Modelling and statistical inference

Let's look at our data again:

A single voxel plotted in time

slices

a brain volume

Time

0 20 40 60 80 100 120 140 160 180

103 104 105 106 107 108 109

0 20 40 60 80 100 120 140 160 180

103 104 105 106 107 108 109
How do we model the data?

Time series of a single voxel

\[ \text{Time series of a single voxel} = A \text{ model (stimulus)} + \text{Constant (mean of the data)} + \text{Measurement noise} \]

A model (stimulus)

Over whole brain

A volume of parameters
If we have two effects of interest and a baseline condition

<table>
<thead>
<tr>
<th>Base</th>
<th>Eff1</th>
<th>Eff2</th>
<th>Base</th>
<th>Eff1</th>
<th>Eff2</th>
</tr>
</thead>
</table>

What would we expect the data to look like?

A voxel not activated at all:

A voxel activated by eff1 only:

A voxel activated by eff2 only:

A voxel activated by eff1 and eff2 differently:
How could we model the two effects?

\[
\text{Data} = a_1 \cdot \text{Effects of interest} + a_2 \cdot \text{Effect of no interest} + \text{Constant} + \text{measurement noise}
\]
What would we expect the sizes of the parameters ‘a1’ and ‘a2’?

A voxel not activated at all:

A voxel activated by eff1 only:

A voxel activated by eff2 only:

A voxel activated by eff1 and eff2 differently:
  eff1 > eff2:
  eff1 < eff2:
How to describe the problem mathematically?

The single effect problem:

\[ y = x_1 \times a + x_2 \times b + \varepsilon \]
Assume we have 60 data points (N=60): 

\[ y = x_1 \cdot a + x_2 \cdot b + \varepsilon \]

\[(60\times1) \quad (60\times1) \cdot \text{scalar} \quad (60\times1) \cdot \text{scalar} \quad (60\times1)\]

\[ y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} + \varepsilon \]

\[(60 \times 2) \quad (2 \times 1)\]

\[ y = X \cdot \beta + \varepsilon \]

\[ \text{Data} \quad \text{Design matrix (underlying model)} \quad (60 \times 2) \quad (2 \times 1) \quad \text{Parameter vector to be estimated} \]
If we have two effects of interest, then

\[ y = x_1 \ast a_1 + x_2 \ast a_2 + x_3 \ast b + \varepsilon \]

Effect 1

Effect 2

Effect of mean

noise

\[ y = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \ast \begin{bmatrix} a_1 \\ a_2 \\ b \end{bmatrix} + \varepsilon \]

Data

Design matrix (underlying model)

(60 x 2)

Parameter vector to be estimated

(2 x 1)
If there are several effects to be investigated:

- simply expand the design matrix and the corresponding parameter vector.

\[
\begin{align*}
    y &= \begin{bmatrix} x_1 & x_2 & \ldots & x_k \end{bmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \end{pmatrix} + \varepsilon \\
    &= X \beta + \varepsilon
\end{align*}
\]