

For $\sigma \notin \Sigma_{o1}$ and $\sigma \in \Sigma_{o2}$

$$\delta^{\mathcal{M}}((x_1, x_2, x_3, x_4), \sigma) = \begin{cases} (\delta^H(x_1, \sigma), x_2, x_3, x_4) \\ (x_1, \delta^H(x_2, \sigma), \delta^H(x_3, \sigma), \delta^G(x_4, \sigma)) \\ (\delta^H(x_1, \sigma), \delta^H(x_2, \sigma), \delta^H(x_3, \sigma), \delta^G(x_4, \sigma)) \\ d, \end{cases} \text{ if } (*)$$

For $\sigma \in \Sigma_{o1}$ and $\sigma \notin \Sigma_{o2}$

$$\delta^{\mathcal{M}}((x_1, x_2, x_3, x_4), \sigma) = \begin{cases} (x_1, \delta^H(x_2, \sigma), x_3, x_4) \\ (\delta^H(x_1, \sigma), x_2, \delta^H(x_3, \sigma), \delta^G(x_4, \sigma)) \\ (\delta^H(x_1, \sigma), \delta^H(x_2, \sigma), \delta^H(x_3, \sigma), \delta^G(x_4, \sigma)) \\ d, \end{cases} \text{ if } (*)$$

For $\sigma \in \Sigma_{o1}$ and $\sigma \in \Sigma_{o2}$

$$\delta^{\mathcal{M}}((x_1, x_2, x_3, x_4), \sigma) = \begin{cases} (\delta^H(x_1, \sigma), \delta^H(x_2, \sigma), \delta^H(x_3, \sigma), \delta^G(x_4, \sigma)) \\ d, \end{cases} \text{ if } (*).$$

For $\sigma \in \Sigma$, $\delta^{\mathcal{M}}(d, \sigma)$ is undefined.

The state d is reachable from the initial state in \mathcal{M} if and only if $\mathcal{L}(H)$ is co-observable with respect to $\mathcal{L}(G)$, Σ_{o1} , Σ_{o2} , Σ_{c1} , and Σ_{c2} .

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2DOF Controller Parametrization for Systems With a Single I/O Delay

Leonid Mirkin and Qing-Chang Zhong

Abstract—This note puts forward a parametrization of all stabilizing two-degrees-of-freedom controllers for (possibly unstable) processes with dead-time. The proposed parametrization is based on a doubly coprime factorization of the plant and takes the form of a generalized Smith predictor (dead-time compensator) feedback part and a finite-dimensional feedforward part (prefilter). Some alternative dead-time compensation schemes and disturbance attenuation limitations are also discussed.

Index Terms—Dead-time compensators, disturbance rejection, set-point tracking, time-delay systems.

I. INTRODUCTION

In many applications, control objectives include both disturbance (load) attenuation and reference (command or set-point) tracking. It is well known [1] that these objectives can be handled independently by the use of so-called two-degrees-of-freedom (2DOF) controller configurations. The idea is to use the feedback component of the controller to attenuate *unmeasured* disturbances (as well as to cope with modeling uncertainty) and the feedforward part of the controller (prefilter) to obtain the desired response to *measured* command signals.

Yet the classical 2DOF configuration of [1] does not completely separate feedback and feedforward characteristics. Although the prefilter in such a configuration obviously does not affect the "feedback properties," the choice of the feedback part does affect the command response. A "true" 2DOF configuration was proposed in 1985 in [2], [3]. These works showed that the set of all stabilizing controllers can be parametrized via two independent parameters affecting only the responses to measured and unmeasured inputs, respectively, thus separating feedforward and feedback characteristics. For finite-dimensional systems, such a parametrization, which is based upon the doubly coprime factorization of the plant, is constructive. Conceptually, the same parametrization can in most cases be applied to various infinite-dimensional systems, including systems with dead time, analogous to the finite-dimensional case, see, e.g., [4]. Yet in practice this might not always be straightforward. The obstacle lies in the difficulty in obtaining an implementable and understandable doubly coprime factorization of the infinite-dimensional plant.

For dead-time systems, i.e., systems with loop delay, the difficulty described above resulted in a situation where virtually none of the available dead-time compensation schemes fully exploits the 2DOF configuration, see [5], [6] and the references therein. In some recent papers [7]–[11], the classical 2DOF configuration in which a stable prefilter enters the loop before the feedback part of the controller (generalized Smith predictor) was studied. Yet, as mentioned above, such a configuration does not achieve complete separation between closed- and open-loop properties. A notable exception is paper [12], where a truly 2DOF dead-time compensation scheme is proposed. This scheme includes a sophisticated feedforward part, which enters the feedback loop

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twice, before and after the feedback controller. The scheme in [12], however, is custom built for processes consisting of an integrator and dead-time and not readily applicable to other systems.

In this note, we derive a 2DOF parametrization of the set of all stabilizing controllers for systems with a single I/O delay. The parametrization is based on a doubly coprime factorization of the plant. The main contribution of this note lies in the fact that the resulting parametrization has the form of a generalized Smith predictor (dead-time compensator), so that it can be easily understood and implemented. Compared with the scheme of [12], the proposed configuration has two advantages. First, it is applicable to general dead-time processes rather than to an integrator only. Second, the feedforward part of the proposed parametrization is finite dimensional, whereas in the parametrization of [12] it contains a delay. The latter advantage is achieved by entering the prefilter “in the middle” of the feedback controller. To the best of our knowledge, this has not yet been exploited in the delay literature.

The note is organized as follows. In Section II preliminary results are collected for the doubly coprime factorization of DT systems. The parametrization itself is presented in Section III and some of its variations are discussed in Section IV. Finally, Section V addresses some limitations imposed on the disturbance attenuation by the delay.

II. PRELIMINARIES: COPRIME FACTORIZATION OF DT SYSTEMS

Since the construction of the doubly coprime factorizations plays a key role in the theory of 2DOF control systems, we start with a brief review of some recent results, mainly from [13], [14], on the doubly coprime factorization of the following systems with a single I/O delay:

$$P(s) = P_r(s)e^{-hs} \quad (1)$$

where P_r is a proper rational transfer matrix and h is the dead-time. The problem is to find stable (i.e., belonging to H^∞) transfer matrices $N, M, \tilde{N}, \tilde{M}, X, Y, \tilde{X},$ and \tilde{Y} satisfying the Bézout identity

$$\begin{bmatrix} \tilde{X} & \tilde{Y} \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & -Y \\ N & X \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \quad (2)$$

and such that $P = NM^{-1} = \tilde{M}^{-1}\tilde{N}$.

In principle, “ N ” and “ M ” parts can easily be derived from the right and left coprime factorizations of the rational part of P . Indeed, let

$$P_r = N_r M_r^{-1} = \tilde{M}_r^{-1} \tilde{N}_r \quad (3)$$

be (right and left) coprime factorizations of P_r . It can then be shown (see, e.g., [14, Lemma 3.2]) that $N = N_r e^{-hs}$ and $M = M_r$ are right coprime, $\tilde{N} = \tilde{N}_r e^{-hs}$ and $\tilde{M} = \tilde{M}_r$ are left coprime, and, moreover, they constitute strongly coprime factorizations [15] of P . The generation of the “ X ” and “ Y ” parts is less evident, though the proof of Lemma 3.2 in [14] is constructive. The Lemma below, which is essentially from [13], gives a method for generating these transfer matrices (to make the presentation self-contained, the proof is also given):

Lemma 1: Let P_a be a rational transfer matrix so that

$$\Pi(s) \doteq P_a(s) - P_r(s)e^{-hs} \in H^\infty \quad (4)$$

and let $N_a, M_a, \tilde{N}_a, \tilde{M}_a, X_a, Y_a, \tilde{X}_a,$ and \tilde{Y}_a constitute the doubly coprime factorization of P_a . Then

$$\begin{bmatrix} M & -Y \\ N & X \end{bmatrix} = \begin{bmatrix} I & 0 \\ -\Pi & I \end{bmatrix} \begin{bmatrix} M_a & -Y_a \\ N_a & X_a \end{bmatrix} \quad (5a)$$

and

$$\begin{bmatrix} \tilde{X} & \tilde{Y} \\ -\tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} \tilde{X}_a & \tilde{Y}_a \\ -\tilde{N}_a & \tilde{M}_a \end{bmatrix} \begin{bmatrix} I & 0 \\ \Pi & I \end{bmatrix} \quad (5b)$$

constitute a doubly coprime factorization of P .

Proof: Since $\Pi \in H^\infty$, the right-hand sides of (5) also belong to H^∞ . Equality (2) is verified by the direct substitution. Now, to complete the proof one only needs to show that $NM^{-1} = P = \tilde{M}^{-1}\tilde{N}$. The first equality follows from

$$NM^{-1} = (N_a - \Pi M_a)M_a^{-1} = P_a - \Pi = P$$

and the second equality is verified similarly. ■

Lemma 1 enables one to express a doubly coprime factorization of the DT system P in terms of that of a rational transfer matrix P_a which is only constrained to satisfy (4). Such a P_a can always be found, though its choice is nonunique. Possible choices for P_a will be discussed in Section IV. Here we just note that, as one could expect, P_r and P_a always share their H^∞ denominators in both left and right coprime factorizations [14].

Lemma 2: Let P_a be any transfer matrix satisfying (4) and let P_r be coprimely factorized as in (3). Then:

$$P_a = (P_a M_r)M_r^{-1} = \tilde{M}_r^{-1}(\tilde{M}_r P_a)$$

are coprime factorizations of P_a . Moreover, in this case Lemma 1 yields that $M = M_r$, $N = N_r e^{-sh}$, $\tilde{M} = \tilde{M}_r$, and $\tilde{N} = \tilde{N}_r e^{-sh}$.

Proof: We show the result for the right coprime factorization only; the left coprime case follows by similar arguments.

As follows from (4), $P_a M_r = \Pi M_r + N_r e^{-sh}$, so it is stable. Then $P_a M_r$ and M_r are right coprime if

$$\begin{bmatrix} M_r \\ P_a M_r \end{bmatrix} = \begin{bmatrix} I & 0 \\ \Pi & e^{-sh} I \end{bmatrix} \begin{bmatrix} M_r \\ N_r \end{bmatrix}$$

has full column rank in $\mathbb{C}^+ \cup \infty$. Yet this follows by the right coprimeness of N_r and M_r and the invertibility of $M_r(\infty)$. The results for M and N are now followed by direct substitution. ■

III. PARAMETRIZATION OF 2DOF CONTROLLERS

Now we are in position to derive the 2DOF parametrization of all stabilizing controllers for the DT process (1).

It is known [15] that there exists a stabilizing controller C for a plant P iff the latter has a strongly coprime factorization in H^∞ and then C (internally) stabilizes P iff $C = -(\tilde{X} + Q\tilde{N})^{-1}(\tilde{Y} - Q\tilde{M})$ for some $Q \in H^\infty$. Moreover, if a (stable) open-loop transfer matrix (prefilter) K is added as shown in Fig. 1, then [2], [3] the parametrization has two degrees of freedom in the sense that the choice of K does not affect the closed-loop characteristics (disturbance attenuation, robustness, etc) and the choice of Q does not affect the nominal command response characteristics. In particular, the closed-loop transfer matrices from r to y and u are

$$\begin{bmatrix} G_{yr} \\ G_{ur} \end{bmatrix} = \begin{bmatrix} N \\ M \end{bmatrix} K \quad (6a)$$

and the transfer matrices from d to y and u are

$$\begin{bmatrix} G_{yd} \\ G_{ud} \end{bmatrix} = \left(\begin{bmatrix} X \\ -Y \end{bmatrix} + \begin{bmatrix} N \\ M \end{bmatrix} Q \right) \tilde{N}. \quad (6b)$$

Theorem 1 provides a DT version of this result.

Theorem 1: Consider the DT process(1). For any rational P_a satisfying(4), the set of all 2DOF stabilizing controllers can be presented in the form depicted in Fig. 2, i.e.,

$$u = (\tilde{X} + Q\tilde{N})^{-1} \left(K r + (\tilde{Y} - Q\tilde{M})(y + \Pi u) \right)$$

where $\tilde{N}_a, \tilde{M}_a, \tilde{X}_a,$ and \tilde{Y}_a are taken from the doubly coprime factorization of P_a and Q and K are arbitrary stable transfer matrices. Moreover, the closed-loop transfer matrices are then

$$\begin{bmatrix} G_{yr} \\ G_{ur} \end{bmatrix} = \begin{bmatrix} N_r e^{-hs} \\ M_r \end{bmatrix} K \quad (7a)$$

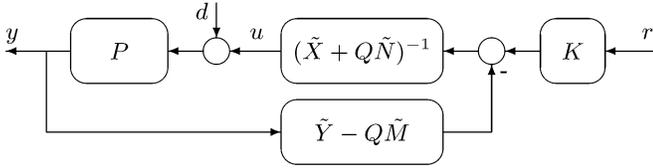


Fig. 1. Standard 2DOF controller parametrization.

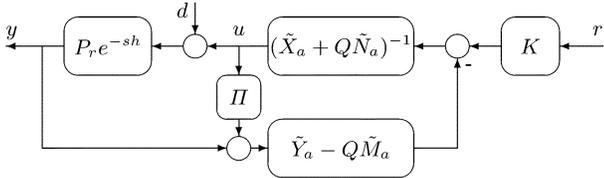


Fig. 2. 2DOF controller parametrization for DT systems.

and

$$\begin{bmatrix} G_{yd} \\ G_{ud} \end{bmatrix} = \left(\begin{bmatrix} X_a + \Pi Y_a \\ -Y_a \end{bmatrix} + \begin{bmatrix} N_r e^{-sh} \\ M_r \end{bmatrix} Q \right) \tilde{N}_r e^{-sh} \quad (7b)$$

where N_r , M_r , and \tilde{N}_r are the coprime factorization as in (3) satisfying $M_r = M_a$ and $\tilde{M}_r = \tilde{M}_a$.

Proof: Assume now that the doubly coprime factorization of P in (1) is chosen according to Lemma 1. Then, for every Q

$$-\tilde{Y} + Q\tilde{M} = -\tilde{Y}_a + Q\tilde{M}_a$$

and

$$\tilde{X} + Q\tilde{N} = \tilde{X}_a + Q\tilde{N}_a - (-\tilde{Y}_a + Q\tilde{M}_a)\Pi$$

from which the configuration in Fig. 2 can be obtained.

Formulae (7) then follow by substituting expressions (5) to (6) and using Lemma 2. ■

The advantage of the configuration in Fig. 2 stems from the fact that all the components of the controller except the internal feedback Π may be finite dimensional.¹ Π can be thought of as a generalized Smith predictor or *dead-time compensator* (DTC). With this rationale in mind, we shall refer to the (possibly) rational component of the feedback part of the controller, i.e., $(\tilde{X}_a + Q\tilde{N}_a)^{-1}(\tilde{Y}_a - Q\tilde{M}_a)$, as the *primary part of DTC*.

Remark 1: It can be shown that in the 1DOF context (i.e., if $K = 0$) the parametrization in Fig. 2 coincides with that in [16], which was derived using different arguments.

Remark 2: The previous remark implies that the robustness analysis of the system in Fig. 2 against (unstructured and even structured) additive and multiplicative uncertainty in P_r can be completely reduced to that for the delay-free plant P_a , see [14], [16], [17]. For example, the complementary sensitivity function for the system in Fig. 2 is

$$T = N(-\tilde{Y} + Q\tilde{M}) = N_r(-\tilde{Y}_a + Q\tilde{M}_a)e^{-hs}.$$

Since the delay transfer function is all-pass, it does not affect $|T(j\omega)|$ and, hence, it also does not affect the robustness radius against multiplicative uncertainty.

IV. CHOICE OF P_a

When P_r is stable, P_a can be any stable transfer matrix. Natural choices are then $P_a = P_r$ and $P_a = 0$. The former yields

$$\Pi = \Pi_{\text{SP}} \doteq P_r - P_r e^{-sh}$$

¹Provided Q and K are finite dimensional. Yet these assumptions may be quite natural as Q and K are designed for the finite-dimensional plant P_a .

which brings the feedback part of the structure in Fig. 2 to the classical Smith predictor [18] form. The latter choice yields

$$\Pi = \Pi_{\text{IMC}} \doteq -P_r e^{-sh}$$

which produces the IMC controller structure [19]. In both cases, one can choose $X_a = I$ and $Y_a = 0$, resulting in the following closed-loop transfer matrices:

$$\begin{bmatrix} G_{yd} \\ G_{ud} \end{bmatrix} = \left(\begin{bmatrix} I \\ 0 \end{bmatrix} + \begin{bmatrix} P_r e^{-sh} \\ I \end{bmatrix} Q \right) P_r e^{-sh}.$$

When P_r is unstable, the choice of P_a is more restrictive. The authors of [14], [17] proposed to use $P_a = P_r F_r$, where F_r is rational and such that

$$\Pi = \Pi_{\text{MSP}} \doteq P_r (F_r - e^{-hs} I)$$

is stable (modified Smith predictor). Such an F_r can be found through interpolation. Yet for systems with more than one unstable pole, this procedure increases the order of P_a and, consequently, the complexity of the doubly coprime factorization of P .

Alternatively, the choice proposed in [13] and [16] (see also [6]) can be used. The idea there is to choose P_a by the completion of the impulse response of P to the whole positive semi-axis, which results in an FIR (finite impulse response) Π . More precisely, bring in a state-space realization of the delay-free part of (1)

$$P_r = \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array} \right] = C(sI - A)^{-1}B$$

where (A, B) is stabilizable and (C, A) is detectable. Then, P_a (having the same order as P_r) can be chosen as

$$P_a = \left[\begin{array}{c|c} A & B \\ \hline C e^{-Ah} & 0 \end{array} \right] = C e^{-Ah} (sI - A)^{-1} B \quad (8)$$

so that

$$\Pi = \Pi_{\text{FIR}} \doteq C e^{-Ah} (I - e^{-h(sI - A)})(sI - A)^{-1} B \quad (9)$$

is an entire function of s . Actually, it can be verified that Π_{FIR} has a finite impulse response with support in $[0, h]$, which implies that it is stable² irrespective of the poles of P_r .

With this choice of Π , the doubly coprime factorization of P_a can be easily performed in the state space. To this end, let F and L be any matrices of appropriate dimensions such that $A + BF$ and $A + LC$ are Hurwitz (such F and L exist by the stabilizability and detectability assumptions). Then, using the standard state-space formulas of [21] with the gains F and $e^{Ah}L$ and the fact that A and e^{Ah} commute, we have

$$\begin{bmatrix} M_a & -Y_a \\ N_a & X_a \end{bmatrix} = \left[\begin{array}{c|cc} A + BF & B & -e^{Ah}L \\ \hline F & I & 0 \\ C e^{-Ah} & 0 & I \end{array} \right] \quad (10a)$$

and

$$\begin{bmatrix} \tilde{X}_a & \tilde{Y}_a \\ -\tilde{N}_a & \tilde{M}_a \end{bmatrix} = \left[\begin{array}{c|cc} A + LC & -e^{-Ah}B & L \\ \hline F e^{Ah} & I & 0 \\ C & 0 & I \end{array} \right]. \quad (10b)$$

²Note that such a Π (at least, its part containing unstable poles of P_r) must be implemented as a finite-memory integration rather than the difference $P_a - P_r e^{-sh}$, see [5] and [6]. It is also worth stressing, that although Π_{FIR} is infinite dimensional, it can be safely implemented; see [20].

A. Integral Action in the Controller

One potential inconvenience in the design with Π_{FIR} in (9) is that its presence might alter the low frequency gain of the primary part of DTC. In particular, since $\Pi_{\text{FIR}}(0) \neq 0$ in general, an integral action is not preserved when implemented in the DTC form. This is unlike the Smith predictor case, where $\Pi_{\text{SP}}(0) = 0$ and therefore the static gain of the primary part of DTC is kept unchanged.

To overcome this problem, one may impose an additional constraint upon the DTC block, namely $\Pi(0) = 0$. Apparently, the simplest way to ensure this is to add a static gain to P_a so that $\Pi(0) = 0$. For the FIR block this modification would result in the following P_a :

$$P_a = -C \int_0^h e^{-A\eta} d\eta B + C e^{-Ah} (sI - A)^{-1} B. \quad (11)$$

As a matter of fact, the Π producing this P_a is similar to the prediction block in [6], where it is derived from plant augmentation arguments. Our arguments, however, appear to be somewhat more natural and lead to an interesting extension discussed later.

A potential drawback of the choice in (11) is that the resulting P_a is no longer strictly proper. This might give rise to some well-posedness problems in the design of the primary part of DTC (which is designed for P_a). Yet this problem can be easily resolved by adding strictly proper dynamics to the “static gain corrector,” i.e., as follows:

$$P_a = -C \int_0^h e^{-A\eta} d\eta B \frac{1}{\tau s + 1} + C e^{-Ah} (sI - A)^{-1} B$$

for some $\tau > 0$. Moreover, if τ can be chosen equal to one of the P_r stable time constants, then the resulting P_a is of the same order as P_r .

This can be illustrated by the following simple example. Consider the unstable plant

$$P = \frac{e^{-sh}}{s(\tau s + 1)} = \left(\frac{1}{s} - \frac{\tau}{\tau s + 1} \right) e^{-sh}. \quad (12)$$

It can be easily verified that the application of (8) results in

$$P_a = \frac{1}{s} - \frac{\tau e^{h/\tau}}{\tau s + 1}.$$

This leads to $\Pi(0) = \Pi_{\text{FIR}}(0) = h - \tau(e^{h/\tau} - 1) \neq 0$ for all $h > 0$. Yet if this P_a is modified by adding the *stable* first-order transfer function with the time constant τ (in order to keep the order of P_a) and the gain $\tau(e^{h/\tau} - 1) - h$, then the resulting P_a

$$P_a = \frac{1}{s} - \frac{\tau + h}{\tau s + 1} = \frac{1 - hs}{s(\tau s + 1)} \quad (13)$$

guarantees that Π is stable and $\Pi(0) = 0$. Therefore, an integral action in the primary part of DTC is preserved. It is worth stressing that the resulting

$$\Pi = \frac{1 - e^{-sh}}{s} + \frac{1 - sh + \tau e^{-sh}}{\tau s + 1}$$

is no longer FIR. Its “unstable” part (the first term previously mentioned), however, is still in the FIR form, while the second term contains only stable dynamics and can thus be easily implemented.

Note that P_a in (13) can be thought of as a rational approximation of P in (12). This suggests that the rationale behind the choice of P_a may be to seek a “good” approximation of $P_r e^{-sh}$ in a required frequency range, typically in the low frequencies range up to the required crossover frequency, under constraint (4). Such an approach potentially has the following two advantages.

- 1) Arguably, the underlying idea behind the use of DTC is to reduce the controller design for the (infinite-dimensional) plant P to that for the (finite-dimensional) auxiliary plant P_a . This replacement always preserves closed-loop stability yet might not preserve

closed-loop performance. Indeed, the designed loop gain $L_a = P_a K_a$ is different from the actual loop gain $L = P K_a (I - \Pi K_a)^{-1}$ (here K_a stands for the primary part of the DTC). It can be shown that

$$S = (I - \Pi K_a) S_a$$

where $S_a \doteq (I - L_a)^{-1}$ and $S \doteq (I - L)^{-1}$ are the designed and actual sensitivity functions, respectively. Hence, “small” Π might be crucial to preserve small sensitivity function (cf. the discussion in [19, Sec. 6.2 2]).

- 2) Smith predictor (DTC) schemes are sometimes regarded as non-robust, especially with respect to uncertain dead time. We believe that the reason lies in the delay-free nature of the design of the primary controller: in the delay-free design one might be tempted to choose a controller that is too aggressive, e.g., a controller with too high bandwidth. In other words, design limitations imposed by the delay do not show up explicitly in the design of the primary controller. Yet DTC schemes do not enable one to circumvent inherent limitations imposed by the delay. Rather, they are just intended to simplify the design procedure. From this point of view, the choice of P_a as an approximation of $P_r e^{-sh}$ (even when P_r is stable) might be of value as this might capture at least some of the delay limitations, thus preventing excessively aggressive design of the primary controller.

A thorough study of these issues goes beyond the scope of this note and is, therefore, not further pursued here.

V. CONSTRAINTS ON THE DISTURBANCE ATTENUATION

In this section, we consider in more detail the properties of the transfer matrix $\hat{G}_{yd} \doteq G_{yd} e^{hs}$ from $\hat{d} \doteq e^{hs} d$ to the output y for the system in Fig. 2.

A. Disturbance Response

From (7a), we have

$$\begin{aligned} \hat{G}_{yd} &= (X_a + \Pi Y_a + N_r Q e^{-hs}) \tilde{N}_r \\ &= (X_a + P_a Y_a - (P_r Y_a - N_r Q) e^{-hs}) \tilde{N}_r \\ &= P_r - (P_r Y_a - N_r Q) \tilde{N}_r e^{-hs}. \end{aligned}$$

One can see that in the interval $[0, h]$ the controller (i.e., Q) does *not* affect the disturbance response. It is then natural to exclude that part of \hat{G}_{yd} from further analysis. To this end, let us introduce the transfer matrix

$$P_h = \left[\begin{array}{c|c} A & B \\ \hline C e^{Ah} & 0 \end{array} \right] = C e^{Ah} (sI - A)^{-1} B.$$

It can be easily verified that the transfer matrix

$$\Delta_h \doteq P_r - P_h e^{-hs} = C(I - e^{-h(sI - A)})(sI - A)^{-1} B$$

is stable (it is FIR). The transfer matrix \hat{G}_{yd} can then be written as

$$\hat{G}_{yd} = \Delta_h + (G_h + N_r Q \tilde{N}_r) e^{-hs} \quad (14)$$

where

$$G_h \doteq P_h - P_r Y_a \tilde{N}_r.$$

Since \hat{G}_{yd} is stable by construction, so is G_h . For example, if Π is chosen to be equal to Π_{FIR} as given by (9), then (10) leads to the following state-space realization of G_h :

$$G_h = \left[\begin{array}{cc|c} A + BF & e^{Ah} LC & 0 \\ 0 & A + LC & B \\ \hline -C & C e^{Ah} & 0 \end{array} \right].$$

It follows from (14) that a norm (i.e., its energy or pick) of y is always bounded from below by the corresponding norm of the signal $y_h \doteq \Delta_h d$ truncated to $[0, h]$. In other words, the latter quantity imposes a strict limitation of the achievable disturbance attenuation performance in any dead-time system regardless the choice of the controller.

It is worth stressing that the design of Q should in general depend not only upon N_r and \tilde{N}_r (which are delay independent) but also upon G_h , which does depend on the delay unless $A = 0$. Moreover, unless³ y_h is zero outside the interval $[0, h]$, the choice of Q should also take Δ_h into account. Therefore, the design of the rational part of the controller in Fig. 2 should in general be *delay dependent*. This point is illustrated below.

B. "Ideal" Attenuation of Step Disturbances

Assume that the plant P is "square" in the sense that its input and output dimensions coincide and that the disturbance is the unit step signal, i.e., $\hat{d}(s) = 1/s$.

Following the aforementioned reasoning, let us consider the response y separately in intervals $[0, h]$ and (h, ∞) . In the first interval, the control input does not react on \hat{d} and hence the response is just the step response of P_r . In the second interval, the contribution of Δ_h to the disturbance response, y_h , is constant

$$y_h(t) = \Delta_h(0) = C \int_0^h e^{A\eta} d\eta B \quad \forall t \in (h, \infty).$$

Hence, the disturbance attenuation problem can be reduced in this case to the pure rational problem of "minimizing" (in whatever sense) the transfer matrix

$$G_\epsilon \doteq \frac{1}{s} \left(\Delta_h(0) + G_h + N_r Q \tilde{N}_r \right).$$

The asymptotically perfect rejection of d in all directions is then equivalent to $G_\epsilon \in H^\infty$ which, in turn, is equivalent to the interpolation condition

$$Q(0) = -N_r(0)^{-1} \left(\Delta_h(0) + G_h(0) \right) \tilde{N}_r(0)^{-1}.$$

This condition actually implies that the integral action is added to the controller. Obviously, such a Q can always be chosen provided that the rational part of the plant, P_r , has no transmission zeros at the origin.

The disturbance response in the presence of delay is thus never zero due to the open-loop behavior in the interval $[0, h]$. The best the controller can achieve is to eliminate the effect of d for $t > h$. This could be done by the choice $Q = Q_*$, where

$$Q_* \doteq -N_r^{-1} \left(\Delta_h(0) + G_h \right) \tilde{N}_r^{-1}.$$

This choice would lead to what we call the *ideal attenuation* of the disturbance signal d . By "ideal" we mean here that the disturbance response is zero whenever the controller has information about the disturbance, i.e., in the interval (h, ∞) . Yet the obvious problem here is that under our assumption that P_r is strictly proper, $Q_* \notin H^\infty$. On the other hand, if P_r is minimum phase, then the almost ideal disturbance attenuation can be achieved by augmenting Q_* with some low-pass filters, for example, by $Q = (1/(\lambda s + 1)^r) Q_*$, where r is chosen to guarantee the properness of Q and $\lambda > 0$ is "small enough."

VI. CONCLUDING REMARKS

In this note the parametrization of all stabilizing 2DOF controllers for DT systems has been derived. The derivation uses a simple form of

³Roughly, if \hat{d} acts in $[0, \infty)$, then y_h might be zero at (h, ∞) only if $\hat{d}(t) = \delta(t)$.

the doubly coprime factorization for DT systems. The closed-loop part of the proposed controller takes the form of the DTC, whose rational part has the same form as in the delay-free case. The proposed 2DOF parametrization enables one to analyze the set-point and disturbance responses separately, without the need to compromise them (as in conventional Smith predictor schemes).

The note has discussed some DTC schemes and suggested a novel approach to the choice of the dead-time compensation block that preserves the low-frequency properties of the rational part of the controller (including the static gain, so that this approach can be thought of as an extension of the ideas of [6]). This conceptually simplifies the design of the rational part of the controller and also suggests that the internal feedback (predictor) block of the DTC should be "small" in the frequency range where the rational part of the controller has high gain.

The proposed parametrization has also been used to analyze the disturbance attenuation properties of DT systems. It has been shown that there always exists a strict constraint on the achievable disturbance attenuation level and that the controller attempting to achieve this level inherently depends on the delay. This fact is somewhat surprising, taking into account the conventional trend to use DTC schemes in order to eliminate delay from the analysis.

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Adaptive Neural Network Control of Nonlinear Systems With Unknown Time Delays

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Abstract—In this note, adaptive neural control is presented for a class of strict-feedback nonlinear systems with unknown time delays. Using appropriate Lyapunov–Krasovskii functionals, the uncertainties of unknown time delays are compensated for such that iterative backstepping design can be carried out. In addition, controller singularity problems are solved by using the integral Lyapunov function and employing practical robust neural network control. The feasibility of neural network approximation of unknown system functions is guaranteed over practical compact sets. It is proved that the proposed systematic backstepping design method is able to guarantee semiglobally uniformly ultimate boundedness of all the signals in the closed-loop system and the tracking error is proven to converge to a small neighborhood of the origin.

Index Terms—Adaptive control, neural networks, nonlinear time-delay systems.

I. INTRODUCTION

Recent years have witnessed great progress in adaptive control of nonlinear systems, which plays an important role due to its ability to compensate for parametric uncertainties. To obtain global stability, some restrictions had to be made to system nonlinearities such as matching conditions [1], extended matching conditions [2], or growth conditions [3]. To overcome these restrictions, recursive and systematic backstepping design was developed in [4]. The over-parametrization problem was then removed in [5] by introducing the concept of tuning function. Several adaptive approaches for nonlinear systems with triangular structures have been proposed in [6], [7]. Robust adaptive backstepping control has been studied for uncertain nonlinear systems with unknown nonlinear functions in [8] and [9], and among others.

Recently, stabilization of nonlinear systems with time delays is receiving much attention [10], [11]. The existence of time delays may make the task more complicated and challenging, especially when the delays are not perfectly known. One way to ensure stability robustness with respect to this uncertainty is to seek for delay-independent solutions. Lyapunov–Krasovskii functionals have been used early as checking criteria for time-delay systems' stability in [12] and [13]. In

[13], the system considered was a class of nonlinear time-delay systems with a so-called "triangular structure", which was later commented that it could not be "constructively obtained" in [14]. The uncertainties from unknown parameters or unknown nonlinear functions are yet to be discussed, especially when the virtual control coefficients are unknown nonlinear functions of states with known signs. With the aid of neural networks parameterization, stable controllers have been constructed in [15] and [16] (to name just a few) and a family of novel integral Lyapunov functions have been developed in [17] and [18] to avoid controller singularity problem commonly encountered in adaptive feedback linearization control.

In this note, we present an adaptive neural controller for a class of strict-feedback nonlinear systems with unknown time delays and unknown virtual control coefficients. Both integral functions and Lyapunov–Krasovskii functionals are used to construct the Lyapunov function candidates such that the controller singularity problem is avoided and the uncertainties from unknown time delays are removed. Semiglobally uniformly ultimate boundedness (SGUUB) of all the signals in the closed-loop system can be guaranteed and the control performance can be improved by appropriately choosing the design parameters. The proposed method expands the class of nonlinear systems that can be handled using adaptive control. The main contributions of the note lie in: 1) the use of integral Lyapunov functions to avoid controller singularity problem commonly encountered in feedback linearization control; 2) the introduction of appropriate Lyapunov–Krasovskii functionals to compensate for the unknown time delays for a time-delay independent controller design; 3) the introduction of practical robust control to avoid possible singularity problem due to the appearance of $1/z_i$ in the controller; and 4) the use of neural networks as function approximators with its feasibility being guaranteed over the practical compact sets for the first time in the literature.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of single-input–single-output (SISO) nonlinear time-delay systems

$$\begin{aligned} \dot{x}_i(t) &= g_i(\bar{x}_i(t))x_{i+1}(t) + f_i(\bar{x}_i(t)) + h_i(\bar{x}_i(t - \tau_i)) \\ 1 &\leq i \leq n - 1 \\ \dot{x}_n(t) &= g_n(\bar{x}_n(t))u + f_n(\bar{x}_n(t)) + h_n(\bar{x}_n(t - \tau_n)) \end{aligned} \quad (1)$$

where $\bar{x}_i = [x_1, x_2, \dots, x_i]^T$, $x = [x_1, x_2, \dots, x_n]^T \in R^n$ and $u \in R$ are the state variables and system input respectively, $g_i(\cdot)$, $f_i(\cdot)$, $h_i(\cdot)$ are unknown smooth functions, and τ_i are unknown time delays of the states, $i = 1, \dots, n$. The control objective is to design an adaptive controller for (1) such that the state $x_1(t)$ follows a desired reference signal $y_d(t)$, while all signals in the closed-loop system are bounded. Define the desired trajectory $\bar{x}_{d(i+1)} = [y_d, \dot{y}_d, \dots, y_d^{(i)}]^T$, $i = 1, \dots, n - 1$, which is a vector of y_d up to its i th time derivative $y_d^{(i)}$. We have the following assumptions for the system's signals, unknown functions and reference signals.

Assumption 2.1: The system states $x(t)$ and their partial time derivatives, $\dot{x}_{n-1}(t)$, are all available for feedback.

Remark 2.1: Note that the requirement for $\dot{x}_{n-1}(t)$ is a constraint but realistic for many physical systems as we are not requiring \dot{x}_n which is directly influenced by the control.

Assumption 2.2: The signs of $g_i(\bar{x}_i)$ are known, and there exist constants g_{i0} and known smooth functions $\bar{g}_i(\bar{x}_i)$ such that $g_{i0} \leq |g_i(\bar{x}_i)| \leq \bar{g}_i(\bar{x}_i)_{\infty}, \forall \bar{x}_i \in R^i$.

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