Problem Class Questions for PHY008 Atomic and Nuclear Physics

The following pages contain a large number of questions to help understand the topics covered in each of the chapters. Some questions have fully worked answers and others, which should be attempted in the problems classes, do not.

Chapter 1

Q1. A small drop of oil of mass m which carries a charge Q may be held stationary in a vertical electric field of intensity E.

a) Write down the expression for the forces on the drop which are then in balance (you can neglect up-thrust from the surrounding air).

b) Describe how this apparatus can be used to measure Q.

c) Explain how the results of such an experiment indicate that all charges consist of an integral number of basic units of charge, e.

Answer to part (a) is \( F_e = F_g = mg = qE \)

A1. a) The force due to gravity \( F_g = mg \) where m is the mass of the droplet and g is acceleration due to gravity. The force due to the electric field \( F_e = qE \) where q is the charge on the droplet and E is the electric field. If the oil droplet is stationary then these forces must balance. So we say \( F_e = F_g = mg = qE \).

b) Initially the oil drops are allowed to fall between the plates with the electric field turned off. They very quickly reach a terminal velocity at which point the gravitational force is balanced by the frictional force due to air resistance. A single oil droplet is selected and its terminal velocity \( v_1 \) is measured. The drag force acting on the drop can be worked out using Stokes’ law: \( F_{\text{drag}} = 6\pi r \eta v_1 \) where \( v_1 \) is the terminal velocity of the falling drop, \( \eta \) is the viscosity of the air, and \( r \) is the radius of the drop.

The volume of the droplet is simply \( V = \frac{4}{3} \pi r^3 \) and the density is \( \rho = \frac{M}{V} \) and so the mass of the droplet is \( M = \rho V = \frac{4}{3} \pi r^3 \rho \) and the weight is \( W = Mg = \frac{4}{3} \pi r^3 \rho g \).

Since \( F_{\text{drag}} = Mg \) when the droplet is at terminal velocity we can write: \( F_{\text{drag}} = 6\pi r \eta v_1 = \frac{4}{3} \pi r^3 \rho g \) and by rearranging we can say: \( r^2 = \frac{9 \eta v_1}{2 \rho g} \). Once \( r \) is calculated, \( W \) can easily be determined.

Now the electric field is turned on and the droplet experiences a force due to the electric field of \( F_{\text{electric}} = qE \) where \( q \) is the charge on the oil droplet and \( E \) is the electric field between the plates. The electric field is then adjusted until the oil droplet remains steady, meaning that the force due to the electric field is equal to the weight of the droplet. So \( F_{\text{electric}} = qE = W \) and therefore \( q = \frac{W}{E} \).

Since the weight of the droplet has already been found, the charge can be calculated.

c) Doing this for a huge number of droplets we find that the values calculated for the charge \( q \) on each droplet are all integer multiples of \(-1.60 \times 10^{-19}\) C. We therefore state that this is the charge of the electron and that the droplets contain multiple numbers of electrons, i.e. \( q = n \times (\text{electron charge}) \) with \( n \) an integer number of electrons.
Q2. A charged plastic sphere of mass $3 \times 10^{15}$ kg is held at rest between two horizontal parallel metal plates as shown in the diagram. The distance between the plates is $5 \times 10^{-3}$ m and a potential difference of 310 V is applied across them.

   a) What is the electric field between the plates?
   b) Calculate the charge on the sphere?
   c) If the polarity is suddenly reversed, calculate the initial acceleration of the sphere?

   Answers are (a) 62 KVm$^{-1}$ (b) $4.8 \times 10^{-19}$ C and (c) 19.6 ms$^{-2}$

Q3. Electrons (mass $9.1 \times 10^{-31}$ kg) are emitted from the cathode in an evacuated tube. The electrons start from rest and are accelerated through a potential difference of 1150 V. Some of the electrons pass through the anode into a region where the electric field is perpendicular to their velocity as shown. Calculate:

   a) The speed of the electrons when then arrive at the anode?
   b) The deflection $y$ that they undergo after passing between two parallel plates which are $2 \times 10^{-2}$ m long and $1 \times 10^{-2}$ m apart, and between which a potential difference of 250V is maintained.

   Answers (a) $2 \times 10^7$ m/s and (b) $2.2 \times 10^{-3}$ m.

Q4. An electron beam, in which the electrons are travelling at $1 \times 10^7$ m/s$^{-1}$, enters a magnetic field in a direction perpendicular to the field direction. It is found that the beam can pass through without change of speed or direction if an electric field of strength $1.1 \times 10^4$ Vm$^{-1}$ is applied in the same region at a suitable orientation. Calculate:

   a) The strength of the magnetic field?
   b) If the electric field were to be switched off, what would be the radius of curvature of the electron path?

   (electron charge is $1.6 \times 10^{-19}$ C and electron mass is $9.1 \times 10^{-31}$ kg).

   Answers are (a) $1.1 \times 10^{-3}$ T and (b) 0.051 m.

Q5. A scientist wishes to make a measurement of the electron charge using Millikan’s oil droplet method. His experiment consists of two parts:

   Part 1: With no electric field present, a drop of oil of mass $10^{-14}$ kg is observed to fall with constant velocity 400 µm s$^{-1}$.

   Part 2: A potential difference of 1.5 kV is applied between horizontal parallel plates 12 mm apart. When the field is switched on, the droplet rises with constant velocity 80 µm s$^{-1}$.

   a) Draw two diagrams showing all forces on the droplet once it is in dynamic equilibrium (i.e. has achieved a constant velocity) for experiment parts 1 and 2. (Numerical values for the forces are not required, but you should clearly label the origin of the force).
b) How many electron charges are there on the droplet? (You may assume that the drag force due to air resistance is proportional to the velocity of the droplet and that the air buoyancy may be neglected).

*Answer is* 6 electron charges.

**A5. a)**

![Diagram showing forces on droplet](image)

**b)** In experiment part 1: At constant velocity the forces between gravity and air resistance must balance. If the drag force due to air resistance is proportional to the velocity then we can say that \( mg = kv \) where \( m \) is the mass, \( g \) is the acceleration due to gravity, \( v \) is the velocity and \( k \) is the constant of proportionality. Since we are told the velocity and the mass then:

\[
10^{-14} \times 9.81 = k \times 400 \times 10^{-6} \quad \text{so} \quad k = 2.45 \times 10^{-10}
\]

In experiment part 2: Again at constant velocity the forces must be equal and since the electric field is now present and the droplet is travelling upwards we must write \( mg + kv = qE \) where \( q \) is the charge on the droplet and \( E \) is the electric field. Electric field \( E = \frac{V}{d} = \frac{1500}{12 \times 10^{-3}} = 125 \times 10^3 \text{Vm}^{-1} \).

If \( mg + kv = qE \) then \((10^{-14} \times 9.81) + (2.45 \times 10^{-10} \times 80 \times 10^{-6}) = q \times 125 \times 10^3\).

So \( q = 9.44 \times 10^{-19} \text{ C} \) and so the droplet has a charge of \[
\frac{9.44 \times 10^{-19}}{1.6 \times 10^{-19}} = 6 \text{ electrons.}
\]

**Q6.** In Millikan’s experiment an oil droplet of mass \( 1.92 \times 10^{-14} \text{ kg} \) is stationary in the space between the two horizontal plates which are 20mm apart, the upper plate being earthed and the lower one at a potential of - 6 KV.

**a)** State, giving a reason, the sign of the electric charge on the droplet

**b)** Calculate the magnitude of the charge? (You may neglect the buoyancy of air).

**c)** With no change in the potentials of the plates, the drop suddenly moves upwards and attains a uniform velocity. Explain why (i) the drop moves, and (ii) the velocity becomes uniform?

*Answers are* (a) negative, (b) \( 6.28 \times 10^{-19} \text{ C} \)

**A6. a)** The droplet is repelled from the negative plate so the charge must be negative.

**b)** Electric field \( E = \frac{V}{d} = \frac{-6000}{20 \times 10^{-3}} = 300 \times 10^3 \text{Vm}^{-1} \). When forces balance \( qE = mg \) and so

\[
q = \frac{mg}{E} = \frac{1.92 \times 10^{-14} \times 9.81}{300000} = 6.28 \times 10^{-19} \text{ C.}
\]

**c)** (i) Before the droplet moved, the forces due to gravity and electric field balanced. The droplet moves upwards without any change to the electric field. Therefore either the charge on the droplet...
has increased or the weight of the droplet has decreased. (Both would most likely have occurred during a collision with another droplet). NB. A pure collision with another droplet in which there was no loss of mass or increase in charge would certainly make the droplet move. However it would not attain constant velocity, but rather would slow down due to air drag.

(ii) The velocity only becomes uniform when forces balance. The forces involved are gravity (down), air resistance (down), electrostatic attraction (up). The loss of mass or the gain of charge created an imbalance of forces and therefore an initial vertical acceleration. As the velocity increased the force due to air resistance opposing the motion also increased until the sum of this force combined with the force due to gravity were of equal magnitude to the electrostatic force. At this point the droplet velocity remained constant.

Q7. A charged oil droplet remains stationary when situated between two parallel horizontal metal plates 25 mm apart if a potential difference of 1000 V is applied to the plates.

a) Calculate the electric field present.

b) Find the charge on the droplet if it has a mass of $5 \times 10^{-15}$ kg (assume $g = 9.81 \text{ ms}^{-2}$).

Answers are (a) $40 \times 10^3 \text{ Vm}^{-1}$ (b) $1.23 \times 10^{-18}$ C.

Q8. a) Calculate the radius of a droplet of oil, density $900 \text{ kg m}^{-3}$ which falls with a terminal velocity $2.9 \times 10^{-4} \text{ m s}^{-1}$, through air of viscosity $1.8 \times 10^{-5} \text{ N s m}^{-2}$, ignoring the density of air and taking the drag force as $F = 6\pi \eta a v$ where $\eta$ is the viscosity, $a$ is the radius of the droplet, and $v$ is the velocity?

b) If the charge on the drop is $-3e$, where $e$ is $1.6 \times 10^{-19}$ C, what potential difference must be applied between two parallel horizontal plates 5 mm apart positioned on either side of the droplet in order to render the droplet stationary?

Answers (a) $1.63 \times 10^{-6}$ m and (b) $1672\text{V}$.
**Chapter 2**

Q9. An electron of mass $m$ travelling with speed $u$ collides with an atom and its speed is reduced to $v$. The speed of the atom is unaltered, but one of its electrons is excited to a higher energy level and then returns to its original state, emitting a photon of light. Show that the frequency of this photon is given by $\frac{m(u^2 - v^2)}{2h}$.

Q10. In the photoelectric effect, the maximum kinetic energy of the emitted electrons depends on which of the following variables….work function of the metal, photon energy, light intensity? (you can choose more than one).

A10. $E_{\text{kinetic energy}} = hf - E_\phi$ and so the kinetic energy depends only on the work function and the photon energy.

Q11. When light of frequency $5.4 \times 10^{14}$ Hz is shone on a metal surface the maximum energy of the electrons emitted is $1.2 \times 10^{-19}$ J. If the same surface is illuminated with light of frequency $6.6 \times 10^{14}$ Hz the maximum energy of the electrons emitted is $2.0 \times 10^{-19}$ J. Use this information to calculate a value for $h$ Plank’s constant.

*Answer is $6.626 \times 10^{-34}$ Js.*

Q12. Draw a sketch showing the energy levels of the electron in the hydrogen atom. Indicate on your diagram the ground state of the atom, the first excited state, and the ionisation energy?

Q13. A photo-emissive metal will only emit electrons if the frequency of the incident light exceeds $5 \times 10^{14}$ Hz.

a) What is the value of the work function of the metal?

b) What would be the maximum kinetic energy of the emitted electrons if the incident light were of wavelength 330nm?

Answers are (a) $3.3 \times 10^{-19}$ J and (b) $2.7 \times 10^{-19}$ J

Q14. a) Light of wavelength 500nm incident on a metal surface ejects electrons with kinetic energies up to a maximum value of $2 \times 10^{-19}$ J. What is the energy required to remove an electron from the metal in eV?

b) If a beam of light causes no electrons to be ejected, however great its intensity, what can be said about its wavelength?

Answers are (a) 1.23 eV (b) $> 1 \times 10^{-6}$ m.

Q15. A clean surface of potassium in a vacuum is irradiated with light of wavelength $5.5 \times 10^{-7}$ m and electrons are found to just emerge, but when light of wavelength $5 \times 10^{-7}$ m is incident, electrons emerge, each with energy $3.62 \times 10^{-20}$ J. Estimate the value for Plank’s constant $h$?

*Answer is $6.626 \times 10^{-34}$ Js.*
Q16. Deduce the kinetic energy of the photoelectrons emitted from (a) a copper surface and (b) a caesium surface, when irradiated by light of wavelength $5 \times 10^{-7}$ m given that the work functions of copper and caesium are, respectively, $6.4 \times 10^{-19}$ J and $3.2 \times 10^{-19}$ J?

*Answers are (a) no emission (b) $7.76 \times 10^{-20}$ J.*

Q17. Light of photon energy 3.5 eV is incident on a photocathode of work function 2.5 V. Find the maximum kinetic energy of photoelectrons emitted from the cathode expressing your answer in eV?

*Answer is 1eV.*

A17. $E_{\text{kinetic energy}} = h\nu - E_\Phi$ so if work function and photon energy $h\nu$ are quoted in eV then we can say straight away that $E_{\text{kinetic energy}} = h\nu - E_\Phi = 3.5 - 2.5 = 1$ eV.

Q18. Calculate the de Broglie wavelength of an electron which has been accelerated from rest through a potential difference of 10,000V?

*Answer is $1.23 \times 10^{-11}$ m.*

Q19. The atomic nucleus may be considered to be a sphere of positive charge with a diameter very much less than that of the atom. Discuss the experimental evidence that supports this view. Discuss briefly how this experimental evidence has been obtained?

A19. Atoms are electrically neutral, so if they contain electrons they must also contain some positive charge, but where? Thomson assumed that this was uniformly distributed like a soup throughout the atom but this was only conjecture. To find out, in 1909 Rutherford fired positively charged helium ($\text{He}^{2+}$) ions with high energy at gold atoms in a thin gold film. Based on the theory that positive and negative charges were spread evenly within the atom and that therefore only weak electric forces would be exerted on the high energy ions passing through the thin foil, he expected to find that most of the ions travelled straight through the foil with little deviation.

What he found, to great surprise, was that whilst most passed straight through the foil, a small percentage (about 1 in 10000) were deflected at very large angles and some even bounced back toward the ion source. Because helium ions are about 8000 times the mass of an electron and impacted the foil at very high velocities, it was clear that very strong forces were necessary to deflect these particles. (Imagine firing bullets at soup. Even if just one ricocheted back it would be surprising!!)

The $\text{He}^{2+}$ positive ions had clearly been repelled by an incredibly large positive charge within the atom, this charge concentrated in a dense region also containing most of the mass. This work led in 1913 to Rutherford declaring the atom to contain a very small nucleus of high positive charge (equal to the number of electrons in order to maintain neutrality) and to the ‘solar-system-like’ model of the atom, in which a positively charged nucleus is surrounded by an equal number of electrons in orbital shells.

From purely energetic considerations of how far helium ions of positive charge and known velocity would be able to penetrate toward the central positive charge of the gold nucleus, Rutherford was able to calculate that the radius of the gold nucleus would need to be less than $3.4 \times 10^{-14}$ metres (the modern value is only about a fifth of this). The radius of the entire gold atom was known at the time from diffraction measurements to be $10^{10}$ metres or so in radius. This then implied that the
diameter of the nucleus, containing all the positive charge and almost all of the mass, was less than $1/3000^{th}$ the diameter of the atom.

Detailed analysis of scattering data for various elements has since found the diameter of the nucleus to range from 2 to $9 \times 10^{-15}$ m from hydrogen to uranium, and the diameter of the nucleus to be almost $10^{-5}$ the diameter of the atom. We now also know the nucleus contains both protons (positively charged) and neutrons (electrically neutral) particles of almost the same mass. Scientists at this time had no idea about the existence of neutrons.

Q20. The work function for caesium is 1.35 eV.

a) What is the longest wavelength that can cause photo-emission from a caesium surface?

b) What is the maximum velocity with which photoelectrons will be emitted from the caesium surface illuminated with light of wavelength $4 \times 10^{-7}$ m? (mass of electron is $9.1 \times 10^{-31}$ kg).

Answer (a) $9.2 \times 10^{-7}$ m (b) $7.9 \times 10^5$ ms$^{-1}$.

A20. a) $E_{\text{kinetic}} = hf - \varphi$. But if the frequency is too low so that $hf < \varphi$ then no electrons will be ejected. The minimum frequency is $hf = \varphi$ but $c = f \lambda$ and so $\frac{hc}{\lambda} = \varphi$

$\lambda = \frac{hc}{\varphi} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.35 \times 1.6 \times 10^{-19}} = 9.2 \times 10^{-7}$ m.

b) $E_{\text{kinetic}} = hf - \varphi$ and $E_{\text{kinetic}} = \frac{mv^2}{2}$ and so $\frac{mv^2}{2} = hf - \varphi$ but $c = f \lambda$ and so $\frac{mv^2}{2} = \frac{hc}{\lambda} - \varphi$

Therefore $\frac{mv^2}{2} = \frac{hc}{\lambda} - \varphi = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{4 \times 10^{-7}} - (1.35 \times 1.6 \times 10^{-19}) = 2.81 \times 10^{-19}$ joules.

$v = \sqrt{\frac{2 \times 2.81 \times 10^{-19}}{9.1 \times 10^{-31}}} = 7.9 \times 10^5$ ms$^{-1}$.

Q21. The diagram shows three energy levels for the atom of a particular substance, the energies being in electron-volts (eV). A beam of electrons passing through this substance may excite electrons from the various energy levels.

a) What is the minimum potential difference through which the beam of electrons must be accelerated from rest to cause any excitation between two of these energy levels?

b) At what speed would these electrons be travelling?

Answers are (a) 2V (b) $8.4 \times 10^4$ ms$^{-1}$
Chapter 3

Q22. Given that the atomic mass of $^{14}_7N$ is 14.003074u and that the sum of the atomic masses of $^1_1H$ and $^{13}_6C$ is 14.011179u, will the reaction $^1_1H + ^{13}_6C \rightarrow ^{14}_7N$ result in emission of energy and if so, how much? (Give your answer is MeV) (1u = $1.66 \times 10^{-27}$ kg)

*Answers are YES and 7.55 MeV.*

Q23. When a deuteron ($^2_1H$) of mass 2.0141u and negligible kinetic energy is absorbed by a lithium nucleus ($^{6}_4Li$) of mass 6.0155u, the product disintegrates spontaneously into two alpha-particles, each of mass 4.0026u. Calculate the energy, in joules, given to each alpha particle (1u = $1.66 \times 10^{-27}$ kg).

*Answer is $1.825 \times 10^{-12}$ J*

Q24. Calculate the binding energy per nucleon for $^4_2He$ and $^3_2He$? (Mass of $^1_1P = 1.00783$u, mass of $^1_0n = 1.00867$u, mass of $^3_2He = 3.01664$u, mass of $^4_2He = 4.00387$u, 1u = 931.5 MeV).

*Answers are 6.8MeV for helium-4 and 2.4MeV for helium-3.*

Q25. Calculate the energy released when the $^{64}_{29}Cu$ nucleus undergoes negative beta decay to $^{64}_{30}Zn$? (atomic mass of $^{64}_{29}Cu = 63.92976$u, atomic mass of $^{64}_{30}Zn = 63.92914$u, neglecting the mass of the electron, and 1u corresponding to 931.5MeV).

*Answer is 0.58MeV.*

Q26. a) Draw a graph of a typical X-ray spectrum and explain how the characteristic and continuous parts of the spectrum are formed?

b) Why is there a definite minimum wavelength of the X-rays produced?

c) Calculate the value of the minimum wavelength if the electrons are accelerated through a potential difference of 12 kV?

*Answer (c) $1.04 \times 10^{-10}$ m.*

Q27. A beam of protons is accelerated from rest through a potential difference of 2 kV and then enters a uniform magnetic field which is perpendicular to the direction of the proton beam. If the flux density is 0.2 T calculate the radius of the path which the beam describes? (Mass of proton is $1.673 \times 10^{-27}$ kg).

*Answer is 3.3cm.*

Q28. Electrons, accelerated from rest through a potential difference of 3 kV, enter a region of uniform magnetic field, the direction of the field being at right angles to the motion of the electrons. If the flux density is 0.01 T, calculate the radius of the electron orbit? (mass of electron is $9.1 \times 10^{-31}$kg).

*Answer is 1.8cm.*
A28. The charge on an electron is \( 1.6 \times 10^{-19} \) C. When the electrons are accelerated by 3kV, since \( \frac{mv^2}{2} = eV \) then velocity is \( v = \sqrt{\frac{2 \times 3000 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 3.25 \times 10^7 \text{ ms}^{-1} \)

The force they experience due to the magnetic field is directed towards the centre of the curve and is given by: \( F_{\text{magnetic}} = Bev \)

As they start to curve they also feel a centrifugal force directed in the opposite direction given by: \( F_{\text{centrifugal}} = \frac{mv^2}{R} \). Travelling in a circle of constant radius \( R \), these two forces must balance and so we can say:

\[
Bev = \frac{mv^2}{R} \quad \text{and therefore} \quad R = \frac{mv}{Be} = \frac{9.1 \times 10^{-31} \times 3.25 \times 10^7}{0.01 \times 1.6 \times 10^{-19}} = 0.018m.
\]

Q29. a) Explain the meaning of the term mass difference and state the relationship between the mass difference and the binding energy of the nucleus?

b) Sketch a graph of nuclear binding energy per nucleon versus mass number for the naturally occurring isotopes and show how it may be used to account for the possibility of energy release by nuclear fission?

A29. a) According to special relativity, a mass \( m \) is equivalent to an amount of energy \( E \) where:

\[
E = mc^2
\]

where \( c \) is the speed of light. It follows that whenever a reaction results in the release of energy, there must have been an associated decrease in the mass of the products. The relative atomic mass is defined as:

\[
\text{RAM} = \frac{\text{the average mass of the atom}}{\text{One twelfth the mass of a } ^{12}_6 C \text{ atom}}
\]

The atomic mass unit is defined such that the mass of a \( ^{12}_6 C \) atom is 12u exactly. It follows that the mass of an atom expressed in u is numerically equal to its relative atomic mass. The unified atomic mass unit has a mass of:

\[
1u = 1.66 \times 10^{-27} \text{ kg}
\]

And this can be converted into an energy of \( 1.66 \times 10^{-27} \times (3 \times 10^3)^2 = 1.49 \times 10^{-10} \text{ joules} \). If this energy is written in electron-volts we can state:

\[
1u = 931.5 \text{ MeV}
\]

The mass of a nucleus is always less than the total mass of its constituent protons and neutrons, the difference in mass called the mass defect.

\[
\text{Mass defect} = (\text{Mass of separate nucleons and electrons}) - (\text{Mass of atom})
\]

The reduction in mass arises because the act of combining the nucleons to form the nucleus causes some of their mass to be released as energy in the form of gamma rays. This has the effect of binding the nucleons together since any attempt to separate them would require energy to be supplied. This energy (the added energy needed to take a nucleus apart into its constituent protons and neutrons, or conversely the energy released when all the nucleons come together) is called the binding energy of the nucleus.

\[
\text{Binding energy (MeV)} = 931.5 \times \text{Mass defect (u)}
\]
b) The rising of the binding energy curve at low mass numbers, tells us that energy will be released if two nuclides of small mass number combine to form a single middle-mass nuclide. This process is called nuclear fusion. The eventual dropping of the binding energy curve at high mass numbers tells us on the other hand, that nucleons are more tightly bound when they are assembled into two middle-mass nuclides rather than into a single high-mass nuclide. In other words, energy can be released by the nuclear fission, or splitting, of a single massive nucleus into two smaller fragments.

Q30. A typical fission reaction is: \[^{235}_{92}U + {}^1 \alpha \rightarrow ^{95}_{42}Mo + ^{139}_{57}La + 2 {}^1\alpha + 7 {}^0\alpha\]

Calculate the total energy released in joules by 1 g of \(^{235}_{92}U\) undergoing fission by the reaction above, neglecting the masses of the electrons.

(mass of neutron is 1.009u mass of \(^{95}_{42}Mo\) is 94.906u mass of \(^{139}_{57}La\) is 138.906u

mass of \(^{235}_{92}U\) is 235.044u 1u has a mass of \(1.66 \times 10^{-27}\) kg \(6 \times 10^{23}\) atoms per mole).

Answer is \(8.5 \times 10^{10}\) J.
Chapter 4

Q31. The half-life of a certain radioactive element is such that \( \frac{7}{8} \) of a given quantity decays in 12 days. What fraction remains undecayed after 24 days? (show your working)
Answer is \( \frac{1}{64} \).

Q32. Two radioactive isotopes P and Q have half-lives of 10 minutes and 15 minutes respectively. Freshly prepared samples of each isotope initially contain the same number of atoms as one another. After 30 minutes what is the ratio of \( \frac{\text{no. of atoms of P}}{\text{no. of atoms of Q}} \) ?
Answer is 0.5.

Q33. A radioactive isotope decays by alpha and \( \beta^- \) particle emission. Over a number of decays the nucleon number decreases by 4 while the proton number is unchanged. How many alpha and \( \beta^- \) particles have been emitted?
Answer is 1 alpha and 2 betas.

Q34. Here is some information about a radioactive isotope which might be used as a fuel in nuclear power stations:

<table>
<thead>
<tr>
<th>Name</th>
<th>Particle emitted</th>
<th>Half-life in years</th>
<th>Particle energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strontium-90</td>
<td>Beta</td>
<td>27</td>
<td>0.54 MeV</td>
</tr>
</tbody>
</table>

(a) A sample of Strontium-90 has an initial activity of 400 curies (1 curie corresponds to \( 3.7 \times 10^{10} \) decays per second). Calculate the maximum theoretical power output of this source giving your answer in watts?
(b) What is the maximum theoretical power output of the source after 54 years?
Answers are (a) 1.29 watts per decaying atom, (b) 0.32 watts

Q35. Sketch a graph showing how the number of neutrons \( N \) varies with the number of protons \( Z \) for stable nuclei. Draw the line \( N = Z \) and indicate approximate numerical values of \( N \) and \( Z \) on the axes. What can be deduced from the graph concerning the numbers of neutrons and protons in stable nuclei?

A35. If we plot the number of neutrons against the number of protons for stable nuclei only, it is evident that heavy elements have more neutrons than protons. Small nuclei require approximately equal numbers of neutrons and protons for stability. When we look at larger and larger nuclides having more and more positive charge within the nucleus, we see that a higher percentage of neutrons are needed to shield the repulsive positive charges from one another. Finally, there comes a point where no amount of neutrons can make the nuclide stable. There are just too many positive charges. There is no element with an atomic number \( Z > 82 \) (lead) which has any stable isotopes. \( ^{208}_{82} \text{Pb} \) is the heaviest stable isotope. Based on its position on the plot, it is possible to predict the most likely form of radioactive decay for an isotope in order for it to become more stable, as shown on the figure. We will now describe the four types of radioactive decay.
Q36. A radioactive isotope decays at a rate of 200 counts per second at 1 p.m. in the afternoon. If the decay rate has reduced to 50 counts per second at 5 p.m. in the same day, what is the half-life of the isotope?

*Answer is 2 hours*

A36. If we start with 200 counts per second and this drops to 50 counts per second in 4 hours then we can say that in 2 hours it will have dropped to 100 and in another 2 hour it will have dropped to 50 counts per second. So the half life is 2 hours.

Q37. An alpha particle is emitted when a nucleus of polonium Po disintegrates to form a lead nucleus Pb, according to the relation,

\[ ^{210}_{84}Po \rightarrow ^{4}_{2}He + ^{206}_{82}Pb \]

Deduce the atomic number and mass number of the lead created?

Q38. The reading of a detector of radiation falls from an average of 1600 counts per minute to an average of 400 counts per minute in a period of 30 minutes.

a) What is the half-life of the radioactive source?

b) In what further period of time would the count rate fall to 100 counts per minute?

*Answers are (a) 15 minutes (b) 30 minutes.*

A38. a) Half life means that the decay rate drops to half its present value in this time. In the question the rate has dropped to \( \frac{1}{4} \) of its original value. We could use the familiar equations 

\[ t_{\text{half-life}} = \frac{0.693}{\lambda} \]

and 

\[ N = N_0 \exp(-\lambda t) \]

but it’s quicker to say that the half life must be 15 minutes to account for the decay rate dropping to 800 and then to 400 in a 30 minute period.

b) This would correspond to another 2 half-lives as we would need the rate to be \( \frac{1}{4} \) of the rate at that time. Therefore it would have dropped after a further \( 15 \times 2 = 30 \) minutes.
Q39. Explain the liberation of energy produced by fission?

A39. Nucleons are more tightly bound when they are assembled into two middle-mass nuclides rather than into a single high-mass nuclide. In other words, energy can be released by the nuclear fission, or splitting, of a single massive nucleus into two smaller fragments. The fission process releases energy because of the difference between the binding energies of different nuclei which can be either positive or negative depending on the reaction.

Fission can be spontaneous, but only for very heavy nuclei such as uranium-238, and even then, the rate is very low. Fission of a heavy nucleus such as uranium-235 \( \left( ^{235}_{92} \text{U} \right) \) can alternatively be induced for example by the absorption of a neutron, producing krypton-92, barium-141, three neutrons, and 174 MeV of energy:

\[
^{235}_{92} \text{U} + _0^1 \text{n} \rightarrow ^{92}_{36} \text{Kr} + ^{141}_{56} \text{Ba} + 3_0^1 \text{n} + 174 \text{MeV}
\]

Following absorption of the initial neutron, the uranium-236 nucleus splits into two daughter products and releases three additional neutrons. These neutrons quickly cause the fission of other uranium-235 atoms, thereby releasing additional neutrons and initiating a self-sustaining series of nuclear fissions, or a chain reaction shown below, which results in continuous release of nuclear energy. For the same mass, a single nuclear fission reaction releases 10 million times as much energy as is released in the burning of fossil fuels.

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Q40. In a radioactive decay series there are several examples in which a nucleus emits an alpha particle followed by two \( \beta^- \) particles.

a) Show that the final nucleus is an isotope of the original one?

b) What is the change in mass number between the original and final nuclei?

Answer (b) four fewer nucleons.

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Q41. A radioactive source has a half-life of 20 days. Calculate the activity of the source after 70 days have elapsed if its initial activity is \( 10^{10} \) decays per second?

Answer is \( 8.8 \times 10^8 \) counts per second

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Q42. The radioactive isotope carbon-14 is absorbed by all living material such as plants in the form of carbon dioxide. Experiments show that the decay rate of carbon-14 in living plants is 19 counts per minute per gram. The half-life of the isotope is 5600 years. When a plant dies, no more of the isotope is absorbed. Measuring the current decay rate can provide information about its age. Suppose the current decay rate in a piece of ancient wood is 14 counts per minute per gram.

a) Deduce the decay constant \( \lambda \) for carbon-14?

b) Calculate the age of the wood?

Answers are (a) \( 3.924 \times 10^{-12} \text{ seconds}^{-1} \) (b) 2467 years
Summary questions covering entire course

Q43. By considering its energy, show that an electron accelerated through a potential difference of $V$ volts is travelling at a velocity $u$ given by:

$$u = \sqrt{\frac{2eV}{m_e}},$$

where $e$ is the electron charge and $m_e$ the electron mass.

An electron of mass $9.1 \times 10^{-31}$ kg is accelerated through 25 kV. How fast is it going?

*Answer is $9.4 \times 10^7$ ms$^{-1}$*

Q44. Show that an electron travelling at velocity $u$ at right angles to a magnetic field $B$ moves is a circle radius $R$ given by:

$$R = \frac{m_e u}{Be}.$$

What value of $B$ causes an orbit radius of 450 mm if $u = 3.0 \times 10^7$ m s$^{-1}$ and the mass of an electron is $9.1 \times 10^{-31}$ kg?

*Answer is $3.8 \times 10^{-4}$ T.*

Q45. Light of wavelength 500 nm falls on a metal which has a work function of 1.5 eV. What is the maximum energy in joules of the emitted photoelectron?

\[c = 3 \times 10^8\ m\ s^{-1},\ h = 6.626 \times 10^{-34}\ Js\]

*Answer is $1.6 \times 10^{-19}$ joules.*

Q46. Molecules, each with an atomic weight of $133.6 \times 10^{-27}$ kg and carrying a charge of $1.60 \times 10^{-19}$ C are accelerated through 150 V. What:

a) is their velocity;

b) their momentum;

c) their wavelength?

*Answers are $19000$ ms$^{-1}$, $2.53 \times 10^{-21}$ kg m s$^{-1}$, $2.6 \times 10^{-13}$ m.*

Q47. An electron with energy $-5 \times 10^{-19}$ J moves around an atomic nucleus in a circular orbit.

a) What is its wavelength, and

b) what is the radius of the orbit into which just two wavelengths fit?

*Answers are (a) 0.692 nm and (b) 0.22 nm.*