Metals, Semiconductors and Insulators.

2nd half of the course by
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Topics to cover
1. Optical transitions
2. Doping
3. Electron statistics
4. Hall effect
5. Cyclotron resonance
6. Scattering and mobility
7. Plasma oscillations
8. Amorphous materials
9. Quantum wells
Electronics, computing

45nm transistors. Intel web site

Integrated circuit

Data storage (cd, dvd, blu-ray)

Lighting, displays

Telecommunications, internet

Multi-colour LED strip light

Telecommunications laser: Bookham

http://www.auxrux.biz/tag/integrated-circuits/
Nanostructures. Single quasiparticle Quantum bit=> novel data storage/memory elements, quantum computing, quantum cryptography.....

Graphene- one monolayer of Graphite. Much faster Carbon-based nanoelectronics.

Nanophotonics, plasmonics. Integrated optical nanocircuits etc, etc and etc…….
Topic 10: Optical absorption

1. Optical Absorption-onset at band gap.


3. Absorption coefficient proportional to density of states. For a bulk material $\sim E^{1/2}$.


5. Role of phonons for indirect gap semiconductors. Temperature dependence of absorption spectra.

6. Optical absorption in doped semiconductors (the shift of the absorption edge- the Moss-Burstein effect)
Optical absorption in semiconductors

In semiconductors valence band is full and conduction band is empty. If a photon with energy $\hbar \omega$ propagates through the material it may be absorbed exciting electron from conduction to valence band.

In a direct gap semiconductor (maxima and minima of valence and conduction bands coincide in momentum space) onset of optical absorption occurs when the energy of photon $\hbar \omega$ is greater than energy band gap $E_g$: $\hbar \omega > E_g$

Energy band gaps in various materials

GaN > 3 eV
GaInP 2-2.3 eV
GaAs 1.4 eV
Si 1 eV
Ge 0.7 eV
InSb 0.2 eV

Schematic diagramm of electron energy spectrum

Conduction band (C-band)
Valence band (V-band)
Optical absorption in direct gap semiconductors

During absorption of photon $\hbar \omega$ electron is created in conduction band and hole (empty state) is left in the valence band. Conservation of energy and momentum must be fulfilled:

$$\hbar \omega = E_g + \frac{\hbar^2 k_e^2}{2m_e^*} + \frac{\hbar^2 k_h^2}{2m_h^*}$$

$$k_{ph} = k_e + k_h$$

$m_{e,h}^*$ - effective electron (e) and hole (h) masses;
$k_{e,h}$ – electron and hole wavevectors
$k_{ph}$ - photon wavevector

The wavevector of electron $k_e$ at the edge of Brillouin zone $\sim \frac{\pi}{a} \sim 10^{10} \text{ m}^{-1}$

Here $a$ is lattice period of a crystal

The wavevector of photon in visible range $k_{ph} = \frac{2\pi}{\lambda} \sim 10^6 \text{ m}^{-1}$

$\Rightarrow$ Electronic transition due to photon absorption is vertical on energetic schematic diagram (see next slide)

$$| k_e | \approx | k_h |$$
Direct transition

Schematic of vertical electronic transition

Example of energy spectrum of GaAs crystal, which is direct gap material (minima of C and V bands coincide)
Energy dependence of absorption coefficient in direct band gap material

Consider electromagnetic radiation of frequency $\omega$ and intensity $I(\omega)$ propagating distance $dx$ through a semiconductor. Absorption coefficient $\alpha(\hbar \omega)$ of the material depends on frequency of the photons. It is defined as ratio of decrease of light intensity $dI(\omega)$ to $I(\omega)dx$ in the limit of $dx->0$

$$\alpha(\hbar \omega) = \frac{dI(\omega)}{dx I(\omega)}$$

$\alpha$ at frequency $\omega$ is proportional to the effective density of electron – hole state states $N(\hbar \omega)$ associated with optical transition at given frequency. It can be shown that the effective density of states involved into the transition is given by the following formula (see Appendix at the end)

$$N(\hbar \omega)d(\hbar \omega) = \frac{8\pi k^2 dk}{(2\pi)^3} = \frac{\left(2 \frac{m_e^* m_h^*}{m_e^* + m_h^*}\right)^{3/2}}{2\pi^2 \hbar^3} (\hbar \omega - E_g)^{1/2} d(\hbar \omega)$$

$$\alpha(\hbar \omega) \sim (\hbar \omega - E_g)^{1/2}$$
The effect of free carriers an doping on optical absorption spectra

The Moss-Burstein effect

Consider semiconductor material, where C band is already partially filled with electrons. This can happen if a semiconductor is heavily doped with donors (n-doped material). Concerning the concept of donors see the next lecture slides.

Conduction band is partially filled with electrons

Only optical transitions from V-band to empty states in C-band allowed due to Pauli principle (no more than one electron can occupy a single quantum state)

From energy and momentum conservation onset of optical absorption occurs at the following energy

$$\hbar\omega_{\text{onset}} = E_g + \frac{\hbar^2 k_F^2}{2m_e^*} + \frac{\hbar^2 k_h^2}{2m_h^*}$$

$$\hbar\omega_{\text{onset}} = E_g + \left(\frac{\hbar^2 k_F^2}{2m_e^*}\right)(1 + \frac{m_e^*}{m_h^*})$$

- \(k_F\) - Fermi wavevector of electron sea in c-band
Indirect optical transitions

In indirect gap materials the maxima of valence and minima of conduction bands are well separated in momentum space. Hence photon on its own, which has a small momentum, cannot be absorbed. Involvement of phonon is required

Schematic of indirect optical transitions

![Schematic of indirect optical transitions](image)
Indirect gap semiconductors

Conduction band minimum not at k=0

Phonon with momentum \( k_{\text{phonon}} \) and energy \( \hbar \Omega \) is involved

Momentum and energy conservation
\( k_{\text{el}} + k_h = k_{\text{phonon}} \); \( \hbar \omega = E_G \pm \hbar \Omega \)

1. Second order process and thus very weak \( \sim 10^4 \) times weaker

1. Phonon energy small compared to \( E_g \)

\[
\begin{align*}
    k_{\text{phonon}} & \approx \frac{51}{a} \sim 10^{10} \text{m}^{-1} \\
    E_{\text{phonon}} & \approx \hbar \Omega = \frac{\hbar v_s}{\lambda} \approx 30 \text{ meV} \\
    v_s & \text{ - sound velocity} \\
    \hbar \Omega & \ll E_g \text{ (energy gap)}
\end{align*}
\]
Indirect gap semiconductors

Low temperature.
Phonons are not present and thus absorption occurs with emission of phonon

The absorption threshold is at \( \hbar \omega = E_G + \hbar \Omega \)

High temperature
Phonons are present and a phonon can be absorbed along with a photon. Threshold energy \( \hbar \omega = E_G - \hbar \Omega \)
Excitons

So far we assumed that electrons interact only with crystal lattice.

Consider attractive interaction between electron and hole.

Formation of stable bound e-h bound state by analogy with Hydrogen atom. This is exciton in semiconductors.

For Wannier-Mott excitons, which have Bohr radius much bigger than period of crystal lattice, the energy levels of bound states (Hydrogenic model) lie below $E_g$

$$E_n = E_g - \frac{\mu e^4}{2\hbar^2 n^2 (4\pi \varepsilon \varepsilon_0)^2} ; \text{ n}=1,2,\ldots$$

$$\frac{1}{\mu} = \frac{1}{m_e^*} + \frac{1}{m_h^*}$$

$m_{e,h}^*$-electron and hole effective masses

$\varepsilon$ - dielectric function of the material
Absorption spectrum vs photon energy revealing bound excitonic levels
Summary:

1. Optical transition in direct and indirect gap materials. Role of phonons at high and low T in case of indirect transitions.


4. Excitonic effects. Energy spectrum of bound e-h state.

Addition to Topic 10 on optical absorption.

Deduction of formula for the joint density of states $N(\hbar \omega)$, which is number of states per unit energy (and per space unit volume), associated with direct optical transition in semiconductors

For direct transition conservation of energy and momentum to be fulfilled:

$$\hbar \omega = E_g + \hbar^2 k^2_e / 2m_e^* + \hbar^2 k^2_h / 2m_h^* \quad \text{Eq. 1}$$

$$k_{ph} = k_e + k_h \quad \text{Eq. 2}$$

$|k_e| = |k_h| \equiv k$

$m_{e,h}^*$—effective electron (e) and hole (h) masses;

$k_{e,h}$—electron and hole wavevectors

$k_{ph}$—photon wavevector

a) Number of states per volume element $d\vec{k} = dk_x dk_y dk_z$ in momentum space (and per unit volume in real space) = $2d\vec{k} / (2\pi)^3$. Factor 2 is for spin.

b) In spherical coordinates $d\vec{k} / (2\pi)^3 = 2k^2 dk \sin \theta d\theta d\phi / (2\pi)^3$

c) Number of states per interval $dk$ in momentum space=

$$2k^2 dk \left[ \int_0^{2\pi} d\phi \int_0^\pi \sin \theta / (2\pi)^3 d\theta \right] = \frac{8\pi k^2 dk}{(2\pi)^3}$$

d) Differentiating Eq.1 obtain $d(\hbar \omega) = \hbar^2 k dk (\frac{m_e^* + m_h^*}{m_e^* m_h^*})$ ;

From Eq.1 obtain $k = (\hbar \omega - E_g)^{1/2} \sqrt{2} \left( \frac{m_e^* m_h^*}{m_e^* + m_h^*} \right)^{1/2}$.

Substituting d) to c) obtain

$$\frac{8\pi k^2 dk}{(2\pi)^3} = \frac{8\pi(\hbar \omega - E_g)^{1/2}}{(2\pi)^3} \left( \frac{2}{\hbar^2 m_e^* + m_h^*} \right)^{1/2} \frac{m_e^* m_h^*}{m_e^* + m_h^*} \frac{d(\hbar \omega)}{\hbar^2} = \frac{(\hbar \omega - E_g)^{1/2}}{2\pi^2 \hbar^3} \left( \frac{2m_e^* m_h^*}{m_e^* + m_h^*} \right)^{3/2} d(\hbar \omega)$$

$\Rightarrow$ number of states per unit energy (and per space unit volume), which is joint density of states associated with the transition:

$$N(\hbar \omega) = \frac{\left( \frac{2}{\hbar^2 m_e^* + m_h^*} \right)^{3/2}}{2\pi^2 \hbar^3} (\hbar \omega - E_g)^{1/2}$$
Previous Lecture on Topic 10, Optical transitions

Direct transition

Indirect transition
Topic11: Doping of Semiconductors. Donors and Acceptors

1. Control of conductivity. Central of microelectronics

2. Analogy with hydrogen atom

3. Derivation of expressions for binding energy and Bohr radius
   \[ E_n = \left( \frac{m^*}{m_e} \right) \left( \frac{1}{\epsilon^2} \right) R_H \quad r_n = \left( \frac{\epsilon m_e}{m^*} \right) a_B \]

4. For GaAs \( E_B \approx 5.8 \text{ meV}, a_B \approx 99 \text{ A} \)

5 Consistent with approximation of macroscopical dielectric constant and effective mass
### Periodic Table, with the Outer Electron Configurations of Neutral Atoms in Their Ground States

The notation used to describe the electronic configuration of atoms and ions is discussed in all textbooks of introductory atomic physics. The letters s, p, d, ... signify electrons having orbital angular momentum 0, 1, 2, ..., in units h; the number to the left of the letter denotes the principal quantum number of one orbit, and the superscript to the right denotes the number of electrons in the orbit.
Concept of donors and acceptors

If atoms from group V of the periodic table (such as phosphorous or arsenic) are added to molten Silicon or Germanium (elements from group IV) they crystallize when the melt is cooled substituting an atom of Silicon or Germanium. As a result after forming four covalent bonds there is an extra electron left, which can occupy states in conduction band. **Atoms of group V will be donors for Silicon and Germanium.**

By contrast, if atom from group III like Boron or Aluminum are added, they provide only 3 valent electrons to form covalent bonds. The missing electron represents a hole in valence band with a tendency to be bound to B\(^{-}\) or Al\(^{-}\) ions. **Atoms of group III will be donors for Silicon and Germanium.**
Doping of semiconductors by donors and acceptors

Group IV semiconductors Si, Ge, (C)

Donors- group V P, As, Sb
Acceptors- group III B, Ga, Al

Group III-V semiconductors GaAs, InSb

Donors –group VI (on As site) S, Se, Te
group IV (on Ga site) Si

Acceptors- group IV (on As site) C, Si
group II ( on Ga site) Zn, Be, Cd
Binding energy of electron(hole) on donor (acceptor)

Use analogy with hydrogen atom

Hydrogen Rydberg

$$R_n = \frac{m_e e^4}{2\hbar^2 (4\pi\varepsilon_0)^2} = 13.6 \text{ eV} \quad \text{ (for n=1)}$$

Electron Bohr radius in Hydrogen

$$r_n = \frac{n^2 \hbar^2}{m_e e^2} 4\pi\varepsilon_0 = 0.53 \text{ } \AA \quad \text{ (for n=1)}$$

$m_e$ – free electron mass; $e$-electron charge;

For semiconductors one needs to take into account electron motion in crystal, rather than in vacuum. Need to consider effective electron mass $m_e^*$ and dielectric constant $\varepsilon$

**Donor bound states**

$$E_n = -\frac{m_e^* e^4}{2\hbar^2 n^2 (4\pi\varepsilon\varepsilon_0)^2} = -\frac{1}{n^2} \left( \frac{m_e^*}{m_e} \right) \frac{1}{\varepsilon^2} \times R_H$$

**Donor Bohr radius**

$$r_n = \frac{\varepsilon n^2 \hbar^2}{m_e^* e^2} 4\pi\varepsilon_0$$
For GaAs  \( m_e^* = 0.067m_e \)
\[ \varepsilon = 12 \]

Donor binding energy (n=1)
\[ E_1 = -\frac{m_e^* e^4}{2\hbar^2 (4\pi\varepsilon\varepsilon_0)^2} = -\left(\frac{m_e^*}{m_e}\right)\frac{1}{\varepsilon^2} \times R_H = -5.8 \text{ meV} \]

Donor Bohr radius
\[ r_n = \frac{\varepsilon n^2 \hbar^2}{m_e^* e^2 4\pi\varepsilon_0} = 98 \text{ Å} \]

The bound state wavefunction of electron on donor extends over many atomic diameters\(\Rightarrow\) approximation of effective mass and dielectric constant is valid
Band gaps at room temperature

Ge 0.66 meV
Si 1.1 eV
GaAs 1.4 eV

Donor binding energies
Ge 10 meV
Si 45 meV
GaAs 5.7 meV

Acceptor binding energies
Ge 11 meV
Si 46 meV
GaAs 27 meV

1. Very much less than band gap
2. Of order $k_B T$ at 300 K

High degree of ionisation at room T
=> Effective doping allows control of conductivity
Make sure you understand (know)

1. Concepts of donor and acceptor
2. Example of donors (acceptors )
3. Hydrogen model for the energy levels of electron (hole)
on donor (acceptor)
4. Why this model works well
5. Binding energies in comparison to thermal energies are small

J.R. Hook and H.E. Hall, Second edition
P136-138
Topic 12: Semiconductor statistics, Intrinsic and Extrinsic Semiconductors

1. Intrinsic semiconductor. Fermi level close to centre of gap.

\[ n_e = n_p \sim \exp \left( \frac{-E_g}{2k_B T} \right) \]

2. Extrinsic conduction, result of doping.


4. Compensation. Compensation ratio. Number of ionised impurities at low and high temperature
Fermi distribution

The probability of occupation of a state with energy $E$ is given by Fermi distribution

$$f(E) = \frac{1}{\exp\left(\frac{E - E_F}{k_B T}\right) + 1}$$

$E_F$-Fermi level. By definition

$$E_F = \left(\frac{\partial F}{\partial N}\right)_{V,T}$$

; F- Free energy

If there is no doping where is the position of Fermi level $E_F$ in band gap?

Consider the case of $T=0$

At final $T$ since populations in bands small, $E_F$ must remain close to the middle of the band gap
Calculation of electron(hole) density as a function of $T$

With increasing temperature electrons are activated from valence to conduction band. Conductivity of the material is proportional to electron concentration $\sigma \sim n_e$.

Electron concentration as a function of temperature

$$n_e = 2\left(\frac{2\pi m^*_e k_B T}{\hbar^2}\right)^{3/2} \exp\left(\frac{E_F - E_g}{k_B T}\right) \quad \text{(Eq. 1)}$$

$$N_c = 2\left(\frac{2\pi m^*_e k_B T}{\hbar^2}\right)^{3/2} \quad \text{-effective density of states}$$

Electron concentration in energy interval $\{E; E+dE\}$

$$n_e = f(E)g(E)dE$$

Electron density of states per unit volume

$$g(E) = \frac{(2m_e^*)^{3/2}}{2\pi^2\hbar^3} (E - E_g)^{1/2}$$
Total electron concentration

\[ n_e = \int_{E_g}^{\infty} f(E) g(E) dE = \begin{vmatrix} \text{here } g(E) - \text{electron density of states;} \\ \text{we also assume } E - E_F >> k_B T \end{vmatrix} = \]

\[ = \left| f(E) \approx \exp\left(\frac{E_F - E}{k_B T}\right) \right| = \]

\[ = \frac{1}{2\pi^2 \hbar^3} \int_{E_g}^{\infty} (2m^*)^{3/2} (E - E_g)^{1/2} \exp\left(\frac{E_F - E}{k_B T}\right) dE = \]

\[ = \frac{(2m^*)^{3/2}}{2\pi^2 \hbar^3} \exp\left(\frac{E_F - E_g}{k_B T}\right) \int_0^{\infty} (E - E_g)^{1/2} \exp\left(-\frac{E - E_g}{k_B T}\right) d(E - E_g) = \]

\[ = \left| \text{new variable } \chi = \frac{E - E_g}{k_B T} \right| = \]

\[ = \frac{(2m^*)^{3/2}}{2\pi^2 \hbar^3} \exp\left(\frac{E_F - E_g}{k_B T}\right) \chi^{3/2} \int_0^{\infty} \chi^{1/2} \exp(-\chi) d\chi = \]

\[ = \int_0^{\infty} \chi^{1/2} \exp(-\chi) d\chi = \frac{\sqrt{\pi}}{2} = \]

\[ = 2\left(\frac{2\pi m^* k_B T}{h^2}\right)^{3/2} \exp\left(\frac{E_F - E_g}{k_B T}\right) \]

\[ N_c = 2\left(\frac{2\pi m^* k_B T}{h^2}\right)^{3/2} \] - Effective number of levels per unit volume in conduction band imagining them concentrated at the bottom of the band \( E = E_g \)
Intrinsic regime. Undoped material

Band to band excitation of electrons and holes. Electron $n_e$ and hole $n_p$ concentrations are equal

\[ n_e = n_p = \sqrt{N_c N_v} \exp\left(\frac{-E_g}{2k_B T}\right) \]

Here $N_c, v$—effective densities of states in conduction and valence bands respectively.

Example:
At 300 K
\[ n_{\text{intrinsic}} = 2 \times 10^{19} \text{ m}^{-3} \text{ (Ge)} \]
\[ = 1 \times 10^{16} \text{ m}^{-3} \text{ (Si)} \]
Extrinsic conduction as a result of doping

Consider material doped only with donors of concentration $N_D$

At very low temperatures $k_B T << E_D$

$E_D$ - donor binding energy

Fermi Level is close to $E_F \approx E_g - E_D / 2$

Precise concentration of electrons

$$n_e = (N_c N_D)^{1/2} \exp\left(\frac{-E_D}{2k_B T}\right)$$
Compensation

Both donors and acceptors are present

1) At high $T$ ($kT \gg E_D$) all impurities are ionised: $n_e = N_D - N_A$

2) At low $T$ ($kT \ll E_D$):

   For $N_D > N_A$,
   
   1) At high $T$ ($kT \gg E_D$) all impurities are ionised: $n_e = N_D - N_A$

   2) At low $T$ ($kT \ll E_D$): $E_F \approx E_g - E_D$

   $n_e \approx N_c \exp\left(-\frac{E_D}{k_B T}\right)$
Temperature dependence of carrier number in doped material

**Fig. 9.5.** The excitation of carriers in a semiconductor as a function of temperature. (Typical values for a specimen of doped germanium.)
Temperature dependence of Fermi level and carrier number in doped material

Fig. 5.6 Variations of (a) the Fermi level $\mu$ and (b) the electron and hole concentrations (note the logarithmic scale) with $1/T$ for an $n$-type semiconductor containing a significant number of acceptor impurities. The figure was calculated for a germanium semiconductor with $N_D = 10^{22} \text{ m}^{-3}$, $E_D = 0.012 \text{ eV}$, $N_A = 10^{21} \text{ m}^{-3}$ and $E_A = 0.010 \text{ eV}$; the scale at the top shows temperature values for this case.
Summary

1. Concept of Fermi level.

2. Position of Fermi level in doped and undoped material.

3. Temperature behaviour of Fermi level

4. Derivation of expression for electron concentration using Fermi distribution

5. Compensation in material doped with donors and acceptors. Electron concentration at low and high T. Number of ionised impurities at low and high T.

J.R. Hook and H.E. Hall, Second edition p139-147
1. Motion of charge carrier in crossed electric and magnetic fields (d.c. $\omega=0$, $\omega\neq0$ corresponds to cyclotron resonance)

2. Solve equation of motion

$$m_e^* \left( \frac{d\vec{V}}{dt} + \frac{\vec{V}}{\tau_e} \right) = -e(\vec{E} + \vec{V} \times \vec{B})$$

3. Hall coefficient $R_H = -1/(ne)$, expressions for mobility, conductivity

4. Hall field exactly balances the Lorentz force

5. Method to determine sign of charge carriers in semiconductor and metals

6. $R_H$ negative for e.g. simple alkali metals. Can be positive for divalent metals due to overlapping bands
Consider geometry of Hall bar

Assume that electrons are the main carriers. Magnetic field $B$ is applied along $Z$. Electric field is applied along $X$, so that carriers move from right to left. Lorentz force acts on electrons and deflects them to the bottom side of the Hall bar. As a result negative and positive charges accumulate on the bottom and the top sides, respectively. The resultant force due to vertical electric field $E_H$ compensates Lorentz force.

**The Lorentz force acting on electrons** $- e \vec{V} \times \vec{B}$

$\vec{V}$ and $\vec{B}$ are electron velocity and external magnetic field

**The Lorentz force is compensated by electric force** $- e \vec{E}_H$

$\vec{E}_H$ is generated Hall electric field
Motion of charge carriers in constant crossed electric and magnetic fields

To derive generated Hall electric field $E_H$ as a function of electron effective mass, scattering time, magnetic field and applied electric field consider general equation of electron motion in constant electric $E$ and magnetic $B$ fields

$$m_e^* \left( \frac{d\vec{V}}{dt} + \frac{\vec{V}}{\tau_e} \right) = -e(\vec{E} + \vec{V} \times \vec{B})$$

$m_e^*$ - electron effective mass

$\tau_e$ - electron scattering time

Electron velocity

$$\vec{V} = (V_x; V_y; V_z)$$
\( \vec{B} \parallel Z \)  
\[ \vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \]  
\[ \vec{V} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ V_x & V_y & V_z \\ 0 & 0 & B \end{vmatrix} \]

\( i, j, k \) are unit vectors along X, Y and Z axes

\[
m_e^* \left( \frac{d\vec{V} + \vec{\tau}}{\tau_e} \right) = -e(\vec{E} + \vec{V} \times \vec{B})
\]

In steady state

\[
\frac{d\vec{V}}{dt} = 0
\]

\[
m_e^* \left( \frac{dV_x + V_x}{\tau_e} \right) = -e(E_x + V_y B)
\]

\[
m_e^* \left( \frac{dV_y + V_y}{\tau_e} \right) = -e(E_y - V_x B)
\]

\[
m_e^* \left( \frac{dV_z + V_z}{\tau_e} \right) = -eE_z
\]

\[
m_e^* \frac{V_x}{\tau_e} = -e(E_x + V_y B)
\]

\[
m_e^* \frac{V_y}{\tau_e} = -e(E_y - V_x B)
\]

\[
m_e^* \frac{V_z}{\tau_e} = -eE_z
\]
Calculation of Hall field

$V_y = 0$  Since current cannot flow out of the bar

Electric field $E_y$ (Hall field) has value, since it exactly counter balances the Lorentz force arising from magnetic field

$$m_e \frac{V_x}{\tau_e} = -eE_x \Rightarrow$$

$$\Rightarrow V_x = -eE_x \frac{\tau_e}{m_e^*}$$

$$0 = -eE_y + eV_x B$$

Hall field

$$E_H \equiv E_y = \frac{-eB \tau_e}{m_e^*} E_x$$
Hall coefficient

Define Hall coefficient \( R_H = \frac{E_y}{j_x B} \)

Here electric current density in X direction \((n_e - \text{electron concentration})\) is given by

\[
j_x = -n_e e V_x = \frac{n_e e^2 \tau_e}{m_e^*} \cdot E_x
\]

\[
R_H = \left( \frac{-e B \tau_e E_x / m_e^*}{n_e e^2 \tau_e E_x B / m_e^*} \right) = -\frac{1}{n_e e}
\]

Electron concentration can be extracted from Hall effect by measuring Hall electric field and current in known magnetic field

Electron mobility \( \mu_e = \frac{e \tau_e}{m_e^*} \)

Conductivity \( \sigma = n_e e \mu_e \)

Measuring both \( \sigma \) and \( n_e \) we can obtain \( \mu_e \)
If both electron and holes are present in the system, it is more complicated

Write equations of motion for both electrons and holes

\[ m^e_e \left( \frac{d\vec{v}_e}{dt} + \frac{\vec{F}_e}{e} \right) = -e\vec{E} - e[\vec{v}_e \times \vec{B}] \]

\[ m^h_h \left( \frac{d\vec{v}_h}{dt} + \frac{\vec{F}_h}{e} \right) = +e\vec{E} + e[\vec{v}_h \times \vec{B}] \]

\[ \vec{F}_e = \{ F_{e_x}, F_{e_y}, F_{e_z} \}, \vec{F}_h = \{ F_{h_x}, F_{h_y}, F_{h_z} \} \]

Lorentz force

Steady state solutions:

\[ \vec{v}_e = -\frac{e\vec{E}}{m^e_e} \times (\vec{E} + [\vec{v}_e \times \vec{B}]) = \mu_e (\vec{E} + [\vec{v}_e \times \vec{B}]) \]

\[ \vec{v}_h = \mu_h (\vec{E} + [\vec{v}_h \times \vec{B}]) \]

\[ \vec{v}_e, \vec{v}_h - \text{electron and hole velocities} \]

\[ \text{Current density: } \Rightarrow \]

\[ \vec{j} = -n_e e \vec{v}_e + n_p e \vec{v}_h = n_e e \mu_e (\vec{E} + [\vec{v}_e \times \vec{B}]) + n_p e \mu_h (\vec{E} + [\vec{v}_h \times \vec{B}]) = \]

\[ \mu_e, \mu_h - \text{electron and hole mobilities} \]

\[ n_e, n_h - \text{concentrations} \]
If the mobility of minority carriers is high these carries may determine the sign of the Hall coefficient.

Final answer:

$$R_H = \frac{\left( n_p \mu_h^2 - n_e \mu_e^2 \right)}{e \left( n_p \mu_h + n_e \mu_e \right)^2}$$

Here $n_e$, $n_p$ are electron and hole concentrations.
RH is negative for e.g. simple alkali metals. Can be positive for divalent metals due to overlapping bands

<table>
<thead>
<tr>
<th>Metal</th>
<th>Group</th>
<th>(-1/(R_H Ne))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na</td>
<td>I</td>
<td>+0.9</td>
</tr>
<tr>
<td>K</td>
<td></td>
<td>+1.1</td>
</tr>
<tr>
<td>Cu</td>
<td>IB</td>
<td>+1.3</td>
</tr>
<tr>
<td>Au</td>
<td></td>
<td>+1.5</td>
</tr>
<tr>
<td>Be</td>
<td>II</td>
<td>−0.2</td>
</tr>
<tr>
<td>Mg</td>
<td></td>
<td>+1.5</td>
</tr>
<tr>
<td>Cd</td>
<td>IIIB</td>
<td>−2.2</td>
</tr>
<tr>
<td>Al</td>
<td>III</td>
<td>+3.5</td>
</tr>
</tbody>
</table>

(Data from the American Institute of Physics Handbook, 3rd edn, McGraw-Hill, New York (1972))
In divalent metals (like Be) current arises from the motion of both electrons and holes (empty states in valence band). As a result Hall coefficient can be positive.
Summary

1. Hall experiment is used to determine carrier concentration and mobility

2. Sign of Hall coefficient is determined by the sign of the main carriers

3. If both electrons and holes are present the mobility of minority carriers may define the sign of Hall coefficient

*J.R.Hook and H.E.Hall, Second edition*  
*p152-154*
Topic 14 Cyclotron Resonance

1. Method to determine effective mass in semiconductors

2. Electron moves in circular orbits in constant magnetic field $B$. The precession angular frequency

$$\omega_c = \frac{eB}{m_e}$$

3. a.c electric field $E \approx \exp(i\omega t); (\omega \approx \omega_c)$ perpendicular $B$ results in strong acceleration of electrons. => effect of cyclotron resonance

4. Condition for observation of cyclotron resonance

$$\omega_c \tau_e \gg 1 \quad \tau_e - \text{electron scattering time.}$$

5. To observe cyclotron resonance one uses low temperature and pure samples to maximise $\tau_e$, and higher magnetic fields to maximise $\omega_c$

6. Quantum mechanical description of cyclotron resonance. Electron spectrum consists of Landau levels. Transition between Landau levels due to photon absorption of energy $\hbar \omega_c$
Electron motion in magnetic field $B$.

Simple treatment

$$m_e^* \frac{d\vec{V}}{dt} = -e\vec{V} \times \vec{B}$$

$$m_e^* \omega^2 r = e \omega r B$$

Electron effective mass $m_e^*$

Electron scattering time $\tau_e$

Electron velocity $\vec{V} = (V_x; V_y; V_z)$

Electron moves in circular orbits about $B$ with frequency of angular motion given by

$$\omega_c = \frac{eB}{m_e^*}$$
Electron motion in crossed d.c magnetic field $B \parallel Z$ and a.c. electric field in X-Y plane

Now we apply a.c. circularly polarised electric field of frequency $\omega$ perpendicular to magnetic field. If $\omega \sim \omega_c$ electric field will rotate about Z axis with the same angular frequency as electrons in semiconductor, causing acceleration of electrons. As a result electromagnetic radiation will be absorbed.

**a.c. Electric field**

$$\vec{E} = (E_x ; E_y) = E(\cos(\omega t); \sin(\omega t))$$

Equation of electron motion in XY plane

$$\frac{dV_x}{dt} + \frac{V_x}{\tau_e} = - \frac{e}{m_e^*}(E_x + V_y B)$$

$$\frac{dV_y}{dt} + \frac{V_y}{\tau_e} = - \frac{e}{m_e^*}(E_y - V_x B)$$

$V_{x,y}$ - Projections of electron velocity on X and Y

$\tau_e$ - Electron scattering time
In order to find a solution we introduce complex velocity

\[ u = V_x + iV_y \]

\[ \frac{du}{dt} + \frac{u}{\tau_e} = -\frac{e}{m_e} (E \exp(i\omega t) - iuB) \]

Looking for solution in the form

\[ u = u_0 \exp(i\omega t) \]

\[ u_0 = \frac{ieE}{m_e (\omega - \omega_c - i/\tau_e)} \]

Cyclotron frequency

\[ \omega_c = \frac{eB}{m^*_e} \]

Maximum amplitude of electron velocity

\[ \sqrt{u_0^*u_0} \]

occurs at \( \omega \approx \omega_c \)

Absorption coefficient is proportional to the current density squared

\[ A \sim j^2 \sim \frac{1}{(\omega - \omega_c)^2 + \frac{1}{\tau_e^2}} \]

Electromagnetic Radiation of frequency \( \omega \approx \omega_c \) will be absorbed
Method to determine effective mass $m_{e,h}^*$ of either electron (e) or hole (h)

Quantitative example

$$\hbar \omega_c = \hbar \frac{eB}{m_e^*}$$

$$\omega_c / 2\pi = 100 \text{ GHz}$$

$$m_e^* = 0.07 m_e \quad \text{Free electron mass } m_e$$

$$B = \frac{m_e^* \omega_c}{e} = 0.247 \text{ T}$$

To observe cyclotron resonance one needs that a particle completes several orbits between collisions

$$\omega_c \tau_e \gg 1$$

$$\tau_e \approx 10^{-12} \text{ s}$$
The higher magnetic field, the higher the experimental frequency and thus the shorter scattering times that can be tolerated.

To observe cyclotron resonance one maximises scattering time by using low temperatures and investigating pure samples.
It is possible to use cyclotron resonance to determine whether charge carriers are electrons or holes.

In the above sign convention for electron cyclotron frequency is positive

\[ \omega_c = \frac{eB}{m_e^*} \]

the electric field used is \( \sigma^+ \) circularly polarised

\[ \vec{E} = (E_x; E_y) = E(\cos(\omega t); \sin(\omega t)) \]

For holes \( \omega_c = -\frac{eB}{m_e^*} \)

is negative, and thus the radiation which can be absorbed is \( \sigma^- \) circularly polarised

\[ \vec{E} = (E_x; E_y) = E(\cos(\omega t); -\sin(\omega t)) \]
Quantum mechanical treatment of cyclotron resonance.

In external magnetic field $\mathbf{B}||\mathbf{Z}$ electron motion is quantised in $xy$ plane.

Quantised Landau levels are formed.

$$E = (n + \frac{1}{2})\hbar \omega_c + \frac{\hbar^2 k_z^2}{2m_e^*}$$

$k_z$ - Momentum in $Z$ direction

Cyclotron energy

$$\hbar \omega_c = \hbar \frac{eB}{m_e^*}$$

Cyclotron resonance corresponds to transition from one Landau level to the next one accompanied by absorption of photon of energy

$$\hbar \omega_c = \hbar \frac{eB}{m_e^*}$$
Quantised cyclotron orbits in k-space

(a) In the absence of a magnetic field, free electrons are uniformly distributed throughout the Fermi sphere at $T = 0$.

(b) A magnetic field causes quantization of the cyclotron orbits such that the electrons lie on a series of concentric cylinders in k-space. The occupied portions of the cylinders are essentially those parts that are inside the original Fermi surface. With increasing field the cylinders expand and move outwards through the Fermi surface. The occupied region of a cylinder shrinks most rapidly when it is just about to pass through the Fermi surface.

Fig. 13.9

A simple square lattice of electron states in this plane in the absence of a field, with the Landau cylinder structure superimposed. Fig. 13.10(b) shows the states on the Landau level cylinders in the presence of a field; each state in Fig. 13.10(a) has moved to the nearest Landau cylinder to leave the average density of states in k-space unchanged. According to Eq. (13.28) the area between successive cylinders in k-space is $2\pi eB/\hbar$, independent of $n$. The density of states per unit area in Fig. 13.10(a) is $(L/2\pi)^2$ so that the number of states $g_n$ associated with the
Few resonances are observed in Si because
1) electron dispersion is anisotropic (electrons travelling in different directions have different mass) and

2) hole valence band is degenerate, consisting of two different energy bands described by different mass.
Summary

• Cyclotron resonance can be used to determine effective mass and sign of carriers

• To observe cyclotron resonance scattering time should be long enough to allow electron to complete few circular orbits

• In quantum mechanical treatment cyclotron resonance corresponds to transition between Landau levels

Topic 15: Mobility and scattering processes in semiconductors.

1. Two dominant scattering mechanisms of electrons affecting conductivity: scattering with ionised impurities (or missing atoms – vacancies and other structural defects) and scattering with phonons.

2. Mobility \( \mu = \frac{e\tau}{m_e} \), \( \tau \) – scattering time.

Mattheisen’s rule: 
\[
\frac{1}{\tau} = \frac{1}{\tau_{ph}(T)} + \frac{1}{\tau_{imp}(T)}
\]

\( \tau_{ph}, \tau_{imp} \) – scattering times with phonons and impurities, respectively

\[
\mu^{-1} = \mu_{imp}^{-1} + \mu_{ph}^{-1}
\]

\( \mu_{imp} \sim \tau_{imp}(T) \sim T^{3/2}; \mu_{ph} \sim \tau_{ph}(T) \sim T^{-3/2} \)

for the mobilities determined by ionised impurities and phonon scattering, respectively

3. Comparison with metals. In semiconductors impurity scattering is temperature dependent since electron velocity is temperature dependent.

In metals impurity scattering is temperature independent.
Scattering by ionised impurities in semiconductors:

Defect (impurity) scattering is temperature dependent, since mean velocity of electrons is temperature dependent.

In case of nondegenerate gas electrons obey Boltzmann distribution.

Average kinetic energy of electrons \( (v - \text{electron velocity}) \)

\[
\frac{m_e v^2}{2} = \frac{3 k_B T}{2}
\]

Scattering by charged impurities significant when thermal energy \( (\sim k_B T) \) of electron is of the order of Coulomb potential of charged impurity acting electron

\[
\sim \frac{1}{r}
\]

\[
k_B T \sim \frac{1}{r} \quad \text{r- distance between electron and charged impurity}
\]
Scattering cross-sectional area of impurity

\[ A \sim r^2 \sim \frac{1}{T^2} \]

Mean free path of electron

\[ l \sim \frac{1}{A} \sim T^2 \]

Electron scattering time

\[ \tau_{\text{imp}}(T) = \frac{l}{\nu} \sim \frac{T^2}{T^{1/2}} \sim T^{3/2} \]

Mobility due to impurity scattering

\[ \mu_{\text{imp}} \sim \tau_{\text{imp}}(T) \sim T^{3/2} \]

Analogy with Rutherford scattering of alpha particles: mean free path associated with such scattering \( \sim \) to the square of electron energy

\[ l \sim \nu^2 \]

\[ \Rightarrow \text{scattering time} \ \ \ \tau_{\text{ph}}(T) = \frac{l}{\nu} \sim \nu^3 \]
Scattering with phonons in semiconductors:

Mobility due to phonon scattering

$$\mu_{ph} \sim \tau_{ph}(T) \sim T^{-3/2}$$

Predominant scattering with longitudinal acoustic phonons (LA), which produce series of compressions and dilations and modulate energies of conduction and valence bands locally.

The LA phonons which scatter electron must have wavelengths $$\lambda_{ph}$$

at least as large as the electron wavelength $$\lambda_e$$
Scattering of electron with energy $k_B T$ by phonon involves a major change of its momentum but not its energy, since the energy of phonon with momentum $2\pi \over \lambda_{ph}$ is small compare to $k_B T$

$\mu_{ph}(T) \sim \tau_{ph}(T) = {l \over \nu} \sim T^{-3/2}$
Fig. 53. Experimental temperature variation of the mobility of a high-purity GaAs VPE sample ($N_D = 4.80 \times 10^{13} \text{ cm}^{-3}$; $N_A = 2.13 \times 10^{13} \text{ cm}^{-3}$) and calculated mobility curves for each scattering process acting separately and for all scattering processes combined. (Reprinted with permission from Elsevier Sequoia S. A., G. E. Stillman and C. M. Wolfe; *Thin Solid Films* 31, 69 (1976).)
Comparison to metals:
In metals at low temperature conductivity and thus mobility is independent of T.

At low T scattering with defects/impurities is dominant. The scattering rate is temperature independent since the electron velocity given by Fermi energy is temperature independent.
Summary

• Two dominant scattering mechanisms of electrons: scattering with ionised impurities (or missing atoms – vacancies and other structural defects) and scattering with phonons.

• Temperature dependence of the mobilities determined by ionised impurity and phonon scattering respectively in the limit of low and high T.

• Comparison to metals. In metals at low temperature conductivity and thus mobility is independent of T.

**Topic 16: Plasma reflectivity, Plasma Oscillations**

1. Collective response of electron gas to electromagnetic field
2. Dielectric constant in a material depends of the frequency of external radiation $\omega$ and wavevector $k$
3. Oscillation of electron density at plasma angular frequency
   \[ \omega_p^2 = \frac{n_e e^2}{m_e \varepsilon_0} \]
   occurs in the ultraviolet for typical metals.

   $n_e$-electron density; $m_e^*$-electron effective mass; $\varepsilon_0$- permittivity of free space

4. Dielectric function:
   \[ \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \]
   Dispersion of plasmon frequency as a function of k-vector derived from
   \[ \omega^2 \varepsilon(\omega, k) = k^2 c^2 \]
5. For $\omega < \omega_p$

   $\varepsilon$ is real and negative, and hence $k$ is imaginary. Wave is damped in metal, which has a very high reflectivity.

6. For $\omega > \omega_p$

   $\varepsilon$ is real and positive , and $k$ is real. wave propagates.

7. Colour of metals e.g. lithium, potassium, silver, copper gold

8. Quantised oscillations of plasma: plasmons. Observed in electron energy loss experiments
Transverse electromagnetic field in a plasma

Collective response of electron gas to an electromagnetic field. a.c. electric field shifts electrons inducing polarisation in metal.

Propagation of electromagnetic radiation through the material is determined by dielectric function in a metal \( \varepsilon(\omega, k) \)

Dielectric function depends on the wavevector and frequency of external radiation.

There is a critical frequency- plasma frequency, which determines reflectivity of metal up to high photon energy (~3 eV)
Derivation of \( \varepsilon(\omega) \) in presence of high density of carriers (electrons)

\[
\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon(\omega) \varepsilon_0 \vec{E}
\]

\( \vec{D} \) - displacement

\( \vec{P} \) - polarisation

Derive electron polarisation from equation of motion of free electrons in a.c. electric field \( E \sim \exp(i\omega t) \)

\[
m_e^* \frac{d^2 x}{dt^2} = -eE; \quad x \text{- electron coordinate}
\]

\( m_e^* \) - effective electron mass

\[
x, E \sim \exp(i\omega t);
\]

\[
-m_e^* \omega^2 x = -eE;
\]

\[
x = \frac{eE}{m_e^* \omega^2}
\]

Dipole moment due to electron displacement

\[
-ex = \frac{e^2 E}{m_e^* \omega^2}
\]

Polarisation

\[
P = -n_e ex = -\frac{n_e e^2 E}{m_e^* \omega^2}
\]

( dipole moment in unit of volume)

\( n_e \) - electron concentration
Frequency dependence of dielectric function

Displacement

\[ \vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon(\omega)\varepsilon_0 \vec{E} \]

\[ \varepsilon(\omega) = \frac{\vec{D}}{\varepsilon_0 \vec{E}} = 1 + \frac{\vec{P}}{\varepsilon_0 \vec{E}} \]

Polarisation

\[ \vec{P} = -\frac{n_e e^2 \vec{E}}{m_e^* \omega^2} \]

\[ \varepsilon(\omega) = 1 - \frac{n_e e^2}{\varepsilon_0 m_e^* \omega^2} \]

Define plasma frequency

\[ \omega_p = \sqrt{\frac{n_e e^2}{m_e^* \varepsilon_0}} \]

Dielectric function

\[ \varepsilon(\omega) = 1 - \frac{\omega^2}{\omega_p^2} \]
Above we derived formula for dielectric function of a metal due to displacement of free electrons from their equilibrium position by a.c. electric field.

To derive dispersion equation (dependence of frequency of light versus \( k \)-vector) we start from wave equation:

1. \( \mu_0 \frac{\partial^2 \mathbf{D}}{\partial t^2} = \Delta \mathbf{E} \)

\( \mu_0 \)– permeability of free space. (we assume our metal is nonmagnetic)

**Propagating electromagnetic field is given by**

2. \( \mathbf{E} \sim \exp(i \mathbf{k} \cdot \mathbf{r} + i \omega t) \)

\( \mathbf{k} \) – wavevector; \( \mathbf{r} \) – coordinate

**Displacement**

3. \( \mathbf{D} = \varepsilon_0 \varepsilon(\omega) \mathbf{E} \)

Substitute 2. and 3. into 1.

\[ \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} (\varepsilon(\omega) \mathbf{E}) = \Delta \mathbf{E} \implies \]

\[ \implies \mu_0 \varepsilon_0 (-\omega^2) \varepsilon(\omega) \mathbf{E} = -k^2 \mathbf{E} \]

\[ \omega^2 \mu_0 \varepsilon_0 \varepsilon(\omega) = k^2 \implies \omega^2 \varepsilon(\omega) = k^2 c^2 \]
Figure 2 Dispersion relation for transverse electromagnetic waves in a plasma. The group velocity $v_g = d\omega/dK$ is the slope of the dispersion curve. Although the dielectric function is between zero and one, the group velocity is less than the velocity of light in vacuum.

Figure 3 Reflectance of indium antimonide with $n = 4 \times 10^{18}$ cm$^{-3}$. (After J. N. Hodgson.)

Table 1 Ultraviolet transmission limits of alkali metals, in Å

<table>
<thead>
<tr>
<th></th>
<th>Li</th>
<th>Na</th>
<th>K</th>
<th>Rb</th>
<th>Cs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_p$, calculated, mass $m$</td>
<td>1550</td>
<td>2090</td>
<td>2870</td>
<td>3220</td>
<td>3620</td>
</tr>
<tr>
<td>$\lambda_p$, observed</td>
<td>1550</td>
<td>2100</td>
<td>3150</td>
<td>3400</td>
<td>---</td>
</tr>
</tbody>
</table>
Consider wave of the form

\[ E \sim \exp(i \omega t + i kr) \]

1. For \( \omega < \omega_p \)

\( \varepsilon'(\omega, k) \) is real and negative, and hence \( k \) is imaginary.
Wave is damped in metal since

\[ E \approx \exp(-kr) \]

In this case metal has high reflectivity.

2. For \( \omega > \omega_p \)

\( \varepsilon'(\omega, k) \) is real and positive, and \( k \) is real.
Wave propagates into metal.

**Example for a metal**

*electron concentration in metal* \( n_e = 10^{28} \text{ m}^{-3} \);

*plasma angular frequency* \( \omega_p = 5.7 \times 10^{15} \text{ rad/sec} \);

*plasma frequency* \( \nu_p = \frac{\omega_p}{2\pi} = 9 \times 10^{14} \text{ s}^{-1} \);

*plasma wavelength* \( \lambda_p = \frac{c}{\nu_p} = 330 \text{ nm (ultraviolet)} \)

However colour of some metals arises from interband absorption.
For Cu, light in with green and blue wavelengths is absorbed and red light is reflected.
The same for Au.
In case of semiconductors, which are heavily doped the plasma effect also become significant.

In this case when calculating plasma frequency one needs to take into account dielectric function due to motion of both free and bound carries

\[ \varepsilon(\omega) = \varepsilon_\infty (1 - \frac{\omega_p^2}{\omega^2}); \quad \omega_p^2 = \frac{n_e e^2}{m_e^* \varepsilon_0 \varepsilon_\infty} \]

\( \varepsilon_\infty \) - dielectric constant due to motion of bound charges (positive ion core background)

**Example for doped semiconductor**

*electron concentration* \( n_e = 4 \times 10^{22} \, m^{-3} \);

*electron effective mass* \( m_e^* = 0.02 m_0 \) (\( m_0 \) - free electron mass)

*plasma angular frequency* \( \omega_p = 1.14 \times 10^{13} \, rad/sec \);

*plasma wavelength* \( \lambda_p \sim 10 \, \mu m \)
Longitudinal Optical modes in Plasma. Plasmons.

Longitudinal modes, where electric field vector is parallel to k-vector cannot be excited by transverse electromagnetic radiation.

Plasmon is a quantum of longitudinal plasma oscillation

In the long wavelength limit of longitudinal plasma oscillations energy of plasmon is given by plasma frequency $\hbar \omega_p$

To observe plasmons, one measures energy loss spectra for electron reflected from metallic films

$$E_{\text{incident \ electron}} = E_{\text{incident \ electron energy}}$$

$$E_{\text{reflected \ electron}} = E_{\text{reflected \ electron energy}} + \hbar \omega_p$$
Figure 8  Energy loss spectra for electrons reflected from films of (a) aluminum and (b) magnesium, for primary electron energies of 2020 eV. The 12 loss peaks observed in Al are made up of combinations of 10.3 and 15.3 eV losses, where the 10.3 eV loss is due to surface plasmons and the 15.3 eV loss is due to volume plasmons. The ten loss peaks observed in Mg are made up of combinations of 7.1 eV surface plasmons and 10.6 eV volume plasmons. Surface plasmons are the subject of Problem 1. (After C. J. Powell and J. B. Swan.)
Summary.

- Transverse optical modes in a plasma.
- Reflection and propagation of transverse mode
- Role of plasma frequency
- Longitudinal modes in a plasma.
- Plasmons.
- Experiments to reveal plasmons

Topic 17 Amorphous Semiconductors

1. No long range order, but local bonding still exists

2. Band edges strongly broadened

3. Electrons scattered by disorder-localised states near band edges

4. Defect states in gap due to dangling bonds

5. Dangling bond states can be removed by hydrogen. Allows material to be doped. Permits applications

6. Growth is fast nonequilibrium process-atoms don’t have time to reach equilibrium positions

7. Contrast with growth of crystals which is a “slow” equilibrium process
Amorphous semiconductors

Local bonding present as in crystalline materials

No long range order. Removes translational symmetry.

Prepared as thin films by evaporation or sputtering

Atom do not have time to achieve crystalline order before positions become fixed on surface
Atoms are knocked out of target by high energy ions. Are then deposited on substrate.
As a result of lack of translational symmetry, momentum selection rule ($\Delta k=0$) is relaxed.

Absorption edges broadened but bands and band gaps still exist—determined in part by local bonding.

Electrons strongly scattered by disorder—> localised states near band edges because disorder is very strong.

In addition there are many unsatisfied dangling bonds—> large density of states in the middle of gap.
Figure 7 Density of electron states as believed to occur in amorphous solids, when states are non-localized in the center of the band. Localized states are shown shaded. The mobility band edges $E_n$, $E'$ separate the ranges of energy where states are localized and non-localized. (After N. Mott and E. A. Davis.)

Fig. 10.16 In a fully coordinated but distorted tetrahedral semiconductor like Si, the disorder causes a spread of the energy levels into the region of the energy gap of the perfect crystal. These so-called tail states arise from the distorted bonds and are considered to be localized and non-conducting.

Fig. 10.17 In practice an amorphous tetrahedral semiconductor contains imperfectly coordinated atoms, leading to uncompensated or 'dangling' bonds. Each such bond produces a localized electron and a localized empty state. On account of their quasi-atomic character, these states are concentrated about the middle of the energy gap; they are sufficiently numerous to lock the Fermi level at $\frac{1}{2}E_g$. This has the consequence that the amorphous semiconductor becomes insensitive to doping.
Density of defect states is very high, up to $10^{25}\text{ m}^{-3}$

Compensates donor or acceptor doping and therefore doping is not effective

However if amorphous Si is grown from SiH$_4$, dangling bonds are satisfied by H ("neutralises" defect states) and doping is effective

Application – large area of displays, solar cells
Very cheap to produce

TFT – thin film transistors in laptops
+ solar cells
Topic 18 Quantum wells

1. Potential wells created by growth of thin layers of crystalline semiconductors

2. Quantum confinement of electron and hole states

3. New bad gap result

4. Many applications as light emitters
Quantum wells (crystalline)

Thin layer of crystalline semiconductor of thickness < de Broglie wavelength of electrons (200 A, 50-100 A typical) leads to quantisation of motion in Z direction.

The layer consists of 20-40 atomic layers only

In 3D electron kinetic energy

\[ E = \frac{\hbar^2}{2m} (k_x^2 + k_y^2 + k_z^2) \]

In case of electron motion in a thin layer z-motion is quantised=>

\[ E = \frac{\hbar^2}{2m} (k_x^2 + k_y^2) + E_n(z) \]

Motion in XY plane of the thin layer is unaffected
Alternating layer of smaller (GaAs) and larger (Al$_x$ Ga$_{1-x}$As) band gap materials.

Creates potential well for electrons and holes.
Typical Quantum well

Potential well formed for electrons and holes

\[ \text{AlGaAs} \quad | \quad \text{GaAs} \quad | \quad \text{AlGaAs} \]

\[ \text{well} \quad \text{barrier} \]

\[ \longleftrightarrow \quad 5\text{-}10\text{nm} \]

\[ \text{AlGaAs} \quad \text{AlGaAs} \quad \text{GaAs} \]

\[ E_g \quad E'_g \]

\[ E_g \quad E'_g \]

\[ \text{AlGaAs} \quad \text{AlGaAs} \quad \text{GaAs} \]

\( E_g \) – energy gap of bulk GaAs material.
In case of quantum well above onset of optical absorption occurs at \( E'_g \), because the band gap is increased due to quantised motion in Z direction.
PARTICLE IN A BOX

ONE DIMENSIONAL CASE: $V_0 = \infty$

$$- \left( \frac{\hbar^2}{2m} \right) d^2 \psi / d z^2 = E \psi$$

$$E_n = \frac{\hbar^2}{2m} \left( \frac{n \pi}{L_z} \right)^2 \quad n = 1, 2, 3 \ldots$$

$$\psi_n = A \sin \left( \frac{n \pi z}{L_z} \right)$$

Fig. 5. Infinitely deep quantum-well energy levels and wave functions. (Reprinted with permission from Friedr. Vieweg & Sohn Verlagsgesellschaft mbH, R. Dingle, Festkoerperprobleme 15, 21 (1975)).
Fig. 6. First two bound energy levels and wave functions in a finite quantum well.
Applications

Laser, Light emitting diodes (LED), detectors

Design of heterostructures based on quantum wells determines wavelength of emitted light:

1.5 μm for fibre optics communications

0.6 μm for DVDs

0.4 μm for Blue-Ray

etc, etc……

VCSEL (Vertical Cavity Surface Emitting Laser)
Quantum cascade lasers based on multiple quantum wells

Laser sources in mid infrared.
Fig. 88. Schematics of a double heterostructure and the associated profiles of band structure, index of refraction, guided optical wave.