Lecture 18: Multiple beam interference, Introduction to Fabry-Perot spectroscopy

Lecture aims to explain:

1. Interference from a plane parallel plate: **multiple beam interference**
2. Experimental set-up for observation of multiple beam interference
3. Coefficient of finesse
4. Analysis of the fringe pattern

Fabry-Perot interferometers or **etalons** are widely used in telecommunications, lasers and spectroscopy to control and measure the wavelengths of light.
Interference from a plane parallel plate: multiple beam interference
Phase modification by a plane parallel plate (etalon)

Transparent plate with parallel flat sides, refractive index $n$, thickness $t$

Phase difference between any two adjacent emerging rays:

$$\delta = \frac{2\pi}{\lambda} \times OPD = \frac{4\pi nt \cos \theta}{\lambda}$$
Multiple beam interference

Etalon with a **transmission coefficient** – $T$ and a **reflection coefficient** – $R$, $A_0$ - amplitude of the electric field of the incident wave

Intensity of the transmitted beam:

$$I \propto \frac{T^4}{1 + R^4 - 2R^2 \cos \delta}$$
Interference fringes

Consider

\[ I \propto \frac{T^4}{1 + R^4 - 2R^2 \cos \delta} \]

The **maxima** for light transmitted through an etalon are observed for angles:

\[ 2nt \cos \theta = m\lambda \]

refractive index \( n \), thickness \( t \)

The **minima** for light transmitted through an etalon are observed for angles:

\[ 2nt \cos \theta = (m + \frac{1}{2})\lambda \]
Basic set-up for observation of multiple beam interference
Two set-ups for multiple beam interference

A wave incident on a plane parallel plate (etalon) can be focused with a lens onto a screen.

Light can be made diffuse by passing through a sheet of ground glass. Then light will be incident on the etalon at all possible angles. A whole interference pattern with radial symmetry will form. For simplicity, figure above shows what will happen for a single angle of incidence.
Coefficient of finesse
The interference pattern due to the multiple beam interference can be described by

\[ A(\delta) = \frac{I}{I_{\text{max}}} = \frac{1}{1 + F \sin^2(\delta/2)} \]

where

\[ F = \frac{4R^2}{(1 - R^2)^2} \]

is the coefficient of finesse. It is a measure of the interference fringe sharpness and contrast.

Fringe contrast increases with \( R \) (more reflections greater contrast!)
Coefficient of finesse $F$ as a function of reflectivity $R$

$$F = \frac{4R^2}{(1 - R^2)^2}$$

F=100 corresponds to $R \approx 0.9$
F=1 corresponds to $R \approx 0.41$
Analysis of the fringe pattern
Dependence of the fringe contrast on the coefficient of finesse $F$

$$A(\delta) = \frac{1}{1 + F \sin^2 (\delta / 2)}$$

$$F = \frac{4R^2}{(1 - R^2)^2}$$

For low reflectivity (low $F$) the image measured in transmission is a bright field with broad dim fringes (right). For high reflectivity (high $F$) the image is essentially dark with sharp bright fringes (left).
Example 18.1
Light with wavelength 1000 nm is incident normally on a Fabry-Perot interferometer consisting of two mirrors with an air gap between them. The initial thickness of the gap is $t = 0.1$ cm. A maximum transmitted light intensity is observed by a detector placed after the interferometer. By gradually changing the thickness of the gap a minimum in intensity is obtained. Find the new thickness of the gap.

Example 18.2
Calculate the intensity (relative to maximum) of the transmitted light exactly in the middle between the adjacent fringes for etalons with $F=1$, 10 and 100.

Example 18.3
Calculate the reflectivity of the mirrors in the Fabry-Perot interferometer if the minimum intensity of the transmitted light is obtained by tuning the wavelength corresponds to 20% of the maximum intensity.
Phase difference between two adjacent rays:

\[ \delta = \frac{2\pi}{\lambda} \times OPD = \frac{4\pi nt \cos \theta}{\lambda} \]

The interference pattern due to the multiple beam interference can be described by:

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where \( F = \frac{4R^2}{(1 - R^2)^2} \)

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