G5 Speed of Sound

Scientific aims and objectives

• To determine the speed of sound in air and three unknown pure gases using the time of flight method. Compare the experimental results to the known values.
• Interpret the results thermodynamically by relating the measured values of the speed of sound to the heat capacity of the gases.

Learning Outcomes

• Being able to calculate the theoretical degrees of freedom for different molecules and be able to estimate their room temperature contribution to heat capacity.
• Being able to use trigger and cursor functions of an oscilloscope to gain a measurement of the time interval between two independent pulse signals.

Apparatus

✓ Gas tubes
✓ Oscilloscope
✓ Transmitter and receiver
✓ Gas supplies, control valves and flow meter
✓ Pulse generator

Safety instructions

Cylinders of three inert and non-toxic gases are provided for this experiment. They do not pose any significant hazard to health in the quantities used in this experiment but, nonetheless, they are not a life-supporting atmosphere. The pressure inside the cylinders is very high and the reduction valves have been set correctly by the Laboratory Technician. You must not adjust them in any way and if there is a problem with the gas supply, ask the technician or demonstrators for assistance.
**Task 1 - Pre-session questions**
The pre-session questions, found at the back of the script, will prepare you for this lab by making you consider both experimental and theoretical aspects of the tasks. Complete them before progressing to make any experimental measurements but ensure that the gas tap to the tube in front of you is turned on to allow the system to equilibrate.

**Task 2 – The experiment**
The speed of sound can be determined in four different and unknown gases by measuring the time taken for a pulse of sound to travel a predetermined distance inside a tube of such gas. By plotting time vs distance an accurate speed can be experimentally determined and associated precision reported by careful estimation of errors associated with $d$ and $t$ and the use of $\chi^2$ spreadsheet.

The experimentally determined values should then be compared with both literature values, as provided in this script and theoretical values calculated using thermodynamics. Gases should be identified with a level of certainty. Deviations from the theoretical, literature and experimental values should be discussed in relation to experimental error, deviation from approximations made in the theory. Make sure that your theoretical, literature and experimental values are all associated with the same temperature – you can apply a simple correction using the velocity – temperature relationship to achieve this assuming that the $\gamma$ factor remains constant.

The number of degrees of freedom of the different possible gas molecules should be calculated assuming an equi-partition of energy between translational, vibrational and rotational energy modes, however some of these modes will inaccessible at room temperature and this should be accounted for.

<table>
<thead>
<tr>
<th>Gas</th>
<th>$\gamma$ from reference (m/s) at 273 K</th>
<th>$\gamma$ from experiment</th>
<th>$\gamma$ from thermodynamics theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2</td>
<td>380</td>
<td></td>
<td></td>
</tr>
<tr>
<td>He</td>
<td>387</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CO$_2$</td>
<td>369</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CO</td>
<td>388</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H$_2$</td>
<td>977</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CH$_4$</td>
<td>430</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ne</td>
<td>435</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N$_2$</td>
<td>341</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O$_2$</td>
<td>315</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Task 3 – Reporting**
You should report your method for measuring the speed of sound paying particular attention to error estimates and the precision of final values that you determine. You should explain how you made theoretical calculations of the speed in the possible gases by estimating the degrees of freedom at lab temperature.
1. **Historical background**

The nature of sound has puzzled mankind since ancient times. Aristotle was the first one to propose that transmission of sound involved the motion of air. However, since this was difficult to prove, Aristotle’s notion was disregarded until Robert Boyle (1660) concluded that sound is transmitted through a medium.

As for its speed, Pierre Gassendi (1635) estimated the speed of sound in air to be around 480 m/s and also observed that the speed was independent of the frequency of sound (Aristotle believed that high notes travelled faster than lower ones). 50 years later, Martin Mersenne, considered to be the “father of acoustics”, corrected the value to 450 m/s. Other scientists of the age suggested that it was approximately 350 m/s. These discrepancies can only be accounted for by the varying conditions in which the experiments were performed.

The first officially precise measurement was made in 1738 by the Academy of Sciences in Paris, using the time of flight method. They fired cannons situated at two ends of a base line 29 km long and measured the interval between seeing the flash from the explosion and hearing the noise. A value of 332 m/s at 0 °C in dry air was obtained. This was a remarkable feat for that time since the most accurate current value is 331.45 m/s.

Laplace was the first to study how the temperature affected the speed of sound. Before his discovery, it was assumed that the elastic motions of air particles take place at constant temperature (isothermal process). Since the motion of molecules is too fast, he reasoned that there are small changes of temperature that lead to an increase in the elasticity of the medium, in an adiabatic way (i.e. no exchange of heat with the environment).

2. **Theoretical Background**

Sound waves require an elastic media for their propagation, for a train of sound waves involves a sequence of elastic displacements of the elements in this medium. Such waves disturb the medium, causing compressions and rarefactions, like the ones shown in Figure 1.

The sound wave produces a vibration in the molecules of the medium and since these do not vibrate in phase, some pressure and density differences arise in the medium.
Since these compressions and rarefactions occur rapidly there is no time for a heat exchange between the molecules in the media and their surroundings, and hence the process is adiabatic.

Figure 1. (a) Elastic medium at rest. (b) Elastic medium disturbed by a sound wave.

The speed of sound, $v$, in a gas is given by the rate of the change of pressure ($P$) by the change of density ($\rho$):

$$v = \sqrt{\frac{dP}{d\rho}}$$

(1)

To compute this ratio, we make use of the equations for an adiabatic compression/expansion of an ideal gas:

$$PV^\gamma = \text{Constant}$$

(2)

Where $\gamma = C_p / C_v$, the ratio between the specific heats of the gas at constant pressure and constant volume.

Since the density is equivalent to the molar mass of the gas ($M$) divided by the partial molar volume $\rho = M / V$ so that $V = M / P$. Thus we obtain:

$$P \rho^\gamma = \text{Constant}$$

(3)

so that,

$$\ln P - \gamma \ln \rho = \text{Constant}$$

(4)

Differentiating with respect to $\rho$, we get,

$$\frac{1}{P} \frac{dP}{d\rho} = -\frac{\gamma}{\rho} \Rightarrow \frac{dP}{d\rho} = \gamma \frac{P}{\rho} = \gamma \frac{PV}{M} = \frac{\gamma RT}{M}$$

(5)
and so, the speed of sound is given by,

$$v = \sqrt{\frac{\gamma RT}{M}}$$  \hspace{1cm} (6)

where \( R \) is the gas constant and \( T \) is the absolute temperature. Hence the velocity is independent of the pressure and proportional to the square root of the temperature.

As it can be seen from equation 6, the value of the speed of sound can be used to determine the value of \( \gamma \), the ratio between the heat capacities. By definition \( C_v \) or \( C_p \) denote the amount of heat energy, which must be absorbed by one mole of a gas at constant volume or pressure to raise the temperature of the gas by one degree. The amount of heat is reflected by an increase in translational, rotational and vibrational energies.

The molecules have quantised modes of translation, vibration and rotation; the number of modes depends on the number of atoms and the symmetry of their bonds. These movements can be described in terms of the degrees of freedom \( f \), which are the independent coordinates required to locate the atoms in space. Therefore, the results of the calculated heat capacities can be discussed in the light of the symmetry and nature of the molecule. The following equations relate the specific heat to the degrees of freedom.

$$C_p - C_v = R$$ \hspace{1cm} (7)

$$\gamma = 1 + \frac{2}{f}$$ \hspace{1cm} (8)

For a monoatomic molecule \( f = 3 \), corresponding to the translational motion in 3 dimensions. A diatomic molecule has these 3 translational plus 2 vibrational (one PE and one KE) and 2 rotational degrees of freedom. A triatomic molecule has 3 translational, 2 rotational and 4 vibrational degrees of freedom if it is linear but 3 of each one if it is non linear. The vibrational modes only come into play at temperatures above 1000 K.

3. Experimental method

There are four sets of equipment, one for measurements with air and the others with pure gases; the tube used for air has no external filling arrangement, whereas the other three are connected to the gas cylinders. Normally the supply of gas will be turned on by the technician beforehand and the only the valves in each table will be used to fill the cylinders with the gases.

Measurements are to be made in all four gases. If there are several teams attempting the experiment, agree a rota for the use of the four sets of experiments in any appropriate order.
Each one of the cylinders contains one moveable receiver or microphone and a high audio frequency transmitter at the end of the tube. Moving the rod modifies the distance between them and the time of flight is measured with the oscilloscope.

**Guidelines**

i. Since the speed of sound is dependent on the absolute **temperature**, make sure you measure it. Since there are no thermometers inside the tubes, you can use the laboratory temperature as a reference.

ii. **Fill the tubes** with the gas by turning on the gas flow meter in each gas line. Introduce the gas at a speed of 5 L / m for 10 minutes to flush the air out of the cylinder. Reduce the flow to 0.5 L / m when you are ready to take the measurements. A moderate flow needs to be maintained to reduce the tendency of the air to diffuse back into the tube. One of the tubes has been fitted with an extra gas level monitor to ensure that the cylinder is properly filled.

iii. **Set the parameters of the equipment.**

   - **Pulse generator:** Set the TGP110 to produce a $100 \, \mu s$ (width) pulse every $Y \, \mu s$ (period), 1 of V amplitude (or a suitable maximum amplitude such that there is no visible distortion of the signal) and no delay. (You are asked to estimate $Y$ in pre-lab question 1)

   - **Oscilloscope:** Set the Digimess DS25 with the following parameters:
     - **Transmitter channel** (CH1 red button): Push the buttons on the side of the screen to set the coupling to DC, the probe to 1X. Turn on the channel and off the inversion.
     - **Receiver channel** (CH2 yellow button): Set coupling to AC and set the other parameters to be the same as for CH1.
     - **Trigger menu** (Trig button): Set the type to edge, the slope to rising, the source to CH1, the mode to normal and the coupling to AC.
     - **Sec/Div knob:** Adjust to have both channels on the screen.
     - **Volts/Div knob:** Adjust to get a suitable height.
     - **Position control:** Position the transmitter pulse near the left hand side of the screen

All of the previous adjustments should produce a couple of traces in the oscilloscope, like the ones shown in figure 2. You can modify the values if necessary.
Figure 2. Typical oscilloscope trace showing transmitter and receiver pulses.

-Cursor menu: Set the parameter type to time and the source to CH1. Set cursor 1 on the leading edge of the transmitter pulse (as shown in picture 2) and cursor 2 to either to the leading edge of the receiver pulse or to the first discernible maxima. Make sure you select a point that is not obscured by the noise. The time between them (delta on the screen) is approximately the time of flight.

Now you are ready to start the experiment. Move the receiver away from the transmitter and observe that the delay between both pulses increases. Use the cursors to record the time of flight at positions from about 100-950 mm; you might find it convenient to use the white tipped supports as an indicator of the position.

2. Analysis of the results

✓ Estimate the speed of sound: With the time of flight given by the oscilloscope and the position of the moveable receiver, estimate the speed of sound.

✓ Identify the gas: Use Table 1 to identify the gases. Correct your measured speeds to 273 K using equation 6.

<table>
<thead>
<tr>
<th>Gas</th>
<th>( v ) (m / s) at 273 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry air</td>
<td>331</td>
</tr>
<tr>
<td>Ar</td>
<td>307</td>
</tr>
<tr>
<td>CO(_2)</td>
<td>259</td>
</tr>
<tr>
<td>CO</td>
<td>338</td>
</tr>
<tr>
<td>H(_2)</td>
<td>1286</td>
</tr>
<tr>
<td>He</td>
<td>972</td>
</tr>
<tr>
<td>CH(_4)</td>
<td>430</td>
</tr>
<tr>
<td>Ne</td>
<td>435</td>
</tr>
<tr>
<td>N(_2)</td>
<td>334</td>
</tr>
<tr>
<td>O(_2)</td>
<td>315</td>
</tr>
</tbody>
</table>

Table 1. The speed of sound for some common gases
✓ **Estimate the ratio of heat capacities:** Compare the values of $\gamma$ calculated by equation 8 with the ones derived from equation 6 and experimental values. Justify the selection of the number of degrees of freedom used in equation 8. It could be useful to make a stick drawing of the molecules and identify the normal modes of translation, vibration and rotation of the molecules according to their symmetry.

✓ **Sources of error:** Identify the sources of systematic and random errors in order to explain the differences between the reported and measured values. Give the confidence limits of your values of the speed of sound and heat capacity ratios in all four gases. Also think about the effect of temperature and humidity of the room and the frequency of sound. (Hint: Does the presence of humidity in the environment modify the molar mass or degrees of freedom of the gases?)
4. **Pre-lab questions**

1. In this experiment the speed of sound is determined by measuring the time taken for discrete pulses of sound to travel down a tube of gas. Estimate how often a pulse should be sent down the tube so as to maximise the number of pulses per second that are sent but such that pulses are not interfered with by the previous pulse?  

   (a) Every 5 µs  
   (b) Every 50 µs  
   (b) Every 500 µs  
   (c) Every 5 ms  
   (d) Every 50 ms  

   Therefore what time base setting should be used for the pulse generator?  

2. Calculate the degrees of freedom of He, O₂, CO₂ and CH₄ by adding up modes from translation, rotation and vibration. Use your first year notes of any online references but if the source is of questionable reputation make sure you cross check it! Which of these degrees of freedom do you think contribute to the heat capacity of these gases at room temperature and at 77 K. Again check your answers in the tables below.

   You might want to use the following to calculate whether there can be occupation of excited rotational or vibrational energy levels at a particular temperature:

   The rotational energy levels of a rigid diatomic molecule are given by
   \[ E_r = B_J(J + 1) \]
   
   Where \( B_J = \frac{\hbar^2}{8\pi^2 c I} \) and \( I \) is the moment of inertia of the molecule about its axis of rotation.

   The vibrational energy levels of a diatomic molecule are given (to a first approximation) by
   \[ E_v = \frac{\hbar \omega}{2\pi} \left( n + \frac{1}{2} \right) \]
   where \( \omega = \frac{1}{2\pi} \sqrt{\frac{K}{\mu}} \), \( \mu = \frac{m_1 m_2}{m_1 + m_2} \) and \( K \) is the force constant of the molecular bond and is around 1 kN/m for O₂.