Lecture 11: Introduction to diffraction of light

Lecture aims to explain:

1. Diffraction of waves in everyday life and applications

2. Interference of two one dimensional electromagnetic waves

3. Typical diffraction problems: a slit, a periodic array of slits, circular aperture

4. Typical approach to solving diffraction problems
Diffraction of waves in everyday life and applications
Diffraction in everyday life
Diffraction in applications

Spectroscopy: physics, chemistry, medicine, biology, geology, oil/gas industry

Communication and detection systems: fibre optics (waveguides), lasers, radars

Holography

Structural analysis: X-ray

Must be taken into account in **applications with high spatial resolution**: imaging (astronomy, microscopy including X-ray, electron and neutron scattering), semiconductor device fabrication (optical lithography), CDs, DVDs, BDs

**Problem:** A propagating wave encounters an obstacle (i.e. a distortion of the wave-front occurs).

**How will this distortion influence the propagation of the wave?**
Interference of two one dimensional (1D) electromagnetic waves
Harmonic wave and its detection

Oscillating electric field of the wave:

\[ E(x,t) = A \sin( kx - \omega t ) \]

In the case of visible light \( \omega \sim 10^{15} \text{Hz} \)

The detectable intensity (irradiance):

\[ I = \left\langle E^2 \right\rangle_T = \frac{1}{T} \int_0^T E^2 \, dt \]
Superposition of waves

Consider two electromagnetic waves:

\[ E_1 = E_{01} \sin(kx_1 - \omega t + \varepsilon_1) \quad E_2 = E_{02} \sin(kx_2 - \omega t + \varepsilon_2) \]

Intensity on the detector:

\[ I = \frac{E_{01}^2}{2} + \frac{E_{02}^2}{2} + E_{01}E_{02}\cos(\delta) \]

Phase shift due to difference in the optical path and initial phase:

\[ \delta = \frac{2\pi}{\lambda}(x_1 - x_2) + (\varepsilon_1 - \varepsilon_2) \]
Dependence of intensity on the optical path length difference

\[ I = \frac{E_{01}^2}{2} + \frac{E_{02}^2}{2} + E_{01}E_{02}\cos(\delta) \]

For the waves with the same initial phase, the phase difference arises from the difference in the optical path length:

\[ \delta = \frac{2\pi}{\lambda} (x_1 - x_2) = \Delta OP \]

Figure shows the dependence of intensity measured by the detector as a function of the optical path difference between the two waves of the same amplitude.
Typical diffraction problems
Diffraction by a slit or periodic array of slits (or grooves)

Use in *spectroscopy*: analysis of spectral (“colour”) composition of light

\[ \Delta \theta = \frac{2\lambda}{b} \]

\( \lambda \)- wavelength, \( b \)- slit width

If many slits arranged in a **periodic array**, sharp maxima will appear at different angles depending on the wavelength: spectral analysis becomes possible.
Diffraction by a circular aperture

Important in high resolution imaging and positioning: sets limitations to spatial resolution in astronomy, microscopy, optical lithography, CDs, DVDs, BDs, describes propagation of laser beams

$$\Delta \theta = \frac{2.44 \lambda}{D}$$

$\lambda$- wavelength, $D$-aperture diameter

The smallest angular size which can be resolved is given by

This also defines the smallest size of a laser spot which can be achieved by focussing with a lens (see images on the left): roughly $\sim \lambda$
Typical approach to solving diffraction problems

Huygens’ principle (Lecture 1):
‘Each point on a wavefront acts as a source of spherical secondary wavelets, such that the wavefront at some later time is the superposition of these wavelets.’
Extended coherent light source

Each infinitely small segment (each “point”) of the source emits a spherical wavelet. From the differential wave equation, the amplitude decays as $1/r$ (see Hecht 28-31):

$$E_i = \frac{\varepsilon_L}{r_i} \Delta y_i \sin(\omega t - kr_i)$$

$\varepsilon_L$ source strength per unit length

Contribution from all points is:

$$E = \varepsilon_L \int_{-D/2}^{D/2} \frac{\sin(\omega t - kr)}{r} dy$$
**Fraunhofer and Fresnel diffraction limits**

**Fraunhofer case:** distance to the detector is large compared with the light source $R \gg D$. In this case only dependence of the phase for individual wavelets on the distance to the detector is important:

$$E = \frac{\mathcal{E}_L}{R} \int_{-D/2}^{D/2} \sin(\omega t - kr) dy$$

Where $r \approx R - y \sin \theta$

**Fresnel case:** includes “near-field” region, so not only phase but the amplitude is a strong function of the position where the wavelet was emitted originally

$$E = \mathcal{E}_L \int_{-D/2}^{D/2} \frac{\sin(\omega t - kr)}{r} dy$$
**SUMMARY**

Diffraction occurs due to superposition of light waves. It is used in **spectroscopy, communication and detection systems** (fibre optics, lasers, radars), **holography, structural analysis** (X-ray), and defines the limitations in **applications with high spatial resolution**: imaging and positioning systems.

The detectable intensity (irradiance) for a quickly oscillating field:

$$I = \left< E^2 \right>_T = \frac{1}{T} \int_0^T E^2 \, dt$$

For the waves with the same initial phase, the phase difference arises from the difference in the optical path length:

$$\delta = \frac{2\pi}{\lambda}(x_2 - x_1) = \Delta OP$$

Typical approach to solving diffraction problems: use Huygens principle and calculate contribution of spherical waves emitted by all “point” emitters. **Fraunhofer diffraction**: observation from a distant point.