The University Of Sheffield.

Department of Physics and Astronomy
2nd Year Laboratory

G1 Kater's Pendulum

Scientific aims and objectives

- To determine an accurate value for g in the lab here in Sheffield and compare it with an empirically calculated value.
- To compare the accuracy of g measured using three different pendulums
  - Simple pendulum
  - Compound pendulum
  - Kater's pendulum
- To evaluate the validity of the small angle approximation made when deriving the equation of motion of all three pendulums.

Learning Outcomes

- To be able to read a vernier scale
- To fit complex equations by manipulating data to the form $y = mx + c$
- To be able to use χ^2 to fit an errors weighted straight line
- To apply compound errors formulae to complex equations
- To know the historical motivation behind the systematic improvements in experimental methods used to measure g.

Apparatus

- Simple pendulum &
  - Stopwatch
  - Ruler
- Compound pendulum &
  - Stopwatch
  - Ruler
- Kater's pendulum &
  - Electronic timing gate
  - Travelling microscope

Safety instructions

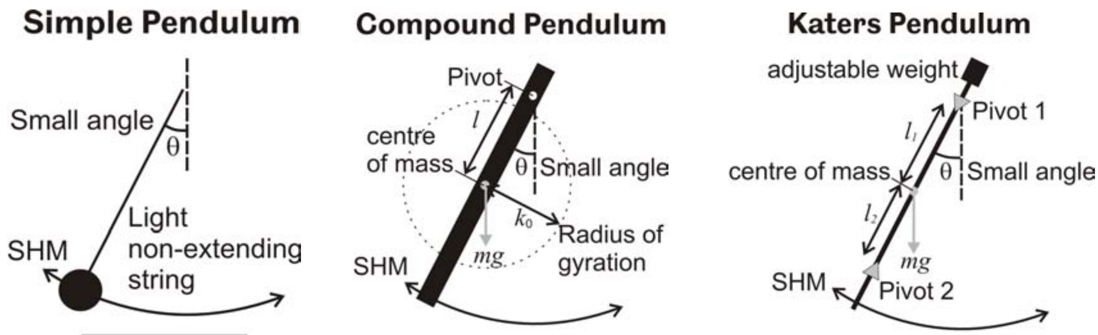
- Always replace the guard on the knife edge suspension point
- Take care when moving Kater's pendulum around.
- Do not climb on the laboratory stools. Always demount the pendulum before adjusting the weight.

Task 1 - Pre-session questions

You are required to complete the online questions found at the back of the script before starting your laboratory work. These questions will prepare you for the experiment by teaching you how to read a Vernier scale; manipulate complex equations to yield a simple straight line fit; and to use compound errors formula. The online questions will give you a context for the experiment by asking you to briefly research the history of the measurement of g using the internet.

Task 2 – The experiment

Three pendulums are provided.



$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T = 2\pi\sqrt{\frac{l^2 + k_o^2}{gl}}$$

$$\frac{8\pi^2}{g} = \frac{T_1^2 + T_2^2}{l_1 + l_2} + \frac{T_1^2 - T_2^2}{l_1 - l_2}$$

The objective is to compare the value and accuracy of g using each pendulum in turn. Kater's pendulum should yield a value precise to three significant figures. The other two pendulums will yield far less precise values and it is important to determine the errors for each pendulum to allow a discussion of the precision of the values of g.

In all cases the small angle approximation has been used to derive the equation of motion. The final part of the experiment is to measure the validity of this approximation by varying the amplitude of oscillation of Katers pendulum and recording the period. These results can be checked against $T(\alpha) = T_0\left(1 + \frac{1}{4}\sin^2\frac{\alpha}{2}\right)$ (where α is the angle of oscillation), as derived from the rigorous equation of motion.

Task 3 – Reporting

Using the Simple and Compound pendulums you should be able to report g to two significant figures and also calculate a value for the radius of gyration of the compound pendulum. Using Kater's pendulum you should be able to report g to at least three significant figures and compare it directly with an empirically calculated value for g in Sheffield. You should discuss the methods that you used to analyse the data recorded from each pendulum, and discuss how you derived the values for errors in each case. You will be likely to need compound errors formula and you should show how you apply them.

Appendix of supporting information

Contents

1. Empirical value for g

The acceleration due to gravity at the surface of the earth can be approximated to the empirical expression (A1) and depends on latitude and height above sea level. (A1) is taken from Kaye and Laby online where the accuracy of the expression is also discussed. http://www.kayelaby.npl.co.uk/general_physics/2_7/2_7_5.html It will be possible to compare your experimental values with that calculated using (A1). The error in your experimental values and in the calculated value should be quoted as part of the comparison.

$$g(\phi,h) = g_e\left(1+\beta_1 \sin^2\phi - \beta_2 \sin^2 2\phi\right) - 3.086\times10^{-6}h \text{ ms}^{-2} \quad \text{(A1)}$$

g_e = 9.7803184 ms^{-2}, β_1 = 0.0053024, β_2 = 0.0000059,

ϕ is the geographical latitude and h is the height in metres above sea-level. g_e is the value of g in ms^{-2} at sea-level at the equator. Note that the first term of this correction multiplies the value of g_e and the second is a subtractive correction. Your experimental value and this calculated value should of course, both be close to the accepted value g = 9.81 ms^{-2}. The latitude of Sheffield is ϕ = $53^0 23'$ North and the height of the Second Year Lab above sea level is 125 m.

2. The simple pendulum

The simple pendulum consists of a weight attached to the end of a light inextensible string of length ℓ. The periodic time T is $T = 2\pi\sqrt{\ell / g}$ for small oscillations, $\theta \approx \sin\theta$ (in radians) and can be derived by considering a restoring force proportional to the displacement of the weight from the equilibrium position. The solution to the equation of motion is only Simple Harmonic when the angular displacement is small and the string is very light and kept tight. In your determination of g using the simple pendulum you should consider the validity of these approximations.

You should make a single measurement of g using the bob pendulum, stopwatch and ruler provided. You should consider the errors in measuring T and l and decide using these error estimates whether the assumptions made when deriving the simple pendulum formula are valid.

3. The compound pendulum

A compound pendulum is a rigid body swinging in a vertical plane about any horizontal axis passing through the body. The resultant force acts through the centre of mass. The periodic time of the compound pendulum is related to the moment of inertia I about the point of suspension.

$$T = 2\pi\sqrt{\frac{I}{mgl}} \tag{A2}$$

I can be expressed in terms of the moment of inertia about the centre of mass,

$$I = mk_0^{\;2} + ml^2 \tag{A3}$$

giving,

$$T = 2\pi\left[\frac{\left(l^2 + k_0^{\;2}\right)}{gl}\right]^{1/2} \tag{A4}$$

where l is the distance between the suspension point and the centre of gravity of the pendulum and k_0 is the radius of gyration about a parallel axis through the centre of gravity. If different values of l are taken, and the corresponding values of T measured, both g and k_0 can be determined.

The radius of gyration k_0 is defined as the radius of a thin uniform hoop rotating about an axis through its centre and perpendicular to its plane, which has the same moment of inertia as the original object.

One way to determine k_0 and g is to make a series of measurements of T with the pivot point at different distances l from the centre of mass. Equation (A4) can then be manipulated into a straight line form and the intercept and gradient together will yield the unknown parameters.

You should determine the experimental error in T and l; ideally by repeat measurements, or alternatively by estimation, and using the appropriate compound errors formulae, determine the error in your X and Y variables for your straight line plot. Using the excel program "χ^2 " you can then determine the error of g and k_0.

In addition you should compare your experimentally determined value for k_0 with a theoretical value found from the definition of the radius of gyration and the integrated value for the moment of inertia of a metal bar $I = \frac{ml^2}{12}$.

4. Kater's pendulum

An improvement in the precision of the measurement of g was developed in 1817 by Kater. He realised that by using a compound pendulum and suspending it from each end in turn the requirement to measure the distance from the centre of mass to the pivot could be removed. He made a very accurate measurement of g in London, a value that was used to define the metre for many years. The version of Kater's reversible pendulum used in this experiment has a knife-edge for suspension at either end: thus there are two distances, l_1 and l_2, and two periods T_1 and T_2. Using equation (A2) it is possible to derive the following expression for g:

$$\frac{8\pi^2}{g} = \frac{T_1^2 + T_2^2}{(l_1 + l_2)} + \frac{T_1^2 - T_2^2}{(l_1 - l_2)} \tag{A5}$$

The distance between the knife-edges, the quantity $L = l_1 + l_2$ can be measured directly, and accurately using the travelling microscope. On the other hand, it is much more difficult to measure the quantity l_1 - l_2 directly, however if the periods T_1 and T_2 are made to be the same then the term $T_1^2 - T_2^2$ goes to zero and the value of l_1 - l_2 is not important.

The objective when using Kater's pendulum is to equalise the period's measured from the two pivots by adjusting the position of the weight on its thread. You can do this roughly by measuring the difference in the periods when the weight is positioned at its extreme limits and then estimating the required position using distance or the number of turns on the thread as a measure. However this methods will only yield an approximate position of the weight for equality between T_1 and T_2 . For a precise measurement of g you should aim to take a series of measurements where T_1 and T_2 vary systematically. Equation (A5) can then be manipulated and approximated to a straight line where $X = T_1^2 - T_2^2$ and $Y = T_1^2 + T_2^2$ assuming that $l_1 - l_2$ is constant. The intercept at $X = 0$ will yield g. Consider carefully the best conditions for this method to work and whether the approximation that $l_1 - l_2$ is constant is valid. Alternative, equally accurate methods are also possible.

You should determine the experimental error in T and L; ideally by repeat measurements, or alternatively by estimation, and using the appropriate compound errors formulae, determine the error in your X and Y variables of your straight line plot. Using the excel program "χ^2 " you can then determine the error of g.

5. Compound errors

f	*Absolute error*	*Proportional error*
x^2	$\Delta f = 2x\Delta x$	$\frac{\Delta f}{f} = \frac{2\Delta x}{x}$
x^n	$\Delta f = nx^{n-1}\Delta x$	$\frac{\Delta f}{f} = \frac{n\Delta x}{x}$
$x + y$	$(\Delta f)^2 = (\Delta x)^2 + (\Delta y)^2$	$\frac{(\Delta f)^2}{f^2} = \frac{(\Delta x)^2 + (\Delta y)^2}{(x+y)^2}$
xy	$(\Delta f)^2 = y^2(\Delta x)^2 + x^2(\Delta y)^2$	$\frac{(\Delta f)^2}{f^2} = \left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2$
x / y	$(\Delta f)^2 = \frac{(\Delta x)^2}{y^2} + \frac{x^2}{y^4}(\Delta y)^2$	$\frac{(\Delta f)^2}{f^2} = \left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2$

6. The small angle approximation

Equations (A2), (A5) and the equation for the period of a simple pendulum, $T = 2\pi\sqrt{\ell / g}$ are all derived assuming that the amplitude of displacement is very small and that $\theta \approx \sin\theta$. Such an approximation has the possibility of introducing both random and systematic errors into the measurement of g and it is therefore necessary to investigate the effect of finite amplitude on the period of oscillation to determine whether the small angle approximation is valid for your measurements.

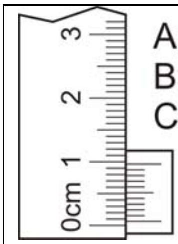
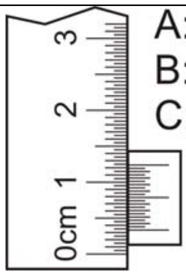
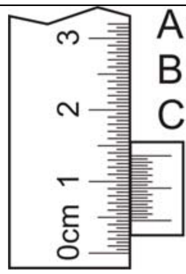
Solving the ***rigorous*** energy equation of motion for a compound pendulum yield a further approximate expression for the variation of period $T(\alpha)$ as a function of angular amplitude α.

$$T(\alpha) = T_0\left(1 + \frac{1}{4}\sin^2\frac{\alpha}{2}\right) \qquad \text{(A6)}$$

You should aim to firstly verify equation (A6), ideally by making a straight line plot of $f(T)$ against $f(\alpha)$ and then utilise your experimental findings of $T(\alpha)$ to conclude whether the small angle approximation is valid in your determinations of g, and if not, how the amplitude correction might influence your determined values.

Pre-lab multiple choice questions

Q1 Read the following vernier scales

1.1	1.2	1.3
		
A: 0.06 cm B: 1.2 mm C: 0.1 cm	A: 3.75 mm B: 3.35 mm C: 3.02 mm	A: 4.75 mm B: 4.55 mm C: 4.02 mm

Q2 Manipulate the following equations to yield $Y = mX + C$ formula that could be fitted as straight-line graphs with $Y = f_1(x, y)$ and $X = f_2(x, y)$ to determine a and b.

		A	B	C
2.1	$\frac{y^2}{x^2 + a} = b$	$Y = y^2$ $X = x^2$	$Y = y^2$ $X = x^{-2}$	$Y = y^2$ $X = ax^{-2}$
2.2	$a^2 = \frac{x^2 + y^2}{a} + \frac{x^2 - y^2}{b}$	$Y = x^2 + y^2$ $Y = x^2 - y^2$	$Y = \frac{x^2 + y^2}{x^2 - y^2}$ $X = x_1^2 + x_2^2$	$Y = x^2 + a^2$ $Y = x^2 - a^2$
2.3	$y = a + b\sin^2\left(\frac{x}{2} + 1\right)$	$X = \sin^2\left(\frac{x}{2} + 1\right)$ $Y = y$	$X = \left(\frac{x}{2} + 1\right)$ $Y = \sqrt{y}$	$X = \left(\frac{x}{2} + 1\right)$ $Y = y$

Q3 Derive expressions for the proportional error $\Delta f / f$ in the following equations.

		A	B	C
3.1	$f(x, y) = x^2 y$	$\sqrt{4\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$	$\sqrt{2\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$	$\sqrt{4\left(\frac{\Delta x}{x}\right) + \left(\frac{\Delta y}{y}\right)}$
3.2	$f(x, y) = \frac{x^2}{y}$	$\sqrt{4\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$	$\sqrt{2\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$	$\sqrt{4\left(\frac{\Delta x}{x}\right) + \left(\frac{\Delta y}{y}\right)}$
3.3	$f(x, y) = x + 1/x$	$\sqrt{\frac{x^4(\Delta x)^2 + (\Delta x)^2}{x^2(x^2 + 1)^2}}$	$\sqrt{\frac{2(\Delta x)^2}{x^2}}$	$\sqrt{\frac{(\Delta x)^2 + (\Delta x)^2}{x^2}}$

Q4 Which of these statements are correct in the context of the historical measurement of g.

4.1	A tool for measuring the acceleration due to gravity is called a gravimeter
4.2	An accurate determination of g can be used to navigate nuclear submarines
4.3	In 1774 an accurate measurement of G was made using a pendulum and a isolated mountain in Scotland
4.4	Reversible pendulums like Kater's were the standard way of measuring the acceleration due to gravity until 1950
4.5	In Kater's original measurement in London he achieved a value of g = (9.81158 +/- 0.00001) m/s^2