Regime Dependent Effects of Inflation Uncertainty on Real Growth: A Markov Switching Approach

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Abstract
We empirically investigate the effects of inflation uncertainty on output growth for the US between 1960 and 2012. Modeling output dynamics within a Markov regime switching framework, we provide evidence that inflation uncertainty exerts a negative and regime dependent impact on output growth. A battery of sensitivity checks confirm our findings.

Keywords: Growth; inflation uncertainty; regime dependency; Markov-switching modeling.
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1 Introduction

Economists agree that sustainable growth, and low and stable inflation constitute two of the fundamental objectives of the policymakers. A reason behind this conviction is that high and unstable inflation leads to an increase in inflation uncertainty distorting the efficient allocation of resources. To that end Friedman (1977) emphasizes that i) an increase in inflation raises inflation uncertainty;\(^1\) and that ii) high uncertainty, distorting the information content of prices, hinders the efficient allocation of resources. Along these lines, Beaudry et al. (2001) argue that during periods of high inflation volatility managers would be unable to detect profitable investment opportunities as it is harder to extract information about the relative prices of goods. Furthermore, during periods of high uncertainty, external funds become prohibitively expensive due to heightened asymmetric information problems causing managers to delay or cancel fixed investment projects. Lower investment, in turn, impedes output growth.

More recently, using structural models, several researchers have begun to examine the channels through which uncertainty could affect real variables. For instance, Bloom (2009) shows that macro uncertainty shocks cause a rapid drop and rebound in aggregate output and employment as firms temporarily pause their investment and hiring. Fernández-Villaverde et al. (2011) show that fiscal volatility shocks reduce output, consumption, investment, and hours worked drop on impact and stay low for several quarters.\(^2\) Basu and Bundick (2012), using a non-competitive one-sector model with counter-cyclical markups, show that in response to an uncertainty shock output, consumption, investment, and hours worked falls. Nakata (2012) using a standard New Keynesian model finds that an increase in the variance of shocks to the discount factor process reduces consumption, inflation, and output. Mumtaz and Theodoridis (2014) find that supply side uncertainty shocks lead to lower output due to precautionary savings. Yet, other researchers, for instance Bachmann and Bayer (2009), point out that risk might

\(^1\)A vast empirical literature provides support for this hypothesis. See for instance Caglayan et al. (2008) and the references therein.

\(^2\)Also see Primiceri (2005) who discusses the persistence of uncertainty regimes.
not be important to generate business cycle fluctuations.

While it is important to examine the uncertainty effects on real variables within the context of structural models, it is equally important to recognize that ignorance of the underlying nonlinearities in the data will lead to biased conclusions. Especially, if the relationship between explanatory variables and the independent variable were to change as the state of the economy varies, linear models would yield biased coefficient estimates and standard errors.\(^3\) Given this particular shortcoming, some researchers have recently begun to implement stochastic volatility models within the context of structural models to examine the impact of uncertainty on real variables.\(^4\) This approach, although attractive, as Fernández-Villaverade and Rubio-Ramírez (2013) point out, in cases where the underlying process has discrete jumps, SV model will anticipate the changes by showing changes in volatility before they happen. This result is due to the fact that estimation methods favor small rather than large changes in the data. Hence, in cases where data present regime shifts, it is advisable to use other approaches which are designed to capture such changes in the data.

In this study, recognizing the presence of regime shifts in inflation and output growth series, we examine the effects of inflation on output growth by implementing a Markov regime switching approach. To pursue our examination, we follow a two step approach. In the first stage we implement a Markov regime switching GARCH model to obtain a proxy for inflation uncertainty. In the second stage, we examine the level and the volatility effects of inflation on output growth using a Markov regime switching framework. One other advantage of this approach is that the model determines the regime switches endogenously. In our investigation, we scrutinize the growth rates of both monthly industrial production and quarterly gross domestic product data for the US. Our findings based on both industrial production and GDP growth rates provide evidence that the impact of inflation uncertainty on industrial production growth is not only significant

\(^3\)Evans and Wachtel (1993) infer that models which do not account for regime changes in the inflation process underestimate not only the extent of uncertainty but also the uncertainty effects on economic growth.

\(^4\)See for instance Fernández-Villaverade and Rubio-Ramírez (2013) and the references therein.
and negative but also it is regime dependent. This is an important finding as it requires the policymakers to consider the state of the economy prior to pursuing a certain policy action. For instance, policy tools that can be successfully used change substantially depending whether the economy is in a deflationary or an inflationary phase.

We also examine an extended model where we estimate inflation and growth rate series simultaneously as we consider the possibility of endogeneity that may emerge between inflation, inflation uncertainty and output growth. To estimate this model we implement a Markov switching model with instrumental variables (MRS-IV) as suggested by Spagnolo et al. (2005). This model also provides firm evidence that the volatility effects of inflation on output growth is regime dependent. Overall, our investigation provides firm evidence that the impact of uncertainty on output growth is negative and significant during the low growth regime yet although the effect is negative it is not significant during the high growth regime. Last but not the least, we examine the sensitivity of our results to the lag structure of the variables in the model and obtain similar observations. The analysis covers the 1960-2012 period.

The remainder of the paper is organized as follows. Section 2 provides a brief summary of the empirical literature. Section 3 presents the Markov switching GARCH methodology, the empirical model and the data. Section 4 reports the empirical results and section 5 concludes the paper.

2 A Brief Review of the Literature

A review of the empirical literature shows that the impact of inflation uncertainty on output growth depends on the approach that one uses to construct measures of uncertainty. For example, Davis and Kanago (1996), and Holland (1988) who use survey based uncertainty measures report that inflation uncertainty affects real economic activities negatively. Although this approach is appealing, survey based uncertainty measures may

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5 Also see Caggiano et al. (2014) and Alessandri and Muntaz (2014) along similar conclusions.
not gauge the true level of uncertainty for it may contain sizable measurement errors (see, for example, Bound et al. (2001)).

Due to its simplicity, researchers have also used the standard deviation or moving standard deviation of the inflation series as a proxy for inflation uncertainty. Findings based on this uncertainty measure are mixed as well. For instance, while Barro (1996) and Clark (1997) fail to provide any significant effects of inflation uncertainty on growth, Judson and Orphanides (1999) stress that inflation and inflation uncertainty are both significantly and negatively correlated with output growth. One major problem with this approach is that it imposes equal weights on all past observations and gives rise to substantial serial correlation in the summary measure.

Separately, researchers have been implementing two alternative approaches to estimate and forecast the volatility in macroeconomic time series. The first route is to utilize a variant of the GARCH methodology (see, for instance, Engle (1982) and Bollerslev (1986)) and the other one is to implement a variant of the Stochastic Volatility (SV) model (see, for instance, Taylor (1986)). Because the SV model is free from the restrictions that an ARCH/GARCH model imposes on the data, one may be tempted to use it because the in-sample fit and forecasts obtained from this model are better in comparison to that from the GARCH methodology.\(^6\) Separately, Fernández-Villaverade and Rubio-Ramírez (2013) argue that the use of SV models provide the researcher with an extra degree of freedom for it allows two shocks whereas the GARCH model allows a single shock to drive the level and volatility dynamics. In this context although SV modeling is preferred when a structural model is constructed, in time series analysis, GARCH approach is often utilized as the GARCH parameters can easily be estimated using maximum likelihood methods.\(^7\)

When we examine the literature, we see that several researchers including Fountas, Ioannidis and Karanasos (2004), Fountas, Karanasos, and Kim (2006) and Bhar and

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\(^6\)See Franses et al. (2008) and the references there in.

\(^7\)Some researchers also indicate that there are no notable differences between the models in terms of sample fit and forecasting (see Lehar et al. (2002)).
Mallick (2010) have initially used univariate ARCH/GARCH models to construct a measure of inflation uncertainty, and, in the second step, they have shown that inflation uncertainty has a negative impact on economic growth. To avoid the two stage modeling approach several other researchers, including Jansen (1989), Elder (2004), Mallik and Chowdury (2011), have implemented bivariate or multivariate (G)ARCH-M or E-GARCH-M models and shown that inflation uncertainty exerts a negative impact on output growth.8

One common weakness of the methodologies discussed above is that none of them considers the presence of regime shifts of the underlying series. In this context, despite its many attractive aspects, ARCH/GARCH methodology is also open to critique because this methodology, in general, assumes a certain economic structure and disregard the potential structural instabilities induced by regime changes. To that end Hamilton and Susmel (1994) and Gray (1996) argue that when regime shifts are overlooked, GARCH models may overstate the persistence in conditional variance.9 At this juncture, although one may be tempted to use stochastic volatility models, if the real process were to have discrete jumps, then the SV model will anticipate the changes by showing changes in volatility before they happen.10 This is because SV estimators favor a sequence of smaller changes over time rather than a jump in the data. To that end Diebold (1986) also shows that ignoring abrupt shifts, SV model may severely bias estimates towards non-stationarities and invalidate inferences.

In this study, we use a two step approach. In the first stage we follow Gray (1996) and compute an inflation uncertainty measure using the Generalized Markov regime switching GARCH methodology. In the second stage, we estimate our Markov switching model to examine the impact of inflation uncertainty on output growth.11 Our approach, allows

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8 The two stage approach could lead to biased coefficient and standard error estimates if the underlying uncertainty measure is gauged with errors.
9 Also see Giordani and Söderlind (2003) who argue that when the underlying series exhibit regime shifts, GARCH models would understate the level of uncertainty.
11 Regime switching models have been extensively used in the literature to examine the behavior of macroeconomic series. This class of models were introduced by Goldfeld and Quandt (1973) which later led to the Markov switching models as suggested by Hamilton (1989).
us to isolate the impact of inflation uncertainty on output growth.\textsuperscript{12}

\section*{3 Data and Econometric Methodology}

To empirically analyze the link between inflation uncertainty and output growth, we use monthly consumer price index (CPI) and monthly seasonally adjusted industrial production index (IPI) for the United States. Data are obtained from the International Financial Statistics of the International Monetary Fund and span the period 1960:01–2012:12. We also examine the association between output growth and inflation uncertainty using quarterly real GDP and CPI series.

We measure output growth ($y_t$) by the first difference of the log of industrial production index $\left[y_t = \log \left( \frac{IPI_t}{IPI_{t-1}} \right) \right]$. Similarly, we compute the inflation rate ($\pi_t$) as the first difference of the log of consumer price index $\left[\pi_t = \log \left( \frac{CPI_t}{CPI_{t-1}} \right) \right]$. We check for the presence of GARCH effects in the inflation series by applying the Lagrange Multiplier test. This test reveals significant GARCH effects in the inflation series. We then estimate a simple GARCH(1,1) model for inflation. As the sum of ARCH and GARCH terms from this model is very close to one, we suspect that the effects of past shocks on current variance is very strong; i.e. the persistence of volatility shocks is high. In this context, Gray (1996) points out that the high volatility persistence may be due to regime shifts in the conditional variance and suggests the use of a model that allows for regime shifts in the data.

Regime shifts in macroeconomic series have been noted earlier by several researchers. To our knowledge, Kim and Nelson (1999) and McConnel and Perez-Quiris (2000) are some of the early studies which report reduction of volatility in the US output. A subsequent study by Stock and Watson (2002) provides further evidence of a widespread volatility decline in macroeconomic series in the US. In particular, since mid-80s, we

\textsuperscript{12}To our knowledge Neanidis and Savva (2013) is the only study that examines the linkages between output and inflation accounting for regime changes within the context of a bivariate smooth transition EGARCH-M model. However, although they use a single step approach, their model is subjected to an identification problem.
observe that volatility measures for employment growth, inflation, consumption and sectoral output have declined sharply with respect to the 70s. For our case, we test for the presence of regime shifts in both inflation and output growth series as we implement Hansen (1992, 1996) tests. In addition, we examine the AIC (Akaike information criteria), Bayesian information criterion (BIC) and three-pattern method (TPM) as suggested by Psaradakis and Spagnolo (2003). These tests suggest in favor of structural break in inflation and output growth series.

3.1 Modeling Inflation Uncertainty Effects on Output Growth

To model the uncertainty effects of inflation on output growth, we implement the following model which accounts for regime changes in the data:

$$y_t = \phi_0 + \sum_{j=1}^{m} \beta_{ji} y_{t-j} + \sum_{j=1}^{k} \varphi_{ji} \pi_{t-j} + \delta_0 \hat{\sigma}_{\pi_{t-1}} + \xi_t,$$

where $y_t$ is the growth rate of output at time $t$ and $\hat{\sigma}_{\pi_{t-1}}$ is the first lag of inflation uncertainty. The model also includes lagged inflation rate and the lagged dependent variable to control for the level effects of inflation and the persistence of output growth. We allow all coefficients of Equation (1), which are indexed by $i$, to vary over the high and low growth regimes. The error term, $\xi_t$, is assumed to be conditionally normal with mean zero and variance $\sigma^2_{0i}$, which is subject to regime shifts. The key coefficients of interest are those associated with inflation uncertainty ($\delta_{01}$ and $\delta_{02}$).

\footnote{The model includes the lagged uncertainty to avoid the endogeneity problem.}
3.2 Measuring Inflation Uncertainty: Markov Switching GARCH Approach

To generate a proxy for inflation uncertainty, we implement the Markov switching GARCH methodology as proposed in Gray (1996). In their earlier work Cai (1994) and Hamilton and Susmell (1994) argue against the use of regime switching GARCH methodology because the model at any point in time depends directly on the unobserved state $S_t$ and indirectly on the history of $\{S_t\}$ (i.e., $\{S_{t-1}, S_{t-2},..., S_1\}$). Gray (1996) solves the path dependence problem as described in equation (2) below. In this model, the conditional mean of inflation follows an AR(p) process:

$$\pi_{it} = \theta_{0i} + \sum_{j=1}^{p} \theta_{ji} \pi_{t-j} + \epsilon_t,$$

and

$$\pi_{it} \mid \Omega_{t-1} \sim \begin{cases} N \left( \theta_{01} + \sum_{j=1}^{p} \theta_{j1} \pi_{t-j}, h_{1t} \right) & \text{w/probability } p_{1t}, \\ N \left( \theta_{02} + \sum_{j=1}^{p} \theta_{j2} \pi_{t-j}, h_{2t} \right) & \text{w/probability } 1 - p_{1t} \end{cases}$$

$$\epsilon_t \mid \Omega_{t-1} \sim N \left( 0, h_{it} \right).$$

where $i$ indicates the regime ($i = 1, 2$), $\pi_t$ represents the inflation process and $h_t$ denotes the conditional variance of inflation. Conditional on the information set available at time $t-1 (\Omega_{t-1})$, $p_{1t} = Pr (S_t = 1 \mid \Omega_{t-1})$ is the probability that the unobserved state variable $S_t$ is in regime 1.

Following Hamilton (1989), regime switches are assumed to be directed by a first-order
Markov process with fixed transition probabilities:\(^{14}\)

\[
\begin{align*}
    Pr [S_t = 1 \mid S_{t-1} = 1] &= P_{11}, \\
    Pr [S_t = 2 \mid S_{t-1} = 1] &= 1 - P_{11}, \\
    Pr [S_t = 2 \mid S_{t-1} = 2] &= P_{22}, \\
    Pr [S_t = 1 \mid S_{t-1} = 2] &= 1 - P_{22}.
\end{align*}
\]

(3)

The conditional variances from the two regimes can be aggregated based on regime probabilities. Note that the aggregate conditional variance is not path dependent and it can be used to compute the conditional variance at the next period. The conditional variance, which follows a GARCH(1,1) process, can be expressed as:

\[
h_{it} = \alpha_{0i} + \alpha_{1i}\varepsilon_{t-1}^2 + \alpha_{2i}h_{t-1}
\]

(4)

where

\[
    \varepsilon_{t-1} = \pi_{t-1} - [p_{1t-1}\mu_{1t-1} + (1 - p_{1t-1})\mu_{2t-1}],
\]

\[
    \mu_{it-1} = \theta_{0i} + \sum_{j=1}^{p} \theta_{ji}\pi_{t-j-1},
\]

and

\[
h_{t-1} = p_{1t-1}\left(\mu_{1t-1}^2 + h_{t-1}\right) + (1 - p_{1t-1})\left(\mu_{2t-1}^2 + h_{2t-1}\right) - \\
p_{1t-1}\mu_{1t-1} + (1 - p_{1t-1})\mu_{2t-1}\right)^2.
\]

The non-negativity of \(h_t\) for all \(t\), is ensured by a set of assumptions that \(\alpha_{0i} \geq 0\), \(\alpha_{1i} \geq 0\) and \(\alpha_{2i} \geq 0\). Note that all parameters of the conditional variance of inflation are state-dependent. Furthermore, as in the case of a single-regime GARCH(1,1) model, the necessary condition for stationarity is that \(\alpha_{1i} + \alpha_{2i} < 1\).

\(^{14}\)For instance, if the economy is in the first state at time \(t-1\) (\(S_{t-1} = 1\)), \(P_{11}\) denotes the probability of switching to the first state at time \(t\) (\(S_t = 1\)).
To estimate the model, we use the maximum likelihood methodology:

\[
L = \sum_{t=1}^{T} \log \left[ \frac{1}{\sqrt{2\pi h_{1t}}} \exp \left\{ -\frac{(\pi_t - \mu_1)^2}{2h_{1t}} \right\} + (1 - p_{1t}) \frac{1}{\sqrt{2\pi h_{2t}}} \exp \left\{ -\frac{(\pi_t - \mu_2)^2}{2h_{2t}} \right\} \right],
\]

where the regime probability \( p_{1t} \) follows a simple nonlinear recursive system:

\[
p_{1t} = P_{11} \left[ \frac{f_{1t-1}p_{1t-1}}{f_{1t-1}p_{1t-1} + f_{2t-1}(1 - p_{1t-1})} \right] + (1 - P_{22}) \left[ \frac{f_{2t-1}(1 - p_{1t-1})}{f_{1t-1}p_{1t-1} + f_{2t-1}(1 - p_{1t-1})} \right].
\]

Assuming conditional normality, the conditional distribution of inflation, \( f_{it} \) where \( i = 1, 2 \), takes the form:

\[
f_{it} = f(\pi_t \mid S_t = i, \Omega_{t-1}) = \frac{1}{\sqrt{2\pi h_{it}}} \exp \left\{ -\frac{(\pi_t - \mu_i)^2}{2h_{it}} \right\}.
\]

We use the conditional variance of the inflation process obtained from the above procedure as a proxy for inflation uncertainty.

It should be noted that the inflation uncertainty measure used in the second stage regression is a generated regressor. Pagan (1984) and Pagan and Ullah (1988) argue that a generated regressor gauges the true unobserved regressor with error. They indicate that the use of a generated regressor measured with error leads to biased coefficient and standard error estimates. Pagan and Ullah (1988) continue to state that the standard instrumental variable approach may not be valid when the endogenous variable is a function of the entire history of the available data. For such cases, they suggest testing the validity of the underlying assumptions of the model that is used to generate the uncertainty proxy and then use the lags of this proxy as an instrument.\footnote{Several researchers implement a similar approach to examine the uncertainty effects on real economic activities. For instance see Ruge-Murcia (2003), Baum et al. (2010) and Caglayan et al. (2013).}

We follow this suggestion and check whether the model we use to generate the uncertainty measure is well specified. After ascertaining that it is the case, we continue with our investigation.\footnote{Specification test results are available upon request from the authors. These tests show that the}
Table 1 reports the maximum likelihood estimates of the Markov Switching GARCH(1,1) model for inflation where the mean inflation rate is modeled as an AR(1) process. Results show that the coefficients of the conditional mean are highly significant for both regimes. In State 1, the implied monthly inflation rate is around 0.13 per cent and in State 2, that the rate is around 0.52 per cent. Thus, State 1 is identified as the low inflation regime and State 2 is recognized as the high inflation regime.

When we inspect the conditional variance of inflation over the two regimes we observe that all parameter estimates are highly significant. Within each regime the GARCH process is stationary as $\alpha_{11} + \alpha_{21} < 1$. Low inflation regime is more sensitive to recent shocks (i.e. $\alpha_{11} > \alpha_{12}$). Moreover, high inflation regime has higher persistence to shocks than low inflation regime (i.e. $\alpha_{22} > \alpha_{21}$). This means that the impact of shocks does not die quickly in the high inflation regime. The estimates of the transition probabilities $P_{11}$ and $P_{22}$ (i.e. $(1 - P_{12})$) are 0.987 and 0.987, respectively, and these estimates suggest the presence of strong persistence of high and low regimes. Within regime persistence of the conditional variance, the sum of the coefficients of ARCH and GARCH terms ($\alpha_{11} + \alpha_{21}$), are 0.622 in State 1 and 0.981 in State 2. A single regime GARCH model would not capture these subtleties.

Figure 1 plots the derived uncertainty measure (IU) along with industrial production growth (IPG) and inflation (INF). The figure shows that inflation and its volatility tend to move together. We also present in Table 2 the periods during which the US economy went through recessionary episodes as announced by the NBER. We see that during the period of our investigation the US has gone through eight recessionary episodes which are shaded in the figure.\(^{18}\)

\(^{17}\)The model choice is based on the SIC criteria.

\(^{18}\)NBER defines an economic recession as: ‘a significant decline in economic activity spread across the country, lasting more than a few months, normally visible in real GDP growth, real personal income, employment (non-farm payrolls), industrial production, and wholesale-retail sales’.

Insert Table 1 and Figure 1 and
To develop a sense to what extent our inflation uncertainty measure relates to the readily available series in the literature, we do the following exercise. We first examine the correlation between our uncertainty measure with the standard deviation of the inflation forecasts based on the University of Michigan inflation expectation series. We find that the correlation is 29% and it is significant at a 1% significance level. We also compute the correlation of our uncertainty proxy with the CBOE Volatility Index (VIX), which is considered as the financial crises index.\textsuperscript{19} Although the correlation coefficient is low at 18%, it is significant at the 5% level. Positive and significant correlations between our uncertainty measure and the two readily available alternatives can be taken as an independent observation that our measure successfully captures the uncertainty in the US price index. Low correlation could be explained by the fact that we allow for regime shifts in computing the inflation uncertainty measure whereas the other two measures do not.

4 Empirical Analysis

This section presents three sets of results based on monthly and quarterly data. Table 3 presents our first set of results based on monthly industrial production data. We then present two additional sets of results to ascertain the validity of our initial observations. In Table 4 results are obtained from quarterly GDP series. We also examine an extended model where we estimate inflation and growth rates simultaneously as we consider the possibility of endogeneity that may emerge between inflation, inflation uncertainty and output growth. To estimate this model we adopt a Markov switching approach with instrumental variables (MRS-IV) as suggested by Spagnolo et al. (2005). We report the results of this model in Table 5. Last but not the least, we estimate our models allowing\textsuperscript{19}This is a relevant comparison because researchers suggest that increased inflation volatility triggers financial crises.
for different lag structure. Results from all models are similar and suggest that inflation uncertainty has a regime dependent effect on output growth as detailed below.

### 4.1 Results Based on Monthly Data

Table 3 provides our basic results for the growth rate of monthly industrial production. When we inspect the coefficient estimates of the model, we observe that the impact of inflation uncertainty in regime one ($\delta_{01}$), the high growth regime, is negative (-0.070) and significant at the 1% level. We also observe that the impact of inflation uncertainty on output in regime two ($\delta_{02}$), the low growth regime, is negative (-0.178) and significant at the 10% level. These observations suggest that the impact of inflation uncertainty on output growth is negative and varies across the business cycle. Moreover, the magnitude of the adverse impact of inflation uncertainty on output growth in the low growth regime is more than twice as much as of that in the high growth regime. We confirm that, based on the likelihood ratio test, the asymmetry of uncertainty effects on output growth between recessions and expansions (the null hypothesis of symmetry ($\delta_{01} = \delta_{02}$) is rejected at the 1% significance level). Table 3 also shows that the impact of inflation on output growth rate is negative and but insignificant for both regimes. These observations provide evidence that inflation uncertainty exerts negative and asymmetric effects on output growth over the business cycle.

To appreciate the use of Markov regime switching approach, it is useful to examine the smoothed probabilities for State 1 (high growth regime) which we provide in Figure 2. This figure shows that the implied turning points match reasonably well with the announced NBER dates. Although the model picks up additional turning points (periods of contraction) than those announced by the NBER, these can be explained by the presence of rapid changes in the output growth series and do not necessarily imply that the model is improperly specified.

Insert Figure 2 and

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20Results from this last exercises are not reported to conserve space but they are available upon request.
4.2 Results Based on Quarterly Data

To verify the observations we have provided, we estimate the model using quarterly GDP series over the period 1960:Q1–2012:Q4. To carry out the analysis we aggregate monthly inflation uncertainty series to quarterly frequency. We measure the growth rate of real GDP in period $t$, $Y_t$, as the first difference of the log of real GDP, $Y_t = \log \left( \frac{RGDP_t}{RGDP_{t-1}} \right)$. Based on the AIC criteria, the model allows for three lags of the dependent variable and one lag for the inflation series. An additional advantage of working with quarterly data is that we can directly compare the estimated dates for low- and high-growth phases of the economy with the business cycle dates announced by the NBER more closely.

The smoothed probability estimates for the quarterly data are shown in Figure 3. Examining this figure, we see that the economic contractions implied by our model largely match with those announced by the NBER dates as summarized in Table 2. Similar to the case of monthly data, the model detects some additional turning points. Following the censoring rule of Harding and Pagan (2002), if we assume that a complete cycle (peak to peak or trough to trough) should last at least five quarters, these rapid movements in the data should not be classified as a period of recession. Furthermore, inspecting the data closely, the additional dates which the model suggests as periods of contraction can be explained by rapid changes in output growth series. Overall, the model appears to successfully predict the business cycle turning points in the US economy.

Table 4 reports our findings. On inspection, we find that the results for the quarterly data are stronger compared to the case of monthly data. This may be due to the fact that industrial production represents only a portion of the output generated in the economy whereas GDP measures the total output generated in the country.
Table 4 shows that during the low growth regime, inflation uncertainty has a negative effect \( \delta_{02} = -0.344 \) and this effect is different from zero at the 1% significance level. We also observe that during the high growth regime inflation uncertainty effects on growth is negative \( \delta_{01} = -0.179 \) and different from zero at the 1% significance level. Ceteris paribus, the adverse impact of inflation uncertainty on economic growth is almost 2 times higher in recessions than that in expansions. These estimates support the view that the impact of inflation uncertainty on output growth over the business cycle is asymmetric. Based on the likelihood ratio test, the null hypothesis of symmetry \( \delta_{01} = \delta_{02} \) is rejected at the 1% significance level. Inspecting the table we also see that inflation has a negative and significant effect on economic growth in both regimes. Furthermore, this effect is regime dependent and the adverse effects of inflation on economic growth is higher in low growth regimes.

### 4.3 Controlling for Endogeneity

In this section, we extend our model and estimate the inflation and growth rates series simultaneously while we consider the possibility of endogeneity that may emerge between inflation, inflation uncertainty and output growth. To estimate this model we implement a Markov switching model with instrumental variables (MRS-IV) as suggested by Spagnolo et al. (2005). The system of equations that we estimate takes the following form:

\[
\pi_t = \theta_{0i} + \sum_{j=1}^{L} \theta_{ji} y_{t-j} + \sum_{j=1}^{N} \eta_{ji} \hat{\sigma}_{\pi t-1} + \psi_{i} \hat{\sigma}_{\pi t-1} + \alpha_i \hat{\sigma}_{y t-1} + \varepsilon_t, \tag{6}
\]

\[
y_t = \phi_i + \sum_{j=1}^{m} \beta_{ji} y_{t-j} + \sum_{j=0}^{k} \varphi_{ji} \hat{\sigma}_{\pi t-1} + \delta_i \hat{\sigma}_{\pi t-1} + \kappa_i \hat{\sigma}_{y t-1} + \xi_t \tag{7}
\]

where \( \xi_t \mid \Omega_{t-1} \sim N(0, \sigma_{\xi_t}^2) \) and \( \varepsilon_t \mid \Omega_{t-1} \sim N(0, \sigma_{\varepsilon_t}^2) \).

In equation (6) output growth and its volatility as well as inflation variability enters the model with higher and single lags, respectively. Similarly, in equation (7) we introduce inflation and inflation uncertainty with higher and single lags, respectively, while we
control for the impact of lagged output growth uncertainty. In both equations we allow for lagged dependent variables to allow for persistence in the data. The error terms in equations (6) and (7) represent structural shocks and they are assumed to be uncorrelated with each other.21

Our extended model is in the same spirit as that used by Mumtaz and Theodoridis (2014) who impose three key assumptions that i) the shocks to the volatility and the level are uncorrelated and ii) the variance covariance matrix of volatility shocks is diagonal and iii) the contemporaneous interaction of the endogenous variables has a recursive structure. They carry out their investigation implementing a structural Vector Autoregressive (SVAR) framework with stochastic volatility and estimate the dynamic interaction between the endogenous variables in the VAR and the time-varying volatility. They follow a one step approach but their approach cannot account for the presence of regime shifts. In contrast, we follow a two step approach and specifically examine the impact of inflation and inflation volatility on output growth over high and low growth regimes.

Insert Table 5 about here

The results obtained for our extended model are reported in Table 5. Observing the table we see that inflation uncertainty has a significant negative impact on output growth in the low growth regime but it has no significant effect in the high growth regime. Once more, the null of symmetry is rejected at the 1% level supporting the claim that the uncertainty effects on output growth are regime dependent. Similar to our earlier findings, inflation has a negative impact on output growth during the low growth regime but this effect is insignificant in both regimes. When we turn to our findings for equation (6), similar to Cukierman and Meltzer (1986) and Cukierman (1992), we see that inflation uncertainty has a positive impact on inflation.22 We find that output growth uncertainty has a significant impact on inflation only during the high growth regime.23

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21 The appendix provides further details of the model.
22 Holland (1995) reports a negative association.
23 Deveraux (1989) reports similar findings.
Overall our results demonstrate the presence of significant regime-dependent asymmetric effects of inflation uncertainty on output growth. Our findings provide evidence that nominal uncertainty retards growth in both low- and high-growth regimes, but more so during periods of low growth. Our results also suggest the use of linear and single regime models inhibit the researcher from observing the differential effects of explanatory variables over the business cycle and can lead to mixed or ambiguous conclusions.

5 Conclusion

In this paper, we examine the impact of inflation uncertainty on output growth over the business cycle. In doing so we account for regime shifts in both output and inflation series by implementing regime switching models as we follow a two step approach. In particular, we utilize the Markov regime switching GARCH model suggested by Gray (1996) to construct our uncertainty measure. Next, we examine the impact of inflation uncertainty on output growth by implementing a Markov switching framework. The investigation uses both monthly and quarterly data sets for the US over the period 1960–2012.

Our findings based on the growth of industrial production show that the impact of inflation uncertainty on industrial production growth is negative in both regimes. Furthermore, we show that the impact is asymmetric and it is statistically significant. To verify our findings, i) we carry out the analysis for quarterly GDP series, ii) impose additional lag structure on the explanatory variables, iii) consider the possibility of endogeneity that may emerge between inflation, inflation uncertainty and output growth. In all cases, the results suggest that uncertainty exerts a negative and regime dependent effect on output growth.

Our results demonstrate the existence of significant negative regime-dependent effects of inflation uncertainty on output growth. Our findings are consistent and supportive of the recent research which suggests that higher uncertainty will cause firms to postpone
investments and hiring, leading to lower economic activity.\textsuperscript{24} It should also be noted that higher inflation uncertainty may induce higher inflation inducing workers and firms to ask for higher wages and prices, respectively.\textsuperscript{25} Based on our findings, we suggest that central banks should implement policies that promote price stability.\textsuperscript{26} In particular, as the adverse impact of uncertainty is much severe in recessions, different from the literature, our results provide us with a firm basis to argue that during low growth periods the merits of economic stability can be higher than previously thought. A wider investigation based on data from other countries on the regime dependent effects of uncertainty on output growth would further expand our knowledge.

\textsuperscript{24}See for example Bloom (2009).

\textsuperscript{25}Workers set higher wages as an insurance against the possibility to be locked in a contractual agreement to increase labour when demand is high. For the same reasoning firms will increase prices.

\textsuperscript{26}For instance, according to Taş (2012), inflation targeting would lead to lower inflation uncertainty.
The extended Model

The structural model presented in equations (6) and (7) can be represented in the matrix form as follows:

\[
\begin{bmatrix}
1 & 0 \\
\varphi_{0i} & 1
\end{bmatrix}
\begin{bmatrix}
\hat{\pi}_t \\
y_t
\end{bmatrix}
= \begin{bmatrix}
\theta_{0i} \\
\phi_i
\end{bmatrix}
+ \sum_{j=1}^{L}
\begin{bmatrix}
\eta_{ji} & \theta_{ji} \\
\varphi_{ji} & \beta_{ji}
\end{bmatrix}
\begin{bmatrix}
\hat{\pi}_{t-j} \\
y_{t-j}
\end{bmatrix}
+ \begin{bmatrix}
\psi_i & \alpha_i \\
\delta_i & \kappa_i
\end{bmatrix}
\begin{bmatrix}
\hat{\sigma}_{\pi_{t-1}} \\
\hat{\sigma}_{y_{t-1}}
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_{ti} \\
\xi_{ti}
\end{bmatrix}
\]

or

\[
B_{0i}x_t = k_i + \sum_{j=1}^{L} B_{ji}x_{t-j} + \Psi_i\sigma_{t-1} + \epsilon_{t,i}
\]

where

\[
x_t = \begin{bmatrix}
\hat{\pi}_t \\
y_t
\end{bmatrix},
\quad k = \begin{bmatrix}
\theta_{0i} \\
\phi_i
\end{bmatrix},
\quad B_{0i} = \begin{bmatrix}
1 & 0 \\
\varphi_{0i} & 1
\end{bmatrix},
\quad \Psi_i = \begin{bmatrix}
\psi_i & \alpha_i \\
\delta_i & \kappa_i
\end{bmatrix},
\quad B_{ji} = \begin{bmatrix}
\eta_{ji} & \theta_{ji} \\
\varphi_{ji} & \beta_{ji}
\end{bmatrix},
\quad \sigma_{t-1} = \begin{bmatrix}
\hat{\sigma}_{\pi_{t-1}} \\
\hat{\sigma}_{y_{t-1}}
\end{bmatrix}
\quad \text{and} \quad \epsilon_{t,i} = \begin{bmatrix}
\varepsilon_{ti} \\
\xi_{ti}
\end{bmatrix}.
\]

The variance covariance matrix assumed to take the following form:

\[
\Sigma_{\epsilon_{i}} = \mathbb{E}(\epsilon_{t,i}\epsilon_{t,i}') = \begin{bmatrix}
\sigma_{\epsilon_{ti}}^2 & 0 \\
0 & \sigma_{\xi_{ti}}^2
\end{bmatrix}
\]

If we pre-multiply (9) by the matrix \(B_{0i}^{-1}\), we obtain an identified reduced form model:

\[
x_t = c_i + \sum_{j=1}^{L} \Phi_{ji}x_{t-j} + \Pi_i\sigma_{t-1} + u_{t,i}
\]

where

\[
c_i = B_{0i}^{-1}k_i, \quad \Phi_{ji} = B_{0i}^{-1}B_{ji}, \quad \Pi_i = B_{0i}^{-1}\Psi_i
\]
and

\[
\mathbf{u}_{t,i} = B_{0i}^{-1} \mathbf{e}_{t,i} = \begin{bmatrix} 1 & 0 \\ -\varphi_{0i} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{ti} \\ \xi_{ti} \end{bmatrix} = \begin{bmatrix} \varepsilon_{ti} \\ \xi_{ti} - \varphi_{0i} \varepsilon_{ti} \end{bmatrix}
\]

and

\[
\Pi_i = \begin{bmatrix} \psi_i & \alpha_i \\ \delta_i - \varphi_{0i} \psi_i & \kappa_i - \varphi_{0i} \psi_i \alpha_i \end{bmatrix}.
\]

We impose the restriction that the first error in the reduced form model coincides with the structural shock of inflation. Furthermore, we can recover the structural shock on output growth as well as the spillover effect across volatilities so long as we have an estimate of \( \varphi_{0i} \). We can estimate \( \varphi_{0i} \) either by regressing \( \xi_t \) on \( \varepsilon_t \) or using the variance covariance matrix of \( \mathbf{u}_{t,i} \) and \( \mathbf{e}_{t,i} \):

\[
\Sigma_{u,i} = E(\mathbf{u}_{it}\mathbf{u}_{it}') = B_{0i}^{-1} \Sigma_{e,i} B_{0i}^{-1}'
\]

or

\[
\Sigma_{u,i} = \begin{bmatrix} \sigma_{\pi ti}^2 & \sigma_{\pi ti, yti}^2 \\ \sigma_{\pi ti, yti}^2 & \sigma_{yt ti}^2 \end{bmatrix} = \begin{bmatrix} \sigma_{\varepsilon ti}^2 & -\varphi_{0i} \sigma_{\varepsilon ti}^2 \\ -\varphi_{0i} \sigma_{\varepsilon ti}^2 & \sigma_{\xi ti}^2 - \varphi_{0i} \sigma_{\varepsilon ti}^2 \end{bmatrix}
\]

where \( \sigma_{\pi ti}^2 \) is the variance of inflation, \( \sigma_{yt ti}^2 \) is the variance of output growth and \( \sigma_{\pi ti, yti}^2 \) is the covariance between output growth and inflation. The left-hand side of Equation (12) includes three independent sources of information while the right hand-side includes three unknown parameters of the structural model. Thus, the model is identified.\(^{27}\) In this context, unlike our model, Neanidis and Savva (2013) do not impose any restrictions on the contemporaneous interaction between inflation and output. As a result, the off diagonal element of their variance covariance matrix, \( \Sigma_{u,i} \), incorporates the contemporaneous impact of output growth on inflation as well as that of output growth volatility.\(^{28}\)

\(^{27}\)It is straightforward to show that:

\[
\sigma_{\varepsilon ti}^2 = \sigma_{\xi ti}^2, \quad \varphi_{0i} = -\frac{\sigma_{\pi ti, yti}}{\sigma_{\pi ti}^2} \quad \text{and} \quad \sigma_{\xi ti}^2 = \sigma_{yt ti}^2 - \sigma_{\pi ti, yti}^2
\]

\(^{28}\)In Neanidis and Savva (2013), for each regime \( \Sigma_{u,i} \) contains three independent source of information.
To estimate the model one can use a recursive algorithm. The conditional probability density function of the data \( w_t = (y_t, \pi_t) \) given the state \( S_t \) and the history of the system can be written as follows:

\[
\text{pdf}(w_t | w_{t-1}, ..., w_1; \nu) = \frac{1}{\sqrt{2\pi}\sigma_{\xi_i}} \exp \left( -\frac{1}{2} \left( \frac{y_t - \phi_i - \sum_{j=1}^{m} \beta_{ji}y_{t-j} - \sum_{j=0}^{k} \varphi_{ji}\hat{\pi}_{t-j} - \delta_i\hat{\sigma}_{\pi_{t-1}} - \kappa_i\hat{\sigma}_{y_{t-1}}}{\sigma_{\xi_i}} \right)^2 \right) \]

\[
\times \frac{1}{\sqrt{2\pi}\sigma_{\epsilon_i}} \exp \left( -\frac{1}{2} \left( \frac{\pi_t - \theta_{0i} - \sum_{j=1}^{L} \theta_{ji}y_{t-j} - \sum_{j=1}^{N} \eta_{ji}\hat{\pi}_{t-j} - \psi_i\hat{\sigma}_{\pi_{t-1}} - \alpha_i\hat{\sigma}_{y_{t-1}}}{\sigma_{\epsilon_i}} \right)^2 \right)
\]

and four unknown parameters. This implies that their model is not identifiable.
References


Figure 1: Inflation, Output Growth and Inflation Uncertainty

Figure 2: Smoothed Probabilities for State 1 (High Growth Regime)–Monthly Data
Figure 3: Smoothed Probabilities for State 1 (High Growth Regime)–Quarterly Data

Table 1: Measuring Inflation Uncertainty: The Markov Switching GARCH Model

\[ \pi_{it} = \theta_{0i} + \sum_{j=1}^{p} \theta_{ji} \pi_{t-j} + \varepsilon_t, \text{ where } \varepsilon_t \mid \Omega_{t-1} \sim N(0, h_{it}), \]

\[ h_{it} = \alpha_{0i} + \alpha_{1i} \varepsilon_{t-1}^2 + \alpha_{2i} h_{t-1} \text{ and } i=1,2 \text{ are regimes.} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{01} )</td>
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<td>0.000</td>
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<tr>
<td>( \theta_{11} )</td>
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<td>0.066</td>
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<tr>
<td>( \theta_{02} )</td>
<td>0.002***</td>
<td>0.000</td>
</tr>
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<td>( \theta_{12} )</td>
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<td>0.053</td>
</tr>
<tr>
<td>( \alpha_{01} )</td>
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Notes: *, **, *** denote significance at the 10%, 5% and 1% levels.
Table 2: NBER Dates of Expansions and Contractions

<table>
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<tr>
<th>Business Cycles</th>
<th>Reference Dates</th>
<th>Duration in Months</th>
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<td></td>
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<td>Trough</td>
</tr>
<tr>
<td>April 1960(II)</td>
<td>February 1961(I)</td>
<td>10</td>
</tr>
<tr>
<td>December 1969(IV)</td>
<td>November 1970(IV)</td>
<td>11</td>
</tr>
<tr>
<td>November 1973(IV)</td>
<td>March 1975(I)</td>
<td>16</td>
</tr>
<tr>
<td>January 1980(I)</td>
<td>July 1980(III)</td>
<td>6</td>
</tr>
<tr>
<td>July 1981(III)</td>
<td>November 1982(IV)</td>
<td>16</td>
</tr>
<tr>
<td>July 1990(III)</td>
<td>March 1991(I)</td>
<td>8</td>
</tr>
<tr>
<td>March 2001(I)</td>
<td>November 2001(IV)</td>
<td>8</td>
</tr>
<tr>
<td>December 2007(IV)</td>
<td>June 2009(II)</td>
<td>18</td>
</tr>
</tbody>
</table>

Source: National Bureau of Economic Research (NBER), Quarterly dates are in parentheses.

Table 3: Inflation Uncertainty Effects on Output Growth: Monthly Data

\[ y_t = \phi_0 + \sum_{j=1}^{m} \beta_{ji} y_{t-j} + \sum_{j=1}^{k} \varphi_{ji} \pi_{t-j} + \delta_0 \sigma_{t-1} + \xi_t, \]

\[ \xi_t \mid \Omega_{t-1} \sim N(0, \sigma_{0i}^2), \text{ and } i=1,2 \text{ are regimes.} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>\phi_{01}</td>
<td>0.002***</td>
<td>0.000</td>
</tr>
<tr>
<td>\beta_{11}</td>
<td>0.124**</td>
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<tr>
<td>\beta_{21}</td>
<td>0.249***</td>
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<td>\beta_{31}</td>
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<tr>
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<td>0.002</td>
</tr>
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<td>\beta_{12}</td>
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<td>0.078</td>
</tr>
<tr>
<td>\beta_{22}</td>
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<td>\beta_{32}</td>
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<td>0.086</td>
</tr>
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<td>\varphi_{12}</td>
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<td>0.000</td>
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<td>P_{11}</td>
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<tr>
<td>P_{12}</td>
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</tbody>
</table>

Log-likelihood 2320.037

Notes: *, **, *** denote significance at the 10%, 5% and 1% levels.
Table 4: Inflation Uncertainty Effects on Output Growth: Quarterly Data

\[ y_t = \phi_0 + \sum_{j=1}^{m} \beta_{ji} y_{t-j} + \sum_{j=1}^{k} \varphi_{ji} \pi_{t-j} + \delta_0 \sigma_{\pi_{t-1}} + \xi_t, \]

\[ \xi_t \mid \Omega_{t-1} \sim N(0, \sigma_{\xi_{it}}^2), \text{ and } i=1, 2 \text{ are regimes.} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_{01} )</td>
<td>0.009***</td>
<td>0.001</td>
</tr>
<tr>
<td>( \beta_{11} )</td>
<td>0.192***</td>
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<tr>
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<tr>
<td>( \beta_{31} )</td>
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<td>0.054</td>
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<tr>
<td>( \varphi_{11} )</td>
<td>-0.128**</td>
<td>0.061</td>
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<tr>
<td>( \delta_{01} )</td>
<td>-0.179***</td>
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<td>( \phi_{02} )</td>
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<td>0.001</td>
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Log-likelihood 741.172

Notes: *, **, *** denote significance at the 10%, 5% and 1% levels.
Table 5: Estimates of Parameters of the Model for Output Growth and Inflation

\[
\pi_t = \theta_0 i_t + \sum_{j=1}^L \theta_{ji} y_{t-j} + \sum_{j=1}^N \eta_{ji} \pi_{t-j} + \psi_i \tilde{\sigma}_{\pi_{t-1}} + \alpha_i \tilde{\sigma}_{y_{t-1}} + \varepsilon_t, \\
y_t = \phi_1 i_t + \sum_{j=1}^m \beta_{ji} y_{t-j} + \sum_{j=0}^k \varphi_{ji} \tilde{\pi}_{t-j} + \delta_i \tilde{\sigma}_{\pi_{t-1}} + \kappa_i \tilde{\sigma}_{y_{t-1}} + \xi_t
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. error</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_1)</td>
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<td>0.001</td>
<td>(\theta_{01})</td>
<td>0.001 ***</td>
<td>0.000</td>
</tr>
<tr>
<td>(\beta_{11})</td>
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<td>0.057</td>
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<td>0.053</td>
<td>(\theta_{21})</td>
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</tr>
<tr>
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<td>(\eta_{11})</td>
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<td>(\alpha_2)</td>
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<td>(\sigma_{\varepsilon_1})</td>
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Log likelihood = 5188.000

Notes: *, **, *** denote significance at the 10%, 5% and 1% levels.