Financial market conditions, real time, nonlinearity and European Central Bank monetary policy: In-sample and out-of-sample assessment

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Abstract

We explore how the ECB sets interest rates in the context of policy reaction functions. Using both real-time and revised information, we consider linear and nonlinear policy functions in inflation, output and a measure of financial conditions. We find that amongst Taylor rule models, linear and nonlinear models are empirically indistinguishable within sample and that model specifications with real-time data provide the best description of in-sample ECB interest rate setting behavior. The 2007-2009 financial crisis witnesses a shift from inflation targeting to output stabilisation and a shift, from an asymmetric policy response to financial conditions at high inflation rates, to a more symmetric response irrespectively of the state of inflation. Finally, without imposing an a priori choice of parametric functional form, semiparametric models forecast out-of-sample better than linear and nonlinear Taylor rule models.

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1. Introduction

Empirical models of monetary policy are widely used to study interest rates and to investigate the objectives of policymakers. The great majority of studies use the Taylor rule model and its extensions (e.g. Taylor, 1993; Clarida et al, 2000), where interest rates relate linearly to the gap between actual and desired values of inflation and output. More recently, however, the focus of the monetary policy literature has been increasingly placed on nonlinear models resulting from either asymmetric central bank preferences (e.g. Nobay and Peel, 2003; Cukierman and Gerlach, 2003; Bec et al, 2002; Orphanides and Wieland, 2000, and Favero et al 1999) or a nonlinear (convex) aggregate supply or Phillips curve (e.g. Dolado et al 2005; Schaling 2004), or still when central banks follow the opportunistic approach to disinflation (Aksoy et al, 2006). Dolado et al (2004) discuss a model which comprises both asymmetric central bank preferences and a nonlinear Phillips curve.

Orphanides (2001) raises another issue previously neglected by the monetary policy literature. He warns that the success of empirical Taylor rule models should be judged by the use of the appropriate information set; he shows that ex post revised data sets (commonly used in the empirical literature) provide a misleading description of the Federal Reserve Bank’s behavior in real time. Focussing on real-time versus ex post revised output gap measures, Orphanides and van Norden (2002) show that empirical estimates of the output gap are subject to significant revisions, whereas Orphanides and van Norden (2005) demonstrate that ex post revised estimates of the output gap significantly overstate the ability of the output gap to predict inflation. They also flag the distinction between “suggested usefulness”, which refers to the assessment of models that use final data and “operational usefulness”, which refers to the assessment of models that use real-time data. In light of the Orphanides and van Norden (2005) findings, it is not surprising that Herrmann et al (2005) reiterate the importance of using real-time data to understand the behavior of policy makers in real time.
The ongoing but nevertheless fading (at the time of writing in autumn 2009) financial crisis has provided an additional challenge to simple Taylor rule models adding to the debate on whether Central Banks can improve macroeconomic stability by targeting financial asset prices (such as exchange rates, house prices and stock prices). For instance, amongst others, De Grauwe (2007) argues that asset prices should be targeted as Central Banks cannot avoid taking more responsibilities beyond inflation targeting. On the other hand, Federal Reserve governor Mishkin (2008) points out that asset price bubbles are hard to identify and even if they are identified, their response to interest rates is far from certain. Amongst others, earlier joint research by Federal Reserve Chairman Bernanke and Gertler (2001) considers the importance of targeting asset prices in an inflation-targeting framework and concludes that “there is no significant additional benefit to responding to asset prices”.

ECB President Trichet (2005) takes a more cautious view. He reiterates that the primary objective of ECB’s policy is the maintenance of price stability (the ECB aims at keeping inflation below but close to 2% over the medium term) and adds that “the ECB’s monetary policy strategy does allow for taking into account [asset price] boom developments without any amendments to the strategy and without providing any additional role to asset prices” (Trichet, 2005). Four years on, and armed with the experience of the financial crisis, ECB Vice President Papademos (2009) continues to express a cautious view, but nevertheless, moves a step closer towards acknowledging the importance of monitoring asset prices as part of ECB’s monetary policy. He notes that ““leaning against the wind” of booming asset prices by raising the policy interest rates would, even in the short to medium term, be compatible with the ECB’s monetary policy strategy aiming at consumer price stability”. He then adds that the “leaning against the wind” policy “would be expected to be more effective in maintaining price stability over the longer term, by helping to prevent the materialisation of deflation risks when the asset bubble bursts” (Papademos, 2009).
Castro (2008) argues that, instead of attempting to target different asset prices, Central Banks could be monitoring asset prices and financial information in the form of a composite financial index. Using such an index, Castro (2008) shows that, in contrast to the Federal Reserve and the Bank of England, ECB policymakers pay close attention to financial conditions when setting the Eurozone interest rate.

Perhaps surprisingly, Taylor-type monetary policy rules have mainly been concerned with in-sample fits of linear and nonlinear models to interest rate data. A notable exception is Qin and Enders (2008) who use US data to compare the in-sample and out-of-sample properties of linear Taylor rules and a nonlinear one driven by large versus small values of past interest rates.

This marks a significant point of departure for our paper: using inflation, output gap and a proxy for financial conditions as the main underlying variables, we examine whether monetary policy in the form of nonlinear Taylor rule models can dominate standard linear Taylor rule models both in-sample and out-of-sample. In particular, we employ an extension of the linear Taylor rule to a regime-switching framework, where the transition from one regime to the other occurs in a smooth way. The switching between regimes is controlled by the state of inflation. This feature of the smooth transition model allows us to test the ability of high against low inflation rates to best describe the nonlinear dynamics of the interest rate in the Eurozone area, also accounting for the information available in the financial conditions index.

To assess the ability of the alternative policy rules to predict ECB interest rates both in-sample and out-of-sample, we use real-time as well as revised data. All models are estimated over expanding windows of data. Recursive estimation of the policy rules provides significant information on how the response coefficients to inflation, output gap and financial conditions have varied across times and across regimes (high against low inflation rates). Out-of sample, we compute one-month-ahead through twelve-months-ahead forecasts. We then consider whether forecast improvement can be achieved by combining across different policy rules. By using sequences of expanding
windows to evaluate ECB monetary policy across individual as well as combined reaction functions, we believe we go some way towards addressing the point made by Bank of England Governor King (2007) that it is impossible to write down any stable reaction function.

Forecasts generated from the Taylor-type models are compared to those of autoregressive and nonparametric/semiparametric models; the latter are flexible in the sense that they can capture the salient features of the data without having to base their inference on the assumptions of linearity and normality. That is, rather than assuming a particular functional form for the object at hand or assuming that the data are drawn from a given probability distribution, the regression technique employs nonparametric/semiparametric kernel-based estimators that implement kernel estimation of conditional mean functions. For this reason, non/semiparametric models are referred to as ‘parameter-free’ or ‘distribution-free’ models. In this respect, nonparametric/semiparametric models provide a challenging alternative (at least in terms of forecasting superiority) to the Taylor rule models currently dominating the monetary policy literature.

We have four main findings. First, amongst Taylor rule models, linear and nonlinear models are empirically indistinguishable within sample and that model specifications with real-time data provide the best description of in-sample ECB interest rate setting behavior. Second, ECB policy-makers pay close attention to the financial conditions index when setting interest rates; the effect of the index remains significant even when nonlinearities are accounted for. Third, the response of monetary policy to the financial conditions index depends on the state of inflation; the response increases when inflation is rising. A plausible explanation is that booming financial conditions trigger an increase in inflationary pressures. On the other hand, the 2007-2009 financial crisis sees a shift from inflation targeting to output stabilisation and a shift, from an asymmetric policy response to financial conditions at high inflation rates, to a more symmetric response irrespectively of the state of inflation. Fourth, semiparametric models are flexible enough to forecast out-of-sample better than any linear or nonlinear Taylor rule model;
semiparametric model forecasts are also superior to pooled forecasts. A semiparametric model that uses final data forecasts at least as well as a semiparametric model with real-time data and better than any other model. This is more so during periods of high inflation rates (associated with large interest rate values).

The paper proceeds as follows. Section 2 summarises the linear, nonlinear and nonparametric models. Section 3 discusses the data. Section 4 reports the in-sample analysis and Section 5 presents our out-of-sample forecasting exercise. Section 6 provides some concluding remarks and offers some policy implications.

2. Monetary policy rules

2.1. Linear and nonlinear Taylor rule models

Existing studies of the impact of inflation and output on monetary policy use a version of the Taylor (1993) rule

\begin{equation}
\hat{i}_t = \hat{i} + \rho_\pi E_t(\pi_{t+p} - \pi^*) + \rho_y E_{t} y_{t+q} + \rho_f E_t \text{fin\_index}_{t+r}
\end{equation}

where \(i^*\) is the desired nominal interest rate, \(\hat{i}\) is the equilibrium nominal interest rate, \(\pi\) is the inflation rate expected at time \((t+p)\), \(\pi^*\) is the inflation target (or desired rate of inflation), \(y\) is the output gap expected at time \((t+q)\), \(\text{fin\_index}\) is a measure of financial conditions at time \((t+r)\), \(\rho_\pi\) is the weight on inflation, \(\rho_y\) is the weight on output gap, \(\rho_f\) is the weight on the financial index, and \(p, q\) and \(r\) may be positive or negative. Allowing for interest rate smoothing (see e.g. Woodford, 2003) by assuming that the actual nominal interest rate, \(i_t\), adjusts towards the desired rate by

\begin{equation}
\tilde{i}_t = \rho_1(L)i_{t-1} + (1 - \rho_1)i_t^*
\end{equation}

we write the empirical Taylor rule as
The theoretical basis of the linear Taylor rule (3) comes from the assumption that policymakers have a quadratic loss function and that the aggregate supply or Phillips curve is linear. Asymmetric preferences, instead, lead to a Taylor rule model in which the response of interest rates to inflation and/or output is different for positive and negative inflation and/or output deviations from their desired level. A nonlinear policy rule also results from assuming a nonlinear Phillips curve; to the extent that nominal wages are downwards inflexible, inflation is a convex function of the unemployment rate (see e.g. Layard et al, 1991). This, by Okun’s law, means that inflation is also convex in the output gap. Combined with a quadratic loss function, the nonlinear aggregate supply leads to a policy rule where the response of interest rates to inflation is higher (lower) when inflation is above (below) target.

The nonlinear policy rule we consider, takes the form

\[ i_t = \rho_i (L) i_{t-1} + (1 - \rho_i) \{ \rho_0 + \rho_E E_t \pi_{t+p} + \rho_y E_t y_{t+q} + \rho_{fin} E_t \text{fin}_{index_{t+r}} \} + \varepsilon_t \]

where, \( \rho_i (L) = \rho_{i1} + \rho_{i2} L + \ldots + \rho_{in} L^{n-1} \) (we can use \( \rho_i \equiv \rho_i (1) \) as a measure of interest rate persistence), \( \rho_0 = \hat{i} - \rho_\pi \pi^* \), and \( \varepsilon_t \) is an error term.

\( \pi_t \) is the inflation rate at time \( t \), \( \rho_i \) is the discount factor, \( E_t \) is the expected value at time \( t \), \( \pi^* \) is the target inflation rate, \( \pi_{t+p} \) is the inflation rate at time \( t+p \), \( y_{t+q} \) is the output gap at time \( t+q \), \( \varepsilon_t \) is the error term, and \( \rho_{fin} \) is the weight of the financial index at time \( t+r \).
than $\tau$ percent. In effect, $M_{1t}$ is a Taylor rule specific to this regime. $M_{2t}$ is a Taylor rule that describes the behaviour of policymakers in the regime where inflation is expected to be more than $\tau$ percent. If $\rho_{1\pi} = \rho_{2\pi}$, $\rho_{1y} = \rho_{2y}$, and $\rho_{1f} = \rho_{2f}$ the model simplifies to the linear Taylor rule in (3). If $\rho_{1\pi} < \rho_{2\pi}$ there is a deflation bias to monetary policy as the response to inflation is greater for larger inflation values. The weight $\theta^\pi_t(E_t, \pi_{t+\rho}; \gamma^\pi, \tau)$ is modelled using the logistic function (see e.g. van Dijk et al, 2002)

\[
\theta^\pi_t(E_t, \pi_{t+\rho}; \gamma^\pi, \tau) = \frac{1}{1 + e^{-\gamma^\pi(E_{t+\rho})/\sigma(E_{t+\rho})}}
\]

where the parameter $\gamma^\pi > 0$ determines the smoothness of the transition regimes. We follow Granger and Teräsvirta (1993) and Teräsvirta (1994) in making $\gamma^\pi$ dimension-free by dividing it by the standard deviation of $E_t, \pi_{t+\rho}$.

We also note that nonlinear policy rules can be defined using the output gap and the financial index as possible transition variables in the weight function (5). This implies that the response of interest rates to inflation, output gap and the financial index depends on output gap and financial conditions regimes, respectively. These nonlinear models were considered in the current paper but were very poorly estimated and for this reason not reported.

2.2. Nonparametric/semiparametric specifications

In our forecasting exercise, forecasts generated by the models discussed above are compared to those of a simple autoregressive model of order $n$ (AR($n$)) and a nonparametric specification; the latter does not impose any distributional condition in modelling the interest rate and is therefore able to reveal structure in data that might be missed by classical parametric linear and nonlinear models.

The paper employs a nonparametric (more precisely a semiparametric model is estimated in the exercise) specification that does not require the researcher to specify a functional form; rather it is local in nature and also based on data-driven techniques for ‘local averaging’. The introduction of the paper
discussed how nonlinear models have challenged linear Taylor rule specifications; in turn, nonlinear models might also be inadequate in uncovering the true data generating process of the Central Bank’s reaction function. Rather than assuming that the functional form is known, nonparametric specifications implement kernel estimation of regression functions and substitute less restrictive assumptions, such as smoothness and moment restrictions. To this end, we carry out the Nadaraya-Watson local constant regression estimator and then consider a more popular extension, namely the local linear regression method (Li and Racine, 2004). A key aspect to sound nonparametric regression estimation is choosing the correct amount of local averaging (bandwidth selection) before passing these bandwidth objects to regression and gradient estimation. We therefore make use of a number of bandwidth selections such as the least-squares cross validation of Hall et al (2004) and the Akaike Information Criterion (hereafter AIC) method of Hurvich et al (1998); our empirical calculations are made in the R np package of Hayfield and Racine (2008). In particular, we employ a semiparametric model which is a compromise between fully nonparametric and fully parametric specifications; this is formed by combining parametric and nonparametric models to reduce the curse of dimensionality of nonparametric models. We employ a popular regression-type model, namely, the partially linear model of Robinson (1988). Adopted to a monetary policy setup, the semiparametric model is

\[ i_t = \rho_t(L) i_{t-1} + f(E_t\pi_{t+1}, E_t y_{t+q}, E_t fin\_index_{t+1}) + \epsilon_t \]

where \( \rho_t(L) \) is the parametric part of the model (i.e., the response to lagged interest rates has often been assumed linear in the literature) and the unknown function \( f(.) \) is the nonparametric part. Without imposing a known functional form for \( f(.) \), the model addresses the difficulties of having a fixed rule or reaction function as implied by Taylor rule models currently dominating the monetary policy literature.
3. Data description
We use Eurozone data for the 1999:M1-2009:M6 period. This covers the period over which the ECB has been operating. The nominal interest rate is the Euro overnight index average lending rate (Eonia). For inflation we use the rate targeted by the ECB (the ECB aims at keeping inflation below but close to 2% over the medium term); this is the annual change in the harmonized index of consumer prices. We use both real-time inflation and revised inflation measures. We use three measures of the output gap series: (i) the difference between real-time industrial production and a Hodrick-Prescott (HP, 1997) trend ¹, (ii) the difference between final industrial production and a HP trend, and (iii) the difference between the economic sentiment indicator and a HP trend. The economic sentiment indicator is based on surveys of firms and consumers at the national level; the index is not subject to revisions. The economic sentiment indicator places a weight of 40% on the industrial confidence indicator, and a weight of 20% on each one of the consumer confidence, construction confidence and retail trade confidence indicators, respectively. The index, which is discussed frequently in the ECB Monthly Bulletins, becomes available earlier than output data and correlates strongly with the Eurozone business cycle (Gali et al, 2004).

The financial index variable pools together relevant information provided by a number of financial variables. As in Castro (2008), the index is constructed as a weighted average of (i) the real effective exchange rate (with the foreign exchange rate in the denominator), (ii) the real house price, (iii) the real stock price, (iv) the spread between the yield on the 10-year government bond and the yield on A or higher rated corporate bonds, and (v) the spread between the 3-month Euribor interest rate futures contracts in the previous quarter and the 3-month Euribor rate. The real effective exchange rate, stock price and house price variables are detrended by a HP filter. ² The constructed financial

¹ The real-time industrial production output gap series has been calculated recursively using the available preceding data points.
² To tackle the end-point problem in calculating the HP trend (see Mise et al, 2005a,b), we applied an AR(4) model (with n set at 4 to eliminate serial correlation) to each of the real-time and revised output measures and the components of the financial index. The AR model was used to forecast twelve additional months that were then added to each of the series before applying the HP filter.
index is expressed in standardised form, relative to the mean value of 2000 and where the vertical scale measures deviations in terms of standard deviations; therefore, a value of 1 represents a 1-standard deviation difference from the mean. The financial components of the index are rarely revised and as such, the index itself is not subject to revisions. The index is also in the spirit of the UK financial conditions index provided by the Bank of England’s *Financial Stability Report* (Bank of England, 2007). All data are seasonally adjusted. \(^3\)

Figure 1 plots the data. From Figure 1a), inflation rose in mid 2007 and then dropped sharply followed by drastic interest rate cuts. There is little difference between real-time and final inflation data (revisions occur only to correct reported errors; see Coenen et al, 2005). From Figure 1b), movements in the economic sentiment data are much more pronounced compared to the industrial production output data. The economic sentiment gap data indicate a much more severe downturn in 2008-early 2009; however, the economic sentiment appears to improve quickly towards mid-2009. Compared to the real-time industrial production data, final data suggest a stronger expansion shortly before the financial crisis, followed by a more severe economic downturn. From Figure 1c), financial conditions deteriorated sharply from mid 2007, having improved steadily over the previous five years. Movements in the financial index have a similar pattern to the interest rate (Figure 1a), which indicates a close link between the two variables.

Using the above information set, we consider six policy rule models. Models 1 to 3 are linear Taylor rule versions of equation (2). Models 4 to 6 are nonlinear Taylor rule versions of equation (4) using the logistic function (5). For forecasting purposes, we consider six more models. Models 7 to 9 are semiparametric versions of equation (6) using real-time inflation and real-time industrial output, revised inflation and industrial output, and real-time inflation

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\(^3\) Real-time data have been collected from the ECB Monthly Bulletins. The European house price index is available from the Financial Times website (www.ft.com). European stock prices refer to the Dow Jones Euro STOXX price index. House and stock prices are deflated by the Consumer Price index. The rest of the data is available from the ECB website (www.ecb.int) and Datastream.
and economic sentiment data, respectively. Model 10 is an AR(4) model (lag length chosen by the AIC). Our preferred specifications allow for one lag of the interest rate, $p=12$ for inflation, $q=0$ for the output gap (the dependence of ECB monetary policy on current rather than expected output gaps agrees with the Euro Area Wide Model in Dieppe et al., 2004), and $r=-1$ for the financial index. Assuming perfect foresight for inflation, we replace forecasts of real-time inflation by real-time realizations of inflation and forecasts of final inflation by final realizations of inflation and then estimate by the Generalised Method of Moments (GMM). 4 The implication of using real-time realizations of inflation values, when these were not available, is that Models 1, 3, 4, 6, 7, and 9 are not truly real-time models, rather, these can be considered as “quasi” real-time models. In our forecasting exercise, we employ two straightforward pooling procedures. First, forecasts are constructed by taking the median forecast value from models that use real-time data, that is, models 1, 3, 4, 6, 7, and 9; we call this Model 11. Second, we use the median forecast from models that use final data, that is, models 2, 5, and 8; we call this Model 12. Our twelve models are summarised in Table 1.

We estimate over expanding windows of data, where the first data window runs from 1999:M1 to 2005:M12, and each successive data window is extended by one observation, hence, the last data window runs from 1999:M1 to 2008:M6 (this setup delivers 31 expanding windows). From a policy point of view, this allows us to identify the evolution of the estimated model parameters over time and across regimes. For forecasting purposes, we generate out-of-sample forecasts for the Eurozone interest rate at forecast horizons $h=1,...,12$. 5 We use sequences of expanding windows in which the sample size for estimation is increased by one observation in each successive window, as opposed to sequences of fixed-length rolling windows, simply

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4 The ECB website (http://www.ecb.int/stats/prices/indic/forecast/html/index.en.html) provides inflation forecasts from the Survey of Professional Forecasters (SPF) on a quarterly basis up to 5 years ahead; to overcome this we assumed a constant inflation forecast for each month within the same quarter. Empirical results using these inflation forecasts were very unsatisfactory both on economic and statistical grounds.

5 Our setup delivers 30 one-step ahead interest rate forecasts, 29 two-step ahead forecasts and so on, up to 19 twelve-step ahead forecasts. This is because we replace $E_t \pi_{t+12}$ by actual values of inflation in our estimated models.
because the larger (increasing) windows help the estimation procedures for the various models which can be quite parameter intensive; this is arguably more so for the semiparametric models that partly use local averaging and therefore require a large number of data-points. For robustness reasons, however, our forecasting exercise also reports results based on a sequence of fixed-length rolling windows where each successive window is constructed by shifting the preceding window ahead by one observation.

4. In-sample analysis
To fix ideas, Table 2 reports estimates of the Taylor rule models 1 to 6 over the first data window, which runs from 1999:M1 to 2005:M12. In all cases, and in line with previous literature (see e.g. Castro, 2008 and Gerdesmeier and Roffia, 2005), the inflation ($\rho_\pi$) and output gap ($\rho_y$) effects are statistically significant. For all models, the inflation effect $\rho_\pi$ is higher than one, satisfying the “Taylor principle” that inflation increases trigger an increase in the real interest rate. Model 1, which uses real-time industrial production and inflation data, records much stronger inflation and output gap effects compared to Model 2 (which uses revised data); a possible explanation is that the magnitude of the response using revised data could suffer from downward bias owing to the errors-in-variables problem. The output gap effect is lower, but nevertheless significant, when the economic sentiment measure is considered (see Model 3). All linear models record a statistically significant response to the financial index ($\rho_f$); in all cases, a one standard deviation increase in the index relative to its mean triggers an interest rate increase in excess of one percentage point; the impact, as with the inflation and output gap ones, is higher for real-time Model 1.

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6 On the expanding window versus fixed-length rolling window issue we note that according to Stock and Watson (2005, p. 26), “recursive forecasts are more accurate than the rolling forecasts” for the representative macroeconomic dataset they study. On the other hand, however, Giacomini and White (2006, p. 1566) find that a “rolling window procedure can result in substantial forecast accuracy gains relative to an expanding window for important economic time series.”
An estimate of the inflation target is derived as $\pi^* = \frac{\hat{i} - \rho_0}{\rho_\pi}$, where we rely on the sample mean of the interest rate (this is equal to 3.04%) as a proxy for the equilibrium nominal interest rate $\hat{i}$. From Table 2, all linear Models 1 to 3 deliver an implied target of approximately $\pi^* = 2\%$, which is consistent with ECB’s aim of keeping inflation below but close to this very figure.

For linear Models 1 to 3, the last three rows of Table 2 report the p-value of Hamilton’s (2001) $\lambda$-test, and the p-values of the $\lambda_\alpha$ and $g$-tests proposed by Dahl and González-Rivera (2003). Under the null hypothesis of linearity, these are Lagrange Multiplier test statistics following the $\chi^2$ distribution. These tests are powerful in detecting non-linear regime-switching behavior like the one considered by Models 4 to 6. All three tests reject linearity.

From Table 2, Models 4 to 6 report time-varying inflation, output gap and financial index effects depending on whether inflation is higher or lower than an inflation threshold; the latter is estimated at $\tau = 2\%$, which is again consistent with ECB’s policy goal. The smoothness parameter $\gamma^2$ has an estimated value of 10, indicating a rather abrupt switch from one regime to another. For Models 4 and 5 (but not for Model 6) we estimate that $\rho_{1x} < \rho_{2x}$; hence, there is some weak evidence of a deflation bias to monetary policy as the response to inflation is larger when inflation exceeds 2%. In contrast to revised-data Model 5, Models 4 and 6 estimate that $\rho_{1y} < \rho_{2y}$, that is, a stronger response to the output gap when inflation exceeds the 2% threshold; for these models, the output response is insignificant at low inflation rates.

All three nonlinear models estimate that $\rho_{1f} < \rho_{2f}$, that is, a much stronger response to the financial conditions index when inflation rises above the 2% threshold. Noting that inflation is positively correlated with the financial conditions index (with a correlation coefficient of 0.43), we shed more light on

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7 We run the tests using Gauss codes obtained from Hamilton’s web page at: http://weber.ucsd.edu/~jhamilto/software.htm#other. To account for the small sample, we report bootstrapped p-values of the three tests based on 1000 re-samples.
their possible link by estimating a Vector Autoregressive (VAR) system of order 2 (the lag length is chosen by the AIC criterion) in inflation and financial conditions index, and then apply Granger-causality tests (Granger, 1969). These tests indicate causality from the financial conditions index to inflation (the $F$-test for testing the null of no causality delivers a $p$-value=0.01) and no evidence of causality from inflation to the financial conditions index (the $F$-test for testing the null of no causality delivers a $p$-value=0.35). Hence, a plausible explanation for the stronger response of monetary policy to the financial index at rising inflation rates is that booming financial conditions trigger an increase in inflationary pressures.

We have also attempted linear and nonlinear versions of Models 1 to 6 that exclude the financial index variable. In statistical terms, these models performed very poorly compared to the models reported here. We therefore conclude that the ECB pays close attention to financial conditions when setting the Eurozone interest rate; moreover, the response to the financial index depends on the state of inflation.

There is very little to discriminate amongst the estimated Taylor rule models in terms of the adjusted $R^2$ and the regression standard error. Model 3 (with real-time data and the economic sentiment variable) records the lowest Akaike Information Criterion (AIC). Amongst the estimated nonlinear models, Model 6 (i.e. the nonlinear version of Model 3) has the best in-sample fit as it records the lowest AIC. Figure 2 plots the recursive AIC values for all models over expanding windows; Model 3 and Model 6 report lower values than the remaining models; however the estimated models (including Models 3 and 6) have overlapping AIC values suggesting that it is hard to distinguish amongst these models within sample. Notice also that the in-sample fit of Model 6 improves considerably as we move into the financial crisis period. Within sample we would expect the fit of such alternative models to be barely distinguishable, given the high correlations between the interest rate and its lags. However, the key distinguishing feature amongst linear and nonlinear

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8 Granger-causality test results using real-time inflation rather than inflation are qualitatively similar.
models lies in their forecast implications, namely that the equilibrium to which the reaction function returns depends on the size of the shocks/inflation states. For the nonlinear model, small shocks/low inflation do not alter the central bank’s reaction function. However, at a low interest rate, large positive shocks to inflation drive the interest rate to a high level consistent with the higher regime reaction function, while at a high interest rate, negative inflation shocks, drive it back to a low interest rate. A linear Taylor type rule model will forecast the interest rate to stay roughly where it is if non-stationary; or, if stationary, to revert to some deterministic equilibrium. Thus the forecast implications of linear as opposed to nonlinear models are quite different. We keep this in mind when forecasting out-of-sample in section 5 below.

To get an idea of how the response parameters $\rho_z$, $\rho_y$, and $\rho_f$ evolve over time, Figure 3 plots their recursive estimates (plus/minus 2*standard errors) over expanding data windows for Model 3 which has the best in-sample fit amongst all models. Figure 4 plots recursive estimates (plus/minus 2*standard errors) of the response parameters $\rho_{jz}$, $\rho_{jy}$, $\rho_{jf}$ ($j=1,2$) for Model 6 which has the second-best in-sample fit amongst all models and the best in-sample fit amongst nonlinear models. We also note that recursive plots of the remaining models are qualitative similar to the ones reported below.

From Figure 3, the inflation response is relatively stable until late 2006 after which it drops sharply and rises again from late 2007 onwards. The response to the output gap is relatively stable; it rises in late 2006 and then reverts slowly towards its earlier values. The response to the financial index remains relatively stable until late 2007, after which it drops slightly. Overall, and compared to the output gap and financial index responses, the inflation response is markedly unstable and statistically insignificant during the financial crisis period; at the same time, the increasingly turbulent period has somewhat widened the confidence intervals of all response estimates. Notice also that the timing of the sharp drop in the inflation response coincides with that of the rise in the output gap response. A tentative economic interpretation (bearing in mind the issue of instability) is that from early 2007,
ECB monetary policy shifted its focus from inflation to output stabilisation, while responding to financial conditions in a relatively consistent manner. We return to this issue shortly.

Figure 4 plots the recursively estimated response coefficients $\rho_{1\pi}$, $\rho_{1y}$, $\rho_{1f}$, $\rho_{2\pi}$, $\rho_{2y}$, and $\rho_{2f}$ for nonlinear Model 6. In this model, the policy response switches from $\rho_{1\pi}$, $\rho_{1y}$ and $\rho_{1f}$ to $\rho_{2\pi}$, $\rho_{2y}$ and $\rho_{2f}$, respectively depending on whether expected inflation is below or above the 2% threshold. The recursively estimated inflation coefficients $\rho_{1\pi}$ and $\rho_{2\pi}$ are fairly similar suggesting neither deflationary nor inflationary bias in ECB monetary policy. From early 2007 onwards and as we move into the financial crisis period, the policy response to inflation becomes smaller and largely insignificant. The response to the output gap at low inflation rates is lower than the output gap response at high inflation rates (i.e. $\rho_{1y} < \rho_{2y}$). The former response is insignificant at the earlier part of the sample, but becomes significant as the financial crisis progresses and takes it toll on the economy; at the same time, monetary policy becomes more responsive to output gap fluctuations irrespectively of the inflation state. The financial index response above the 2% inflation threshold is three times as large as the response below (i.e. $\rho_{1f} < \rho_{2f}$) prior to the financial crisis. As the financial crisis unfolds at the peak of forecasted inflation around mid 2007 and gains pace even with inflation falling, stabilisation of the financial conditions becomes equally important irrespectively of the state of inflation; indeed, the response to the financial index emerges the same by the end of our sample. Our nonlinear estimates therefore indicate that ECB policymakers used notable discretion post 2006 as the financial crisis saw a shift from inflation targeting to output stabilisation and a shift, from an asymmetric policy response to financial conditions at high inflation rates, to a more symmetric response irrespectively of the state of inflation; however, these results should be read with some

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9 The recursively estimated values of the inflation threshold and the smoothness parameter are remarkably similar to those reported in Table 2.
caution as the confidence intervals of the recursive nonlinear responses get relatively wider with the financial crisis unfolding.

5. Forecasting analysis

5.1. Methodological issues

Generating dynamic out-of-sample forecasts from nonlinear models is more complicated compared with generating forecasts from linear models as the expected value of a nonlinear function is different from the function evaluated at the expected value of its argument (see, e.g., Granger and Teräsvirta 1993, and Franses and van Dijk 2000, among others). We tackle this issue by adopting at each step of our forecasting exercise a bootstrap method where errors used at step $h$ ($h > 1$) are the average errors obtained from simulating the nonlinear model at step $h$ one thousand times.

Forecasting performance is evaluated using the Mean Squared Prediction Error (MSPE) and Median Squared Prediction Error (MedSPE) criteria. To compare alternative forecasts, we employ the Diebold and Mariano (1995) test. This is computed by weighting the forecast loss differentials between two competing models $i$ and $j$ equally, where the loss differential for observation $t$ is given by $d_t = [g(e_{it-h}) - g(e_{jt-h})]$, where $g(.)$ is a general function of forecast errors (e.g. MSPE or MedSPE). The null hypothesis of equal accuracy of the forecasts of two competing models, can be expressed in terms of their corresponding loss functions, $E[g(e_{it-h})] = E[g(e_{jt-h})]$, or equivalently, in terms of their loss differential, $E[d_t] = 0$. Let \( \bar{d} = \frac{1}{P} \sum_{t=R+h}^{R+P+h-1} d_t \) denote the sample mean loss differential over $t$ observations, such that there are $P$ out-of-sample point forecasts and $R$ observations have been used for estimation. The Diebold-Mariano test statistic follows asymptotically the standard normal distribution:

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{P}}} \xrightarrow{d} N(0,1),$$
where $N(.)$ is the normal distribution and $\hat{f}_d(0)$ is a consistent estimate of the spectral density of the loss differential at frequency 0. To counteract the tendency of the $DM$ test statistic to reject the null too often when it is true in cases where the forecast errors are not bivariate normal, Harvey et al (1997) propose a modified Diebold-Mariano test statistic:

$$DM^{*} = \left[ \frac{P+1-2h+P^{-1}h(h-1)}{P} \right]^{1/2} \rightarrow t_{(P-1)},$$

where $DM$ is the original Diebold and Mariano (1995) test statistic for $h$-step ahead forecasts and $t_{(P-1)}$ refers to the Student’s $t$ distribution with $P - 1$ degrees of freedom.

Recently, van Dijk and Franses (2003) argued that the uniform weighting scheme employed by the $DM$ and $DM^{*}$ tests may be unsatisfactory for frequently encountered situations in which some observations are more important than others. For example, in an interest rate forecasting exercise, large interest rate observations, $i_t$, generally signal periods of high inflation. van Dijk and Franses (2003) modify the test statistic by weighting more heavily the loss differentials for observations that are deemed to be of greater substantive interest. In their approach, the weighted mean loss differential is given by $\bar{d}_w = \frac{1}{P} \sum_{i=R+1}^{R+P+h} w(\omega_i) d_i$, where $\omega_i$ is the information set available at time $t$. In the case of interest rates, two cases of particular interest are:

$$w_{LT}(\omega) = 1 - \Phi(i_r),$$
$$w_{RT}(\omega) = \Phi(i_l),$$

where $\Phi(i_r)$ is the cumulative distribution function of $i_r$, to focus on the left tail of the distribution of $i_r$, and:

$$w_{LT}(\omega) = \Phi(i_l),$$
$$w_{RT}(\omega) = \Phi(i_r),$$

to focus on the right tail of the distribution of $i_r$. The weighted $DM$ statistic is computed as:
\( W - DM = \frac{\hat{d}_w}{\sqrt{\frac{2\pi \hat{f}_{dw}(0)}{P}}} \),

where \( \hat{f}_{dw}(0) \) is a consistent estimate of the spectral density of the loss differential at frequency 0. The weighted \( DM^* \) test statistic is given by:

\[
(12) \quad W - DM^* = \left[ \frac{P+1-2h+P^{-1}h(h-1)}{P} \right]^{1/2} W - DM.
\]

van Dijk and Franses (2003) propose using the Student's \( t \) distribution with \( P - 1 \) degrees of freedom to obtain critical values for the \( W-DM^* \) test.

In our forecasting exercise, the left-tailed \( W-DM^* \) statistic focuses on the ability of the competing models to forecast small interest rate values, which is generally interpreted as evidence of periods of low inflation. On the other hand, the right-tailed \( W-DM^* \) statistic focuses on the ability to forecast large interest rate values, which is generally interpreted as evidence of periods of high inflation. It should be noted that the literature has challenged \( DM \)-type statistics in two aspects. First, West (1996, 2001) and West and McCracken (1998) analyzed modification of forecast comparison tests in light of the use of estimated model parameters in the computation of such tests. However, van Dijk and Franses (2003) pointed out, that for \( DM \)-type tests under quadratic loss, such parameter estimation uncertainty is asymptotically irrelevant. van Dijk and Franses (2003) went on to argue that corrections of the type suggested by West (1996, 2001) and West and McCracken (1998) are not necessary when examining the statistical significance of MSPE reductions (which is precisely what we are doing in the current paper). Second, under the assumption that the estimation sample size \( R \) and the number of out-of-sample forecasts \( P \) tend to infinity, McCracken (2000) and Clark and McCracken (2001) showed that, if the underlying forecasting models are nested, the asymptotic distribution of the \( DM \) statistic is not standard normal. van Dijk and Franses (2003) noted that these conditions on the parameters \( R \) and \( P \) effectively mean that expanding windows of data are used for
estimation. On the other hand, when $R$ remains finite, as in the case of fixed-length rolling estimation windows, Giacomini and White (2006) showed that the asymptotic distribution of the $DM$ statistic is still standard normal when forecasts are compared from nested models.

5.2. Out-of-sample forecasting comparisons

Columns (i)-(ii) of Table 3 present the average out-of-sample forecasting rankings across the recursive windows and twelve forecast horizons of the twelve models according to two evaluation criteria, the mean squared prediction error (MSPE) and the median squared prediction error (MedSPE); “better” or “higher ranked” forecasting methods have “lower” numerical ranks. In examining the average rank results of Table 3, it is useful to note that if the average rank of Model $i$ is higher than the average rank of Model $j$ according to either the MSPE or the MedSPE, then Model $i$ outperforms Model $j$ according to the particular criterion for more than 50% of the forecast horizons, that is, for at least seven out of the twelve forecast horizons used.

The key result is that the three semiparametric models 7, 8, and 9 are ranked higher than any other model according to both the MSPE and the MedSPE, with Model 8, the semiparametric model with final data, being the top-ranked forecasting model (Model 8 forecasts at least as well as semiparametric Model 7 according to the MedSPE). The AR model is ranked fourth whereas Model 11, which pools forecasts from all models with real-time data, is ranked fifth. According to the MSPE, nonlinear Models 4, 5, and 6 are ranked higher than the corresponding linear Models 1, 2, and 3, respectively, with nonlinear Model 4 (which uses real-time inflation and real-time industrial production data) ranked higher than the remaining linear and nonlinear Taylor rule models. According to the MedSPE, Models 4 and 1 have the same average rank. Models 3 and 6, the models with the best in-sample fit amongst all linear and nonlinear policy rules, have very low out-of-sample forecasting ability compared to the remaining models. According to the MSPE, Models 3 and 6 are ranked ninth and eighth, respectively; according to the MedSPE, these are ranked eleventh and ninth, respectively.
Our modified Diebold-Mariano ($DM^*$) test results appear in Table 4. These examine the statistical significance of MSPE reductions with uniform weight placed on forecast losses. Left-tailed and right-tailed $W-DM^*$ tests in Tables 5 and 6 examine the statistical significance of MSPE reductions with greater weight placed on forecast losses associated with, respectively, low interest rate values and large interest rate values. Recalling that Model 8 is ranked first, we see that its forecasting superiority over the remaining models is much stronger when it comes to predicting large interest rate values. Indeed, as we move from left-tail weighting to right-tail weighting, Model 8 increases its forecasting dominance over seven models (that is, Models 1,3,4,9,10,11, and 12) and reduces its forecasting dominance over only two models (that is, Models 2 and 5). As we move from uniform weighting to right-tail weighting, Model 8 increases its forecasting dominance over five models (that is, Models 1,9,10,11, and 12) and reduces its forecasting dominance over only two models (that is, Models 2 and 5). This observation is most striking by comparing Model 8 with Model 10 (the AR model). Model 8 generates significant MSPE reductions, at the 10% significance level, relative to the AR model (Model 10) at 8.3% of the forecast horizons with left-tail weighting (see Table 5) and at 66.7% of the forecast horizons with uniform weighting (see Table 4). With greater weight given to large interest rate values, however, Model 8 generates significant MSPE reductions relative to the AR model at 75% of the forecast horizons (see Table 6).

Model 10 (the AR model) is the only model to deliver a statistically lower MSPE relative to the top-ranked Model 8. In particular, the MSPE of Model 10 is significantly lower, at the 10% significance level, than the MSPE of Model 8 at 8.3% of the forecast horizons; investigation of these results at the individual forecast steps reveals this significant MSPE reduction occurs at $h=1$ step, that is, at the very short term. This is the case with all uniform, left-tail, and right-tail weightings placed on the forecast loss differentials. Model 11, which pools forecasts from models with real-time data, generates significant MSPE reductions relative to Model 12 (which pools forecasts from models with final data) at 16.7% of the forecast horizons with right-tail weighting (see Table 6). When it comes to predicting low interest rates, however (i.e. with
left-tail weighting), its ability to forecast better than Model 12 increases to 83.3% of the forecast horizons (see Table 5).

To sum up, our forecasting results show that semiparametric models are flexible enough to forecast better than any other linear or nonlinear Taylor rule model; semiparametric model forecasts are also superior to pooled forecasts. Semiparametric model 8, which uses final data, forecasts better (based on the MSPE) or at least as well (based on the MedSPE) as semiparametric Model 7 (which uses real-time data) and better than any other model. This is more so during periods of high inflation rates (associated with large interest rate values). The relative forecasting superiority of models that use final as opposed to real-time data is not uncommon; for instance, Orphanides and van Norden (2005) report similar findings in forecasting the relationship between inflation and the output gap in the US. The forecasting superiority of semiparametric Model 8 with final data might be due to the revision process; real-time data might be subject to "noise" that degrades the accuracy of their out-of-sample forecasts relative to those obtained with final data.

We have also tried other pooled forecasts, such as pooled forecasts from all Taylor rule models (Models 1 through 6) and pooled forecasts from all models (Models 1 through 10). None of these forecasts was ranked any higher than the pooled forecasts reported in the paper.

In the interest of robustness, columns (iii)-(iv) of Table 3 report our forecasting rankings based on sequences of fixed-length rolling windows. According to the MSPE criterion, semiparametric Model 9 is the top-ranked model followed by Model 10 (the AR model) and then by Model 11 (the model that pools forecasts from models with real-time data). According to the MedSPE criterion, semiparametric models 7, 9, and 8 are ranked, first, second, and third, respectively. Therefore, rolling estimates confirm to some extent the forecasting superiority of semiparametric models based on the sequence of expanding windows discussed above.
6. Conclusions

In this paper we explore how the ECB sets interest rates in the context of Taylor-type policy reaction functions. We consider both linear and nonlinear policy functions in inflation, output and a measure of financial conditions. Using both real-time and revised information, we assess policy both in-sample and out-of sample. We find that amongst Taylor rule models, linear and nonlinear models are empirically indistinguishable within sample and that model specifications with real-time data provide the best description of in-sample ECB interest rate setting behavior. We also find that ECB policy-makers pay close attention to the financial conditions index when setting interest rates. In addition, the response of monetary policy to the financial conditions index depends on the state of inflation; the response increases when inflation is rising. A plausible explanation is that booming financial conditions trigger an increase in inflationary pressures. On the other, hand, the 2007-2009 financial crisis witnesses a shift from inflation targeting to output stabilisation and a shift, from an asymmetric policy response to financial conditions at high inflation rates, to a more symmetric response irrespectively of the state of inflation. Finally, semiparametric models, that relax the assumption of a Taylor rule specification and the restriction of a parametric structure on the reaction of monetary policy to observable economic variables, are flexible enough to forecast out-of-sample better than any linear or nonlinear Taylor rule model. This could help in some way to design new nonlinear parametric models to reflect the importance of such determinants.

The response of ECB policy-makers to financial conditions arguably has important policy implications as it might shed some light on why the current downturn in the Eurozone area is less severe than in the US where financial conditions do not feature in the Federal Reserve Bank’s reaction function. According to OECD calculations, annual US real output gap dropped from 0.7% in 2007 to -0.4% in 2008 and is expected to drop to -3.6% in 2009 and to -4.2% in 2010. On the other hand, annual real GDP output gap in the Eurozone area dropped from 0.8% in 2007 to -0.1% in 2008 and is expected
to drop to -2.4% in 2009 and to -3.1% in 2010. Although the Eurozone economic structure is less flexible than the US one, therefore providing more protection against bad economic outcomes (Trichet, 2009), targeting financial conditions might also be an additional reason. Our results offer some preliminary support to this argument. To further assess the importance of targeting financial conditions for economic stability, a more detailed study would allow (both in-sample and out-of-sample) for linear and regime switching behavior in joint estimates of the policy rate, aggregate supply and aggregate demand equations within a structural Vector Autoregressive (VAR) system in the interest rate, inflation, output gap and the financial index. We intend to extend our work to this very direction. Further, it would be interesting to estimate our model using data for different Central Banks in order to investigate the ability of linear, nonlinear and semiparametric models to predict in-sample and out-of-sample fluctuations in interest rates. It would also be interesting to investigate the robustness of our results with respect to the construction and evaluation of both interval and density forecasts; the use of interval and density forecasts may show improved forecasting performance for nonlinear models (Clements and Hendry, 1999). We note, however, the simulation results in Clements et al (2003) which suggest that the Diebold and Mariano test is in fact more powerful than interval and density forecast-based tests in discriminating between linear and nonlinear models. We intend to address these issues in future research.

References

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10 Data available from OECD’s website at http://www.oecd.org/dataoecd/26/40/38785295.htm#O.


Table 1: Model definitions

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Model Equation</th>
<th>Notes</th>
</tr>
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<tbody>
<tr>
<td>Linear model</td>
<td>$i_t = \rho_1 i_{t-1} + (1 - \rho_1) \left{ \rho_0 + \rho_2 E_{\tau_{t+2}} + \rho_3 y_t + \rho_4 \text{fin}<em>\text{index}</em>{t-1} \right} + \epsilon_t$</td>
<td>It uses real-time inflation and real-time industrial production.</td>
</tr>
<tr>
<td>Linear model</td>
<td>$i_t = \rho_1 i_{t-1} + (1 - \rho_1) \left{ \rho_0 + \rho_2 E_{\tau_{t+2}} + \rho_3 y_t + \rho_4 \text{fin}<em>\text{index}</em>{t-1} \right} + \epsilon_t$</td>
<td>It uses real-time inflation and final industrial production.</td>
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<td>Linear model</td>
<td>$i_t = \rho_1 i_{t-1} + (1 - \rho_1) \left{ \rho_0 + \rho_2 E_{\tau_{t+2}} + \rho_3 y_t + \rho_4 \text{fin}<em>\text{index}</em>{t-1} \right} + \epsilon_t$</td>
<td>It uses final inflation and final industrial production.</td>
</tr>
<tr>
<td>Linear model</td>
<td>$i_t = \rho_1 i_{t-1} + (1 - \rho_1) \left{ \rho_0 + \rho_2 E_{\tau_{t+2}} + \rho_3 y_t + \rho_4 \text{fin}<em>\text{index}</em>{t-1} \right} + \epsilon_t$</td>
<td>It uses real-time inflation and economic sentiment.</td>
</tr>
<tr>
<td>Nonlinear logistic model</td>
<td>$i_t = \rho_1 i_{t-1} + (1 - \rho_1) \left{ \rho_0 + \theta^r (E_{\tau_{t+2}}; \gamma^r, \tau) M_{y_\mu} + (1 - \theta^r (E_{\tau_{t+2}}; \gamma^r, \tau)) M_{y_\mu} \right} + \epsilon_t$</td>
<td>It uses real-time inflation and real-time industrial production.</td>
</tr>
<tr>
<td>Nonlinear logistic model</td>
<td>$i_t = \rho_1 i_{t-1} + (1 - \rho_1) \left{ \rho_0 + \theta^r (E_{\tau_{t+2}}; \gamma^r, \tau) M_{y_\mu} + (1 - \theta^r (E_{\tau_{t+2}}; \gamma^r, \tau)) M_{y_\mu} \right} + \epsilon_t$</td>
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<td>Nonlinear logistic model</td>
<td>$i_t = \rho_1 i_{t-1} + (1 - \rho_1) \left{ \rho_0 + \theta^r (E_{\tau_{t+2}}; \gamma^r, \tau) M_{y_\mu} + (1 - \theta^r (E_{\tau_{t+2}}; \gamma^r, \tau)) M_{y_\mu} \right} + \epsilon_t$</td>
<td>It uses real-time inflation and economic sentiment.</td>
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<tr>
<td>Semiparametric model</td>
<td>$i_t = \rho_1 (L) i_{t-1} + f(E_{\tau_{t+2}} y_t, \text{fin}<em>\text{index}</em>{t-1}) + \epsilon_t$</td>
<td>It uses real-time inflation and real-time industrial production.</td>
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<td>Semiparametric model</td>
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<td>Semiparametric model</td>
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<td>Linear Autoregressive model</td>
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<td>It uses real-time inflation and final data, that is, models 2, 5, and 8.</td>
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<td>Median forecast from models</td>
<td>Median forecast from models with real-time data, that is, models 1, 3, 4, 6, 7, and 9.</td>
<td>Median forecast from models with real-time data, that is, models 1, 3, 4, 6, 7, and 9.</td>
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Table 2: Model estimates, 1999:M1-2005:M12

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<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
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Notes: Numbers in parentheses are standard errors. The implied target \( \pi^* \) is derived as \( \pi^* = \frac{\hat{i} - \rho_0}{\rho_\pi} \), where \( \hat{i} = 3.04\% \). AIC is the Akaike Information Criterion. J stat is the \( p \)-value of a chi-square test of the model’s overidentifying restrictions (Hansen, 1982). The set of instruments includes a constant, 1-4, 9, 12 lagged values of inflation, the output gap, the 10-year government bond, M3 growth, and the financial index. The table also reports bootstrapped \( p \)-values of the \( \lambda, \lambda_a, \) and \( g \) tests based on 1000 re-samples.
Table 3: Average out-of-sample forecasting ranks

<table>
<thead>
<tr>
<th>Model $i$</th>
<th>(i) MSPE rank (recursive estimates)</th>
<th>(ii) MedSPE rank (recursive estimates)</th>
<th>(iii) MSPE rank (rolling estimates)</th>
<th>(iv) MedSPE rank (rolling estimates)</th>
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Notes: Columns (i)-(ii) report the average out-of-sample forecasting ranks of Model $i$ across the recursive windows and forecasting horizons $h=1,...,12$, using the Mean Squared Prediction Error (MSPE) and Median Squared Prediction Error (MedSPE) criteria. Columns (iii)-(iv) do the same for rolling windows. See Table 1 for the forecasting model definitions.
### Table 4: Pair-wise out-of-sample forecast comparison using $DM^*$

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Notes: The Table presents pair-wise out-of-sample forecast comparisons for the 12 forecasting models and expanding windows, across forecasting horizons $h = 1, \ldots, 12$, using the modified ($DM^*$) Diebold-Mariano MSPE statistic of Harvey et al. (1997). The entries in the Table show the percentage of forecast horizons for which the $DM^*$ test rejects the null hypothesis that Model $i$'s forecast performance as measured by MSPE is not superior to that of Model $j$ at the 10% significance level. See Table 1 for the forecasting model definitions.

### Table 5: Pair-wise out-of-sample forecast comparison using left-tailed $W-DM^*$

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Notes: The Table presents pair-wise out-of-sample forecast comparisons for the 12 forecasting models and expanding windows, across forecasting horizons $h = 1, \ldots, 12$, using the modified weighted Diebold-Mariano MSPE statistic of van Dijk and Franses (2003) ($W-DM^*$). The entries in the Table show the percentage of forecast horizons for which the left-tailed $W-DM^*$ test rejects the null hypothesis that Model $i$'s forecast performance as measured by MSPE is not superior to that of Model $j$ at the 10% significance level. See Table 1 for the forecasting model definitions.
Table 6: Pair-wise out-of-sample forecast comparison using right-tailed $W$-$DM^*$

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Notes: The Table presents pair-wise out-of-sample forecast comparisons for the 12 forecasting models and expanding windows, across forecasting horizons $h = 1,...,12$, using the right-tailed modified weighted Diebold-Mariano MSPE statistic of van Dijk and Franses (2003) ($W$-$DM^*$). The entries in the Table show the percentage of forecast horizons for which the right-tailed $W$-$DM^*$ test rejects the null hypothesis that Model $i$’s forecast performance as measured by MSPE is not superior to that of Model $j$ at the 10% significance level. See Table 1 for the forecasting model definitions.
Figure 1: Interest rate, inflation, output gap measures and the financial index

a) Interest rate and inflation measures

b) Output gap measures

c) Financial conditions index
Figure 2: Recursive Akaike Information Criterion (AIC) values, Models 1 to 6.
Figure 3: Recursive inflation, output gap, and financial index coefficients, Model 3

a) Inflation coefficient $\rho_\pi$

![Inflation coefficient graph](image)

b) Output gap coefficient $\rho_y$ (economic sentiment measure)

![Output gap coefficient graph](image)

c) Financial conditions index coefficient $\rho_f$

![Financial index coefficient graph](image)
Figure 4: Recursive inflation, output gap, and financial index coefficients, Model 6

a) Inflation coefficients $\rho_{1\pi}$ and $\rho_{2\pi}$

b) Output gap coefficients $\rho_{1y}$ and $\rho_{2y}$ (economic sentiment measure)

c) Financial conditions index coefficients $\rho_{1f}$ and $\rho_{2f}$