The Sonometer

Aim
In this experiment you will study the vibrational modes of a stretched string. The dependence of the fundamental frequency on length and tension is determined. In addition resonance effects and the nature of harmonic modes are investigated.

Theory
For a perfectly flexible, perfectly elastic string, the velocity of wave propagation (v) is given by

\[ v = \sqrt{\frac{T}{\mu}} \]

where \( T \) is the tension (in Newtons) and \( \mu \) is the mass per unit length (in Kgm\(^{-1}\)).

If the string has a length \( L \) and both ends are fixed then only vibrational modes are allowed where an integer number of half-wavelengths equals the string length

\[ n \frac{\lambda}{2} = L \Rightarrow \lambda = \frac{2L}{n} \quad n = 1, 2, 3, 4, \ldots \]

With the addition of the relationship between the propagation velocity, wavelength and frequency \( f \)

\[ v = \lambda f \]

it is possible to calculate the frequencies of the fundamental mode \((n=1)\) and harmonics \((n>1)\) for a given combination of \( T, \mu \) and \( L \).

Equipment details
The sonometer allows accurate variation of the tension in the wire, by hanging a series of masses from the end arm, and the length of the wire by moving the bridges. \( \mu \) may also be varied by using different wire thicknesses. A pickup coil is used to detect the vibrations. A second drive coil can be used to induce vibrations in the wire.

The pickup coil is connected to the input of an amplifier which allows you to hear the vibrations. The output of the amplifier is connected to the CI-6508 input adaptor which in turn is connected to analog channel A of the 700 Interface box.

The computer acts as an oscilloscope allowing the frequency of the vibrations to be recorded. Data acquisition can be started and stopped by clicking on MON and STOP respectively. Within the scope window the scales of the horizontal and vertical axes can be changed by clicking on the small buttons with the saw tooth icons. It is important to change the scales before you acquire data. Changing them between data acquisition and making a measurement can lead to incorrect values being recorded.

Values can be read from the screen by clicking on the cursor button at the bottom of the scope window and then positioning the cursor at the desired point on the waveform. The cursor is switched off by clicking on the mouse.
A total mass of up to 1000g can be placed on the carrier and hung from the end arm. The tension in the wire is given by $kMg$ where $M$ is the mass, $g$ is the acceleration due to gravity ($9.8\text{ms}^{-2}$) and $k$ is a multiplying factor whose value depends on the where on the arm the masses are hung (see following figure). Each time a new mass is hung on the arm it is important to ensure that the arm is horizontal by adjusting the screw at the far end of the wire. Only if the arm is horizontal will the tension be given by $kMg$.

![Diagram showing string tension and masses on the arm](image)

**Task 1**

**Dependence of the fundamental frequency on tension**

Use the above equations to derive a relationship between the frequency of the fundamental mode ($n=1$) and $L$, $\mu$ and $T$.

Remove the drive coil from the sonometer. Using a wire length of approximately 30cm measure the vibrational frequency of the wire for a range of tensions. Initially set the time and voltage scales to 2ms/div and 0.02V/div. You should try to take ~15-20 data points. You should position the detector coil close to the midpoint of the wire and try to pluck the wire such that a clean sinusoidal waveform is produced. A clean waveform can be obtained by plucking the wire relatively strongly and then waiting until the oscillations die down in amplitude to appear clearly on the screen.

Plot your frequency-tension data in a form that confirms your derived equation and determine a value of $\mu$ along with associated errors. Compare you value of $\mu$ with its accepted value as provided by the demonstrator.

**Task 2**

**Dependence of the fundamental frequency on length**

Using a mass of ~750g hung from position 5, measure the fundamental frequency as a function of wire length.

Plot your frequency-length data in a form to confirm the relationship derived in Task 1. Determine a value of $\mu$ and compare this to your previous value.
Task 3  **Resonances**
Ensure that in the signal generator window the generator is switched off and the voltage is set to zero. Connect the Driver coil to the output of the Power Amplifier which in turn should be connected to Analog Channel B of the 700 Interface. If you are unsure what to do ask the demonstrator for help.

Using the maximum possible length of wire place the drive coil near to one end of the wire and use the same tension as in Task 2. Place the detector coil close to the centre of the wire.

Set the signal generator to produce a sinewave, a voltage of 2.0V and click on the ON button. Voltages and frequencies can be changed by clicking on the buttons with arrow icons or by clicking on the numbers and typing in the required value.

Increase the frequency in steps of 10Hz by using the arrow buttons and look for resonances where the wire vibrates under the influence of the driving frequency. Once you have located the approximate position of a resonance you should be able to fine tune the frequency and determine its value to within less than 1Hz.

Close to resonance you should see and hear beats between the frequencies of the wire and the driving coil. It may help to locate the first resonance if you know what the fundamental frequency of the wire is. Because the magnetic field produced by the drive coil produces a force on the wire twice per cycle you should observe the fundamental resonance when the drive frequency equals half the fundamental frequency.

You may need to move the positions of the coils to observe some of the resonances. In addition you will need to vary the voltage applied to the drive coil. The initial resonances will require only a small voltage but higher ones will need the voltage to be increased. Do not exceed 4V. Avoid placing the coils close together as this may result in a direct pickup.

Locate as many resonances as you can and record their frequencies. Do they correspond to the fundamental frequency and integer multiples of this frequency? Try plotting frequency against $n$ where $n$ is the $n^{th}$ resonance observed. What is the gradient of your graph?

Task 4  **Displacement envelope of the vibrational modes**
With the drive coil close to one end of the wire and the signal generator frequency set to excite the fundamental mode slide the detector coil along the wire and look at the amplitude of the detected signal. Is the amplitude zero at any point? Repeat for excitation of the different harmonics as determined in Task 3. For each harmonic sketch how the amplitude of the vibrations varies with distance along the wire. Comment on your results. For some of the lower resonances you should be able to directly observe the wire vibrating.

D J Mowbray, February 2003