Lecture 16: Spectroscopy using diffraction gratings

Lecture aims to explain:

1. Diffraction for oblique incidence

2. Diffraction pattern and spectroscopy using blazed reflecting grooves

3. Diffraction grating

4. Grating spectrometers
Diffracton for oblique incidence
Oblique incidence, the single slit case

\[ \Delta \text{phase}_{\text{total}} = k \Delta z \sin \theta - k \Delta z \sin \theta_i \]

\( z \) - coordinate along the width of the slit

For single slit diffraction we will change

\[ \beta \equiv \frac{(kb/2)}{\sin \theta} \]

\[ \text{to} \quad \beta \equiv \frac{(kb/2)}{(\sin \theta - \sin \theta_i)} \]

Same formula for the diffracted intensity

\[ I(\theta) = 4I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \]

Main maximum at

\[ (kb/2)(\sin \theta - \sin \theta_i) = 0 \]
Oblique incidence, many slits

Phase shift between adjacent slits changes from

\[ \alpha \equiv (ka/2) \sin \theta \]

...to

\[ \alpha \equiv (ka/2)(\sin \theta - \sin \theta_i) \]

Same formula for the diffracted intensity

\[ I(\theta) = I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \left( \frac{\sin N\alpha}{\sin \alpha} \right)^2 \]

Maxima are given by the condition:

\[ \alpha(\sin \theta_m - \sin \theta_i) = m\lambda \]
Diffraction pattern using blazed reflecting grooves
Independent control of interference and diffraction terms?

We could try to direct most of the diffracted intensity in the maximum with \( m \neq 0 \) where the spectral components of the incident light could be separated.

Central maximum does not give spectral resolution, better resolution in higher orders cannot be accessed.

\[
I(\theta) = I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \left( \frac{\sin N\alpha}{\sin \alpha} \right)^2
\]
**Diffraction with blazed reflective grooves**

The “single-slit” diffraction maximum will be determined by the specular direction and is defined by the blaze angle $\gamma$

$$\beta \equiv \frac{kb}{2} [\sin(\theta + \gamma) - \sin(\theta_i - \gamma)]$$

The phase difference from different “slits” (grooves) is independent of the blaze angle and is still determined by the direction of incidence:

$$\alpha \equiv (ka / 2)(\sin \theta - \sin \theta_i)$$

Introducing additional phase shifts using blazed grooves gives us independent control of the diffraction and interference terms.
Using **blazed grooves** the signal in $m \neq 0$ maximum can be strongly enhanced.

Graph shows diffraction pattern for $\theta_i = 0$ and $\gamma = \pi/15$. 

Spectroscopy with blazed grooves
American astronomer David Rittenhouse invented the first man-made diffraction grating in 1785. Fraunhofer also developed a diffraction grating in 1821, which occurred after James Gregory discovered the principles of diffraction grating.
Diffraction grating

A repetitive array of diffracting elements (either apertures or obstacles) that has the effect of producing periodic alterations in the phase, amplitude or both, of an emergent wave is said to be a **diffraction grating**

**Main application of diffraction gratings is for spectroscopy:** to disperse light with a mixture of wavelengths so that its intensity can be measured as a function of wavelength

$$a(sin \theta_m - sin \theta_i ) = m\lambda$$
Ruled diffraction gratings are produced by physically forming grooves (density from 20 to 1800 gr/mm) into a reflective surface with a diamond mounted on a ruling engine. Diffraction gratings can be ruled on a variety of substrates; for example, glass, metal and ceramic. This is the preferred type of grating for the infrared (IR) because of the relatively large spaces between grooves. Most of the gratings used in spectrometers are plastic replicas of a high quality metal or glass “master” grating.

Holographic diffraction gratings are formed when a series of interference fringes, corresponding to the grooves of the desired grating, are recorded on a photosensitive layer, and the subsequent chemical treatment forms a modulated profile on the surface of the blank by selective dissolution. Holographic gratings are the type of choice for the visible and UV.
Grating spectrometers

In 1814, Fraunhofer invented the spectroscope, and discovered 574 dark lines appearing in the solar spectrum. These were later shown to be atomic absorption lines, as explained by Kirchhoff and Bunsen in 1859. These lines are still called Fraunhofer lines in his honour.
Example 16.1: grating spectrometer

Light is diffracted by a 5 cm x 5 cm diffraction grating having 100 slits/mm. Light is then focussed by a lens (or spherical mirror) on a detector array. Find an optimum size of such spectrometer with a maximum possible free spectral range if a typical size of the detector pixel is 20 microns.
More examples and applications can be found at:

www.oceanoptics.com/

www.jobinyvon.com/

www.princetoninstruments.com/products/spec/actonseries/
Maxima in the far-field diffraction pattern for the multiple slit system in the case of oblique light incidence at an angle $\theta_i$ are given by

$$a(sin \theta_m - sin \theta_i) = m\lambda$$

The whole diffraction pattern is still described by the same expression:

$$I(\theta) = I_0 \left( \frac{sin \beta}{\beta} \right)^2 \left( \frac{sin N\alpha}{sin \alpha} \right)^2$$

but with $\beta \equiv (kb / 2)(sin \theta - sin \theta_i)$ and $\alpha \equiv (ka / 2)(sin \theta - sin \theta_i)$

A repetitive array of diffracting elements that has the effect of producing periodic alterations in the phase, amplitude or both, of an emergent wave is said to be a *diffraction grating*. Main application is for *spectroscopy*.

Grating spectrometers are instruments incorporating diffraction gratings. They are used for spectroscopy, have high spectral resolution and large free spectral range.