PHY331
Magnetism
Lecture 3
Last week…

• Derived magnetic dipole moment of a circulating electron.
• Discussed motion of a magnetic dipole in a constant magnetic field.
• Showed that it precesses with a frequency called the Larmor precessional frequency.
This week…

• Discuss Langevin’s theory of diamagnetism.
• Use angular momentum of precessing electron in magnetic field to derive the magnetization of a sample and thus diamagnetic susceptibility.
• Will set the scene for the calculation of paramagnetic susceptibility.
Langevin’s theory of diamagnetism

• We want to calculate the sample magnetisation \( M \) and the diamagnetic susceptibility \( \chi \) (recall \( M = \chi H \))

• Every circulating electron on every atom has a \textit{magnetic dipole moment}

• The sum of the magnetic dipole moments on any atom is zero (equal numbers circulating clockwise and anticlockwise ?)

• The magnetisation \( M \) arises from the reaction to the torque \( \Gamma \) due to the applied magnetic field \( B \) which creates the Lamor precessional motion at frequency \( \omega_L \).
• i) the angular momentum \( L \) of the circulating electron is,
\[
L = mvr = m\omega r^2
\]

• The angular momentum \( L_p \) of the precessional motion is,
\[
L_p = m\omega_L \langle r^2 \rangle
\]

where \( \omega_L \) is the Larmor frequency and \( \langle r^2 \rangle \) is the mean square distance from an axis through the nucleus which is parallel to \( B \)
• ii) We showed last week that the magnetic moment \( m_p \) per electron associated with \( L_p \) is

\[
m_p = -\frac{e}{2m} L_p
\]

• so the total magnetisation \( M \) of the sample must be,

\[
M = N_{\text{atoms}} \times Z_{\text{electrons/atom}} \times m_p
\]

\[
M = N \times Z \times m_p
\]
So substitute for each quantity, in turn,

\[ M = - N Z \frac{e}{2m} L p \]

\[ M = - N Z \frac{e}{2m} m \omega_L \left< r^2 \right> \]

\[ M = - N Z \frac{e}{2m} m \left( \frac{eB}{2m} \right) \left< r^2 \right> \]

so that,

\[ \chi = \frac{M}{H} = - \mu_0 N Z \frac{e^2}{4m} \left< r^2 \right> \]
Now express $<r^2>$ in terms of the mean square radius of the orbit $<\rho^2>$, where

$$\langle r^2 \rangle = \frac{2}{3} \langle \rho^2 \rangle$$

will give

$$\chi = -\mu_0 N Z \frac{e^2}{6m} \langle \rho^2 \rangle$$

The result we wanted!!
What does this mean???

• The atom as a whole does not possess a permanent magnetic dipole moment.

• The magnetisation is produced by the influence of the external magnetic field on the electron orbits and particularly by the precessional motion.

• The diamagnetic susceptibility $\chi$ is small and negative, because $<\rho^2>$ is small.

• Negative susceptibility means that diamagnetism opposes applied magnetic field. (prefix: dia - in opposite or different directions)

• Note that measurements of $\chi$ were once used to get estimates of atomic size through the values of $<\rho^2>$.

Remember - all materials are diamagnetic – but this small effect is swamped if also paramagnetic of ferromagnetic.
**Paramagnetism**

- Paramagnets have a small positive magnetisation $M$ (directed parallel the applied field $B$).
- Each atom has a **permanent magnetic dipole moment**.

**Langevin (classical) theory**

- The paramagnet consists of an array of permanent magnetic dipoles $m$.
- In a uniform field $B$ they have
  
  Potential Energy $= - m \cdot B$
A dipole *parallel* to the field has the lowest energy

- **BUT**, the *B* field causes *precession* of *m* about *B*.
- **However** it can’t alter the *angle* between *m* and *B* (as the *L_z* component is *constant* in the precession equations).

- **For the dipole to lower its energy (and become parallel to the field)** we need a *second mechanism*. This is provided by the thermal vibrations
  - The magnetic field “would like” the dipoles aligned to lower their energy
  - The thermal vibrations “would like” to randomise and disorder the magnetic dipoles
• We can therefore expect a statistical theory, based on a “competition” between these two mechanisms,

\[ k_B T \approx -m.B \]

...to calculate the magnetisation \( M \) of a paramagnet
Calculate the magnetisation $M$ of a paramagnet

- $M$ must be the vector sum of all the magnetic dipole moments

$$M = \sum [1\text{ Resolved component of a dipole in field direction} \times [2\text{ Number of dipoles with this orientation}$$

To do this, use Boltzmann statistics to obtain the number $dn$ of dipoles with energy between $E$ and $E + dE$

$$dn = c \exp \left(-\frac{E}{kT}\right) dE$$

where $k = $ Boltzmann’s constant, $c = $ is a constant of the system and $T = T$ measured in Kelvin.
How to find $c$?

Integrate over all the energies, which must give $N$ the total number of dipoles:

$$N = \int_0^\infty c \exp\left(-\frac{E}{kT}\right) dE$$

Next week we will do this to calculate the susceptibility of a paramagnet. The important result we will get will be

That susceptibility is temperature dependent with

$$\chi = C/T$$

This is Curie’s law.
Summary

• Calculated the susceptibility for a diamagnet

\[ \chi = -\mu_0N Z \frac{e^2}{6m} \langle \rho^2 \rangle \]

• Argued that all materials have a small diamagnetic susceptibility resulting from the precession of electrons in a magnetic field.
• This precession results in a magnetic moment, and thus a susceptibility.
• Then discussed paramagnetism, resulting from atoms having a permanent magnetic moment.
• Sketched out how we will calculate paramagnetic susceptibility.