. Lowe’s definition*
of ‘piece’ at 1998: 73 is different from mine, but we can ignore the difference for present purposes.) Lowe accepts that pieces are countable: we can ask how many drops of water or grains of sand there are. Portions of stuff, however, need not contrast in any way with their surroundings, and can have arbitrary boundaries.

I find it hard to see how portions of water could be strongly uncountable. They are composed of water molecules\(^4\), and water molecules are countable if anything is: even if there is no finite number of them, or no precise number owing to vagueness, we can ask how many there are. (They are ‘pieces’ in Lowe’s sense.) And surely no water molecules can compose more than one portion of water at once. Given these facts, the number of portions ought to be a function of the number and arrangement of molecules.

What function it is depends on two factors, both of which are disputable. The first is the circumstances in which water molecules compose something—where some things, the \(x\)s, compose something \(y\) if and only if each of the \(x\)s is a part of \(y\), none of the \(x\)s share a part, and every part of \(y\) shares a part with one or more of the \(x\)s. That determines how many things composed of water molecules—call them aggregates—there are. The second factor is what is required for an aggregate to count as a portion of water. Maybe a lone molecule could not be a portion of water because it cannot be in a liquid state; maybe similar facts prevent an aggregate of many molecules from being a portion of water if they are too far apart. Then the number of portions of water is the number of aggregates that satisfy the conditions necessary for being a portion of water.

Thus, if any water molecules whatever compose something—if there is universal composition for water molecules—then the number of aggregates is the number of non-empty subsets of the set of water molecules—\(2^n - 1\) if there are \(n\) molecules. If

\(^3\)Lowe in fact shows some hesitation about whether portions of actual stuff are uncountable (1998: 72), though Laycock (1975: 418) does not.

\(^4\)In reality the microstructure of water is more complicated, but the point is unaffected.
some water molecules but not others compose something, the number of aggregates will be something between \( n \) and \( 2^n - 1 \). Either way, subtracting from the number of aggregates the number that don’t count as a portion of water will yield the number of portions of water. Or maybe water molecules never compose anything no matter how they are arranged. In that case the number of portions of water will be equal to the number of water molecules if an individual water molecule counts as a portion of water and zero if not.

Suppose for the sake of illustration that there are exactly three water molecules. If composition is universal and any aggregate of water molecules counts as a portion of water, then the number of portions of water is seven. If no portion of water can be composed of fewer than four molecules, there are none. Other assumptions will yield numbers between zero and seven. But in each case and for any number of molecules there will be a number, even if not a precise one, of portions of water.

Lowe is unconcerned about all this because he is convinced that there could be homogeneous stuffs, every portion of which is composed of smaller portions of the same stuff. The number of portions of such stuff could not be a function of the number of molecules or atoms or any other natural unit of it, because there would be no such units of which all portions of it are composed. David Lewis called such stuff *atomless gunk*. I doubt whether Lowe has any right to be confident that atomless gunk is metaphysically possible. But suppose it is. If it existed, he says, there would be portions of it, but we could not say or even ask how many.

In that case there would be portions of gunk, but there would be no number of them. There would not even be at least one, contrary to the quantification principle. If there were gunk in England and gunk in France (and a gunk-free zone in between), the English gunk would be different from the French gunk--maybe not qualitatively different, but, if we may so speak, numerically different. But there would not be at least two things that are gunk, contrary to the identity principles.
 Portions of gunk, Lowe says, would have determinate identity but not determinate (or even indeterminate) countability: for any portions x and y, either x is y or x is not y; but x’s being y does not entail that x and y are one, and x’s not being y does not imply that x and y are two.

4. Arguments for the uncountability of portions

Why suppose that portions of any sort of stuff would be uncountable? Here is a common line of argument:

When we say that water surrounds our island, or that the water surrounding our island is clear, our discourse is not singular discourse (about an individual) and is not plural discourse (about some individuals); we have no single individual or any identified individuals that we refer to when we use ‘water’. We are talking about some stuff, not a thing or some things.... (McKay 2008: 311)

To say that water surrounds the island, the argument goes, is not to say that a certain thing surrounds the island as a reef might, or that certain things collectively surround it as a lot of jellyfish might. It would be wrong to analyze the sentence ‘water surrounds the island’ in such terms. That’s not what it means. The meaning of the word ‘water’ does not imply that it is made up of bits. In this respect it differs from such mass terms as ‘sand’ or ‘snow’: it is part of their meaning that whatever they apply to must be made up of smallest portions of sand or snow. ‘Water’ has, in Laycock’s apt phrase, no ‘semantic atoms’ (2006: 52). Semantic analysis of sentences about water cannot eliminate mass nouns in favour of count nouns. Talk of water is not talk about countable things. It must therefore be talk about something uncountable.

But it is still a fact, even if not a semantic one, that water is made up of smallest bits, and that for water to surround an island is for such bits to surround it. And
those bits--molecules or elementary particles--are countable. Gunk, of course, is not made up of smallest bits. But it may still be a metaphysical fact that for gunk to surround an island is for a certain number of portions of gunk to surround it, even if this fact does not obtain by virtue of the meaning of the word ‘gunk’.

Peter Simons has proposed an argument suggesting that the truth conditions of certain sentences must appeal to uncountable things (1987: 155--though as far as I know he does not endorse the uncountability thesis). Imagine that most of the world’s gunk has yet to be mined. How could we give truth conditions for this in terms of countable things? It’s no good saying that most of the discrete pieces of gunk have yet to be mined: we may have dug up most of them even though most of the gunk remains underground. Contrariwise we may have dug up most of the gunk even though most of the pieces of it are unmined. And if portions of gunk are countable (that is, not strongly uncountable), it’s no good saying that most of the world’s portions of gunk have yet to be mined either, for if there is any number of unmined portions it will be the same as the number of mined ones: some transfinite cardinal, presumably $2^{\aleph_0}$. But if a statement could be true, and its truth conditions cannot be given in terms of countable things, then they must be given in terms of uncountable things. It must therefore be possible for there to be uncountable things.

Well, these truth conditions look right, and are consistent with the countability of portions of gunk:

There are two sets of portions of gunk $S_1$ and $S_2$ such that

i. no member of either set overlaps any other member of either set;

ii. every portion of the world’s gunk overlaps a member of one of the sets;

iii. all the members of $S_1$ have been mined;

iv. none of the members of $S_2$ have been mined; and

v. the sum of the masses of the members of $S_2$ exceeds the sum of the masses
of the members of $S_1$

(where portions overlap if and only if some portion is a part of both). Those
suspicious about sets could no doubt devise something analogous in terms of
plural quantification. There may, of course, be statements about gunk that resist
truth conditions in terms of countable entities; but that has yet to be shown.

Lowe’s argument for the uncountability of portions of gunk is that because they
are infinitely divisible into smaller such portions, they lack a ‘principle of
individuation’ (1998: 74-76). This means that nothing could count as one portion of
gunk, as opposed to two or some other number. And if nothing could count as one,
nothing could count as any other number either. Let me explain how I think the
argument goes.

In order for things to have a number, it must be possible to count them, or at
least the members of any finite subset of them. And for things to be counted, each
must be picked out uniquely and distinguished from all others: it must be, as Lowe
says, individuated. Otherwise something may get left out or counted twice. But this
is impossible for portions of gunk. A portion of water, by contrast, is composed of
water molecules; and because a water molecule is not itself infinitely divisible into
further water molecules, it can be picked out uniquely. We know what counts as
one of them. Given that no water molecules can compose more than one portion of
water at once, it seems possible to individuate and count portions of water as sums
of molecules. It is even possible to individuate and count some portions of gunk,
namely those that make up material objects that contrast with their surroundings
(‘pieces’ in Lowe’s terminology). If I have a ring made of gunk, I can refer uniquely
to the gunk making it up (which Lowe thinks is distinct from the ring itself). But not
all portions of gunk can be referred to in this way: some are mere arbitrary portions.
Such a thing could only be individuated as a portion of gunk. And that, Lowe says,
cannot be done.
Here is what he says about why not (where a ‘part’ is what I have been calling a portion):

Parts of stuff have no *unity* merely in so far as they are parts of stuff, one consequence of this being that we cannot make direct reference to a part of stuff using a demonstrative noun phrase of the form ‘that part of *S*’, where ‘*S*’ is a mass term denoting the kind of stuff in question. If I point in the direction of some gold and say, ‘That part of gold weighs one ounce’, for example, then I have failed to express a determinate proposition, because I have failed to pick out a determinate object of reference....The problem, once again, is that whenever I point in the direction of some gold, there is never *just one* part of gold that I could be taken to be demonstrating because, as we have seen, any part of gold is always divisible into other parts of gold (once again adopting the fiction that gold is a homogeneous stuff). Nothing whatever *counts* as ‘just one’ part of gold, *simpliciter*, in the way that something counts as just one gold ring.... (1998: 76; see also 2003: 78).

When he says that portions of stuff have no ‘unity’, Lowe means that their parts need not be arranged in any special way: any portions of gunk whatever make up a larger portion. Because of this, and because they are infinitely divisible into smaller portions of gunk, their boundaries are arbitrary and they need not contrast in any way with their surroundings. That prevents us from picking out any one arbitrary portion of gunk as such: no matter how precisely we point, there will never be just one portion of gunk there.

This argument is puzzling in a number of ways. For one thing, the reason we can never point at just one arbitrary portion of gunk would seem to be that wherever we point there will always be many such portions amongst which our pointing does not discriminate--that is, more than one. But ‘more than one’ is a numerical
description: if we can say that whenever we point at gunk we point at more than one arbitrary portion of gunk, then such portions are not strongly uncountable.

The deeper question the argument raises is why our inability to refer uniquely to an arbitrary portion of gunk should imply or even suggest that there is no number of such portions. This inability would seem to be due merely to our limited powers of discrimination—which ought to be irrelevant, seeing as the existence and metaphysical nature of portions of gunk is not supposed to depend on the powers of human beings. Lowe evidently thinks that the inability is not merely contingent, and that no being could pick out one arbitrary portion of gunk uniquely. Not even God could do it. You might think that it must be within the power of an omnipotent being to pick out a precise gunk-filled region of space. And there would have to be exactly one portion of gunk occupying such a region. Thus, a being with infinite discriminatory powers could count portions of gunk by counting gunk-filled regions. But Lowe says that regions cannot be individuated either—not even by God. Nothing could count as just one region of space, or as any other number.

Lowe says little about why regions of space should be strongly uncountable. But here is a way of arguing for it. What would it take to individuate a region of space? To make things simple, think of a two-dimensional region. You could pick one out by doing something analogous to drawing a closed figure on a sheet of paper. Because of the thickness of the line and the vagueness of its boundaries, however, that will not succeed in picking out any one region. Beings with superior discriminatory powers could draw sharper lines. But one could pick out a unique region only by drawing an infinitely sharp line, and maybe not even God could do that. Perhaps God’s power could only consist in this: for any line that could be drawn, he can draw a sharper one. His powers of discrimination could be unlimited, but not infinite. Or one could pick out a region by choosing a point and specifying its boundaries in terms of their precise distance from it. But if God cannot draw an infinitely sharp line, he won’t be able to pick out a unique point either. At
most he might have this power: for any extended region, he can choose a smaller one.\(^5\)

But this reasoning relies on two controversial assumptions: that infinite discriminatory powers are impossible, and that if no being could count things of a certain sort, there is no number of them. I don’t know how to support these claims. Without further argument, then, the case for the uncountability of portions of gunk is inconclusive.

5. The countability of portions

I think we can go further and argue that portions of gunk would be countable.

We can certainly ask how many pieces of gunk there are--how many discrete portions with nonarbitrary boundaries. Suppose I have a cubical piece of gunk, and no other gunk, on my desk. Then there is exactly one piece of gunk there. If I also have a spherical piece of gunk on the floor, then I have two pieces of gunk. The idea that I might have a cubical piece of gunk on my desk and a spherical piece on the floor without having at least two pieces of gunk looks unintelligible.

So pieces are countable. And a piece is a special sort of portion. It follows that at least some portions of gunk, namely those that are pieces, are countable after all. If I have two pieces of gunk, I have at least two portions. We can say something about how many portions there are: at least as many as there are pieces. We can begin counting the portions, even if we can never finish the job. They are not beyond numerical description.

One may reply that if portions of gunk are countable, that is only because a special subclass of them--the pieces--are countable. \textit{Arbitrary} portions--those that are not pieces--remain uncountable, confirming the uncountability thesis.

But the objection can be pressed further. It seems possible for something to be

\(^5\)One might expect Lowe to say that points are uncountable because they cannot be individuated. In fact he denies that there are any points, and understands talk of a point in terms of the limit of a sequence of ever-smaller regions (1998: 75).
a piece of gunk at one time and a mere arbitrary portion at another. If we break a
piece of gunk in two, it looks as if each of the resulting smaller pieces was
previously a mere arbitrary portion. Arbitrary portions can be made into discrete
pieces by detaching the surrounding gunk. Likewise, it seems that a piece of gunk
can cease to be a piece and become a mere arbitrary portion by being fused with
another piece. Suppose I start with ten pieces of gunk, then squash them together
so that they cease to be pieces and become arbitrary portions. If there were initially
ten pieces, and none has ceased to exist or ceased to be a portion, does it not
follow that there are still at least ten portions? And if each of them is now an
arbitrary portion, are there not now at least ten arbitrary portions? It certainly seems
so. A thing’s countability could hardly be a mere temporary feature of it: a thing
cannot be countable at one time and not countable at another. But in that case
even some arbitrary portions of gunk would admit of numerical description.

It is hard to see how even permanently arbitrary portions--those that are never
discrete pieces--could be uncountable. Whatever is potentially or possibly
countable would seem to be actually countable. Suppose that in one possible
situation there are exactly \( n \) items of a certain sort. Now consider any other
possible situation in which all those items exist and are of that sort and there are no
other items of that sort. Then there must be \( n \) items of that sort in the second
situation too. Nothing could be only contingently uncountable. But any arbitrary
portion of stuff could have been a piece: it could be made into a piece by removing
the surrounding stuff of the same kind, even if this never actually happens. So
every portion is possibly countable. It follows that every portion is in fact countable.

One might try to defend the uncountability of portions by denying that any
arbitrary portion could come to be a discrete piece, or that any piece could be made
into an arbitrary portion. It is a necessary truth that every arbitrary portion is
essentially arbitrary and every piece is essentially nonarbitrary. Arbitrary and non-
arbitrary portions of gunk are different fundamental metaphysical kinds.
This would mean that if we take a piece of gunk and break it in two, we create two non-arbitrary portions that did not exist before. And we destroy the original portions whose places the new ones take (but these are not two original portions, since arbitrary portions are uncountable). If we fuse two pieces of gunk to make a larger piece, then again the two smaller pieces do not become arbitrary portions, but cease to exist and are replaced by new portions (though again, not two of them). It is metaphysically impossible for a piece of gunk to survive the process of having another piece of gunk fused to it—if it did, it would change from a countable piece to an uncountable arbitrary portion, which is itself impossible. Nor can an arbitrary mass of gunk survive the removal of the gunk surrounding it.

But I doubt whether anyone would actually say this. If we know anything about what it takes for gunk to persist, we know that the result of breaking a piece of gunk in half is that some of the original gunk goes into one of the two pieces and the rest of it goes into the other piece. We have merely separated one from the other. We haven’t destroyed or created any gunk, but only reconfigured the gunk we already had. And if we have the same gunk, how could we not have the same portion of gunk? A portion of stuff is by definition nothing other than some particular stuff. As long as no gunk is destroyed in the process, then, breaking a piece of gunk in two cannot destroy any portion of gunk.

These arguments rely on the assumption that pieces of gunk, which are countable, are portions of a certain sort. Lowe has recently said that pieces are not portions: a piece of matter is distinct from the portion of matter that makes it up or constitutes it (2009: 50). But this is a mere change of vocabulary. As long as we take ‘piece’ to mean ‘discrete portion with nonarbitrary boundaries’ as originally defined, the arguments are unaffected.

6. Numerical and quasinumerical descriptions

I have argued against the claim that portions of gunk would be strongly
uncountable—though I have not shown that the uncountability thesis is false, or
even that there is no reason to accept it. I will conclude with some remarks about the uncountability thesis in general.

What would it mean if the thesis were true? It might not mean very much. Suppose we grant for the sake of argument that there really are things to which no numerical description applies. That would not by itself prevent us from defining ‘quasinumerical’ descriptions like this:

At least *one* thing is $F_{=_{df}}$ something is $F$.

Exactly *one* thing is $F_{=_{df}}$ something is $F$ and everything that is $F$ is identical to it.

Exactly *two* things are $F_{=_{df}}$ something is $F$ and something distinct from it is $F$ and everything that is $F$ is identical to one or the other of them.

And so on. In that case we could say that there is a *number* of $F$s if and only if there is *one* $F$ or there are *two* $F$s or..., and so on—that is, if and only if one of these quasinumerical statements is true.\footnote{Or at least not definitely false, to allow for vagueness. Defining quasinumerical analogues of transfinite numerical descriptions introduces complications that I must set aside, but I doubt whether it is impossible.} Lowe says that for some values of $F$ there is no number of things that are $F$—for instance, there is no number of portions of gunk—but he does not deny that there is a *number* of such portions. Nor does he deny that we can sensibly ask *how many* portions there are, where this is understood as a request for a quasinumerical description. And the result of replacing the numerical terms of the quantification and identity principles with the corresponding quasinumerical terms would seem to be principles that are not only true, but consistent with the uncountability thesis. There being uncountable things would not rule out the claim that everything is *countable*. And that might lead us to doubt whether the uncountability thesis is as interesting as it appears, or even
whether it is intelligible.

If the definitions of the quasinumerical descriptions make sense, there could be a population of beings who spoke a language identical to ours except that they use quasinumerical terms where we use genuine numerical ones: when they say ‘one’ or ‘two’ or ‘number’, they mean *one* or *two* or *number*. That would make it a serious question whether we ourselves are such beings: whether our own words ‘one’, ‘two’, ‘number’, and so on are numerical or only quasinumerical. Maybe the reason some of us find the uncountability thesis so baffling is that we mean nothing more by ‘one’ and ‘two’ than *one* and *two*, even if others mean something else. In that case there would be no real disagreement between those who say that portions of gunk would be uncountable and the rest of us. There would be no proposition that they accept and we deny or vice versa: they can accept that portions of gunk are *countable*, and we quasinumerates need not affirm that everything is countable in Lowe’s sense. We might even wonder why we quasinumerates should care whether everything is countable in Lowe’s sense--supposing we can even understand it--as long as it’s countable in ours.

Uncountabilists may reply that some things are not even *countable*: it is impossible to ask not only how many there are, but even whether they are the same or different. Such things would not admit even of identity or nonidentity, never mind number. The claim would have to be not only that such things are sometimes neither definitely identical nor definitely distinct, but that the concept of identity has no application to them whatever: it makes no sense even to inquire whether they are identical. But I cannot explore this idea here.

7. The number of things

Let us now ask what it would mean if the uncountability thesis were false. What if necessarily everything were countable? What if the existence of something were necessarily the existence of some number of such things, or at least always implied
some numerical description of them?

It would follow that there is a number of things or entities in general. Or at least it would make sense to ask how many there are; and the question would have an answer, even if it did not take the form of a number. As we noted in §2, it may be that there is no precise number of things owing to vagueness; or it may be that for any number, finite or transfinite, there are more things than that. But the question, How many things are there? would have a unique right answer.

If there is a number of things, then there is a number of red things (though probably not a precise number), contrary to the claims of Geach and Dummett. Of all the things there are, some are red and some are not (indeterminate cases aside). If there is a number of things in general, there would have to be a number of them that are red. Likewise, there would have to be a number of soft things, and a number of things that are not tigers.

This would not mean that the concept red thing determined a principle of individuation and a criterion of identity that would enable us to count red things as such. Even if red things are not countable as red things, each of them would be countable as something or other—under some sortal concept.

The number of red things would be determined by two factors: what things there are, and what it is for a thing to be red. Suppose, to take a simple case, that the only concrete things are material objects. (I assume that only a concrete thing could be red.) And suppose that all material objects are composed of elementary particles. Then the number of material objects will be a function of the number of particles—though for reasons discussed in §3 there is room for debate about what function it is. And the number of red things will be the number of material objects that are red. There will no doubt be a good deal of vagueness and observer-relativity about what it is for a material object to be red, and this will infect the question of how many red things are there, much as it infects the question of how many bald men there are in London. But that does not make the question
unaskable or unanswerable.

The ontology of concrete objects may be more complicated: for instance the red things might include not only material objects composed of elementary particles but also portions of gunk, surfaces, beams of electromagnetic radiation, or mental images. Then the answer to the question of how many red things there are will be more complicated too. But a complicated answer is still an answer.⁷

References


Laycock, H. 1972. ‘Some questions of ontology’, Philosophical Review 81: 3-42


---. 2002. ‘The number of things’, Philosophical Perspectives 12, Realism and Relativism: 176-196

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