PHY331
Magnetism
Lecture 4
Last week…

• Discussed Langevin’s theory of diamagnetism.

• Use angular momentum of precessing electron in magnetic field to derive the magnetization of a sample and thus diamagnetic susceptibility.

• Set the scene for the calculation of paramagnetic susceptibility.
This week…

• Will calculate the paramagnetic susceptibility using an entirely classical approach.

• We will obtain Curie’s Law for a paramagnet.

\[ \chi = \frac{M}{H} = \frac{\mu_0 N m^2}{3kT} = \frac{C}{T} \]

• We will then start to put quantum framework together to obtain a quantum theory of paramagnetism.
The calculation of the magnetisation $M$

Recall the energy of a dipole in a field $E = -m \cdot B$

\[ E = -mB \cos \theta \]  \hspace{1cm} (3)

\[ dE = +mB \sin \theta \, d\theta \]  \hspace{1cm} (4)

The number of dipoles with energy between $E$ and $E + dE$ will be,

\[ dn = c \, \exp(-E/kT) \, dE \]

If we substitute (3) and (4), this is also the number of dipoles with orientation between $\theta$ and $\theta + d\theta$
\[ M = \sum \text{[1] Resolved component of a dipole in field direction} \times \text{[2] Number of dipoles with this orientation} \]

which is (2) in the equation above.

The resolved component of the dipoles with this orientation which is (1) in the equation above, is clearly just,

\[ m \cos \theta \] (1)

therefore the total magnetisation is,

\[ M = \int_{0}^{\infty} m \cos \theta \, dn \] (5)

for convenience, we can write,

\[ M = N \langle m \rangle \] (6)

Where \( N = \) the total number of dipoles
and \( \langle m \rangle = \) their average component in the field direction
hence using (5) and (6)

\[
\frac{\langle m \rangle}{m} = \frac{\int_0^N \cos \theta \, dn}{N} = \frac{\int_0^N \cos \theta \, dn}{\int_0^N dn}
\]

so substituting we get,

\[
\frac{\langle m \rangle}{m} = \frac{\int_0^{\pi} \cos \theta \, c mB \sin \theta \, \exp(mB \cos \theta / kT) \, d\theta}{\int_0^{\pi} c mB \sin \theta \, \exp(mB \cos \theta / kT) \, d\theta}
\]

**note:** the term inside the exponential is positive, because it has one negative sign from the -E and another from the -mBcosθ

also, this equation is one of the general type \[
\langle x \rangle = \int x \, dm / \int dm
\]
to simplify and solve the above, substitute,

\[ x = \frac{mB \cos \theta}{kT} = a \cos \theta \]

which gives,

\[ \frac{\langle m \rangle}{m} = \frac{1}{a} \int_{-a}^{a} x e^x \, dx \]

whose solution is

\[ \frac{\langle m \rangle}{m} = \frac{e^a + e^{-a}}{e^a - e^{-a}} - \frac{1}{a} \]

Langevin function \( L(a) \)

\[ \frac{\langle m \rangle}{m} = L(a) \]

\[ \langle m \rangle = mL(a) \]
Langevin Function

Plot as a function of

\[ a = \frac{mB}{kT} \]
Now \[ M = N\langle m \rangle = NmL(a) \]

where \[ a = \frac{mB}{kT} \]

Can see that Magnetization increases as the applied field \((B)\) increases or, it decreases as the temperature \(T\) increases. This confirms that there is “competition” between these two factors,

\[ k_B T \approx -m \cdot B \]
Calculating magnetic susceptibility

We have,

\[ M = N\langle m \rangle \]

or

\[ M = NmL(a) \]

When \( a \) is small the Langevin function \( L(a) \) can be expanded as a power series.

When \( a \) is small \( mB << kT \)

Thus this approximation works in the **limit of small applied fields and high temperatures**.

\[ a = \frac{mB}{kT} \]
\[ L(a) = \frac{a}{3} + \frac{a^2}{45} + \frac{2a^5}{945} + \ldots \approx \frac{a}{3} \]

so that,

\[ M \approx Nm \frac{a}{3} \]

or

\[ M \approx \frac{Nm^2 B}{3kT} \approx \frac{Nm^2 \mu_0 H}{3kT} \]

and the paramagnetic susceptibility is then,

\[ \chi = \frac{M}{H} = \frac{\mu_0 N m^2}{3kT} = \frac{C}{T} \]

• This is Curie’s Law and \( C \) is Curie’s constant
Quantum theory of paramagnetism

What changes do we need to make to Langevin’s classical theory?

i) detailed changes to the way in which the magnetic dipole moment is defined,

The general definition is $m = m(L)$

Using QM, the angular momentum $L$ is quantised, and *every electron* is now specified by *four* quantum numbers

$(n, l, m_l, s)$
\( n, \text{ principal quantum number} \) (energy of the orbit)
\( n = 1, 2, 3, \)

\( l, \text{ angular momentum quantum number} \)
(elliptical orbits of different eccentricity)
\( l = 0, 1, \ldots \ldots (n-1) \)

\( m_l, \text{ magnetic quantum number} \) (describes the orientation of \( l \) in \( B \) and gives the magnitudes of the aligned components of \( l \))
\( -l < m_l < +l \)

\( s, \text{ spin quantum number} \) (electron spin)
\( s = \pm \frac{1}{2} \)
Quantization of orbital angular momentum. The figure is drawn for the case of \( l = 2 \).
Quantization....

Now, the *angular momentum* is quantised in terms of the *quantum mechanical eigenvalue*

\[ \hbar \sqrt{l(l+1)} \]

The *spin motion* is also quantised in terms of its *quantum mechanical eigenvalue*

\[ \hbar \sqrt{s(s+1)} \]

*Will find that both* \( l \) *and* \( s \) *contribute to magnetic dipole moment*...
Next week

• We will see how we can add angular momentum and spin together to define the total angular momentum of an atom (J).
• This will then be used to obtain the magnetic dipole moment of an atom and thus define paramagnetic susceptibility.
• Result will be similar to classical derivation!
Summary

• We calculated paramagnetic susceptibility using an entirely classical approach and obtained Curie’s Law for a paramagnet.

• We saw that quantization of angular momentum and spin will be important components of a quantum theory of paramagnetism.