USE OF STATISTICAL TABLES

Lucy Radford, Jenny V Freeman and Stephen J Walters introduce three important statistical distributions: the standard Normal, *t* and Chi-squared distributions

PREVIOUS TUTORIALS HAVE LOOKED at hypothesis testing[1] and basic statistical tests.[2–4] As part of the process of statistical hypothesis testing, a test statistic is calculated and compared to a hypothesised critical value and this is used to obtain a *P*-value. This *P*-value is then used to decide whether the study results are statistically significant or not. It will explain how statistical tables are used to link test statistics to *P*-values. This tutorial introduces tables for three important statistical distributions (the standard Normal, *t* and Chi-squared distributions) and explains how to use them with the help of some simple examples.

STANDARD NORMAL DISTRIBUTION

The Normal distribution is widely used in statistics and has been discussed in detail previously.[5] As the mean of a Normally distributed variable can take any value (−∞ to ∞) and the standard deviation any positive value (0 to ∞), there are an infinite number of possible Normal distributions. It is therefore not feasible to print tables for each Normal distribution; however it is possible to convert any Normal distribution to the standard Normal distribution, for which tables are available. The standard Normal distribution has a mean of 0 and standard deviation of 1.

Any value *X* from a Normal distribution with mean μ and standard deviation σ can be transformed to the standard Normal distribution using the following formula:

(1) $$z = \frac{X - \mu}{\sigma}$$

This transformed *X*-value, often called *z* or *z*-score, is also known as the standard Normal deviate, or Normal score. If an average, rather than a single value, is used the standard deviation should be divided by the square root of the sample size, *n*, as shown in equation (2).

(2) $$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

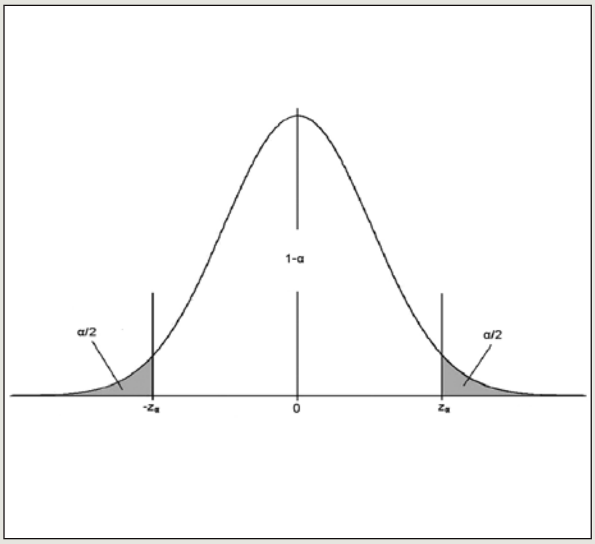

TABLE 1. Extract from two-tailed standard Normal table. Values tabulated are *P*-values corresponding to particular cut-offs and are for *z* values calculated to two decimal places.

TABLE 1

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	1.0000	0.9920	0.9840	0.9761	0.9681	0.9601	0.9522	0.9442	0.9362	0.9283
0.10	0.9203	0.9124	0.9045	0.8966	0.8887	0.8808	0.8729	0.8650	0.8572	0.8493
0.20	0.8415	0.8337	0.8259	0.8181	0.8103	0.8206	0.7949	0.7872	0.7795	0.7718
0.30	0.7642	0.7566	0.7490	0.7414	0.7339	0.7263	0.7188	0.7114	0.7039	0.6965
0.40	0.6892	0.6818	0.6745	0.6672	0.6599	0.6527	0.6455	0.6384	0.6312	0.6241
0.50	0.6171	0.6101	0.6031	0.5961	0.5892	0.5823	0.5755	0.5687	0.5619	0.5552
0.60	0.5485	0.5419	0.5353	0.5287	0.5222	0.5157	0.5093	0.5029	0.4965	0.4902
0.70	0.4839	0.4777	0.4715	0.4654	0.4593	0.4533	0.4473	0.4413	0.4354	0.4295
0.80	0.4237	0.4179	0.4122	0.4065	0.4009	0.3953	0.3898	0.3843	0.3789	0.3735
0.90	0.3681	0.3628	0.3576	0.3524	0.3472	0.3421	0.3371	0.3320	0.3271	0.3222
1.00	0.3173	0.3125	0.3077	0.3030	0.2983	0.2837	0.2891	0.2846	0.2801	0.2757
1.10	0.2713	0.2670	0.2627	0.2585	0.2543	0.2501	0.2460	0.2420	0.2380	0.2340
1.20	0.2301	0.2263	0.2225	0.2187	0.2150	0.2113	0.2077	0.2041	0.2005	0.1971
1.30	0.1936	0.1902	0.1868	0.1835	0.1802	0.1770	0.1738	0.1707	0.1676	0.1645
1.40	0.1615	0.1585	0.1556	0.1527	0.1499	0.1471	0.1443	0.1416	0.1389	0.1362
1.50	0.1336	0.1310	0.1285	0.1260	0.1236	0.1211	0.1188	0.1164	0.1141	0.1118
1.60	0.1096	0.1074	0.1052	0.1031	0.1010	0.0989	0.0969	0.0949	0.0930	0.0910
1.70	0.0891	0.0873	0.0854	0.0836	0.0819	0.0801	0.0784	0.0767	0.0751	0.0735
1.80	0.0719	0.0703	0.0688	0.00672	0.0658	0.0643	0.0629	0.0615	0.0601	0.0588
1.90	0.0574	0.0561	0.0549	0.0536	0.0524	0.0512	0.0500	0.0488	0.0477	0.0466
2.00	0.0455	0.0444	0.0434	0.0424	0.0414	0.0404	0.0394	0.0385	0.0375	0.0366
2.10	0.0357	0.0349	0.0340	0.0332	0.0324	0.0316	0.0308	0.0300	0.0293	0.0285
2.20	0.0278	0.0271	0.0264	0.0257	0.0251	0.0244	0.0238	0.0232	0.0226	0.0220
2.30	0.0214	0.0209	0.0203	0.0198	0.0193	0.0188	0.0183	0.0178	0.0173	0.0168
2.40	0.0164	0.0160	0.0155	0.0151	0.0147	0.0143	0.0139	0.0135	0.0131	0.0128
2.50	0.0124	0.0121	0.0117	0.0114	0.0111	0.0108	0.0105	0.0102	0.0099	0.0096
2.60	0.0093	0.0091	0.0088	0.0085	0.0083	0.0080	0.0078	0.0076	0.0074	0.0071
2.70	0.0069	0.0067	0.0065	0.0063	0.0061	0.0060	0.0058	0.0056	0.0054	0.0053
2.80	0.0051	0.0050	0.0048	0.0047	0.0045	0.0044	0.0042	0.0041	0.0040	0.0039
2.90	0.0037	0.0036	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028
3.00	0.0027	0.0026	0.0025	0.0024	0.0024	0.0023	0.0022	0.0021	0.0021	0.0020

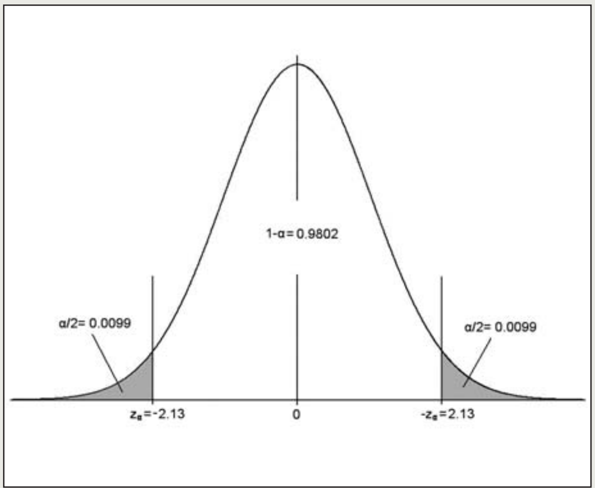

FIGURE 1. Normal curve showing the *Z* values and corresponding *P*-values for the data in example 1.

TABLE 2

	PROBABILITY					
d.f.	0.5	0.1	0.05	0.02	0.01	0.001
1	1.000	6.314	12.706	31.821	63.657	636.619
2	0.816	2.920	4.303	6.965	9.925	31.598
3	0.765	2.353	3.182	4.541	5.841	12.941
4	0.741	2.132	2.776	3.747	4.604	8.610
5	0.727	2.015	2.571	3.365	4.032	6.859
6	0.718	1.943	2.447	3.143	3.707	5.959
7	0.711	1.895	2.365	2.998	3.499	5.405
8	0.706	1.860	2.306	2.896	3.355	5.041
9	0.703	1.833	2.262	2.821	3.250	4.781
10	0.700	1.812	2.228	2.764	3.169	4.587
11	0.697	1.796	2.201	2.718	3.106	4.437
12	0.695	1.782	2.179	2.681	3.055	4.318
13	0.694	1.771	2.160	2.650	3.012	4.221
14	0.692	1.761	2.145	2.624	2.977	4.140
15	0.691	1.753	2.131	2.602	2.947	4.073
16	0.690	1.746	2.120	2.583	2.921	4.015
17	0.689	1.740	2.110	2.567	2.898	3.965
18	0.688	1.734	2.101	2.552	2.878	3.922
19	0.688	1.729	2.093	2.539	2.861	3.883
20	0.687	1.725	2.086	2.528	2.845	3.850
21	0.686	1.721	2.080	2.518	2.831	3.819
22	0.686	1.717	2.074	2.508	2.819	3.792
23	0.685	1.714	2.069	2.500	2.807	3.767
24	0.685	1.711	2.064	2.492	2.797	3.745
25	0.684	1.708	2.060	2.485	2.787	3.725
26	0.684	1.706	2.056	2.479	2.779	3.707
27	0.684	1.703	2.052	2.473	2.771	3.690
28	0.684	1.701	2.048	2.467	2.763	3.674
29	0.683	1.699	2.045	2.462	2.756	3.659
30	0.683	1.697	2.042	2.457	2.750	3.646
40	0.681	1.684	2.021	2.423	2.704	3.551
60	0.679	1.671	2.000	2.390	2.660	3.460
120	0.677	1.658	1.980	2.358	2.617	3.373
∞	0.674	1.645	1.960	2.326	2.576	3.291

TABLE 2. Distribution of *t* (two-tailed) taken from Swinscow & Campbell.[6]

◀ For example, the exam results for the first year of a medical degree are known to be approximately Normally distributed with mean 72 and standard deviation 8. To find the probability that a student will score 89 or more we first need to convert this value to a standard Normal deviate. In this instance, as we have a single value we use equation (1):

$$z = \frac{89 - 72}{8} = 2.13$$

If we wished to find the probability that an average of 10 scores is 75 or more we would use equation (2) to convert to the standard Normal distribution:

$$z = \frac{75 - 72}{8/\sqrt{10}} = 1.19$$

We then use the standard Normal table to find the probabilities of observing these *z* values, or values more extreme given that the population mean and standard deviation are 72 and 8 respectively.

Standard Normal tables can be either one-tailed or two-tailed. In the majority of hypothesis tests the direction of the difference is not specified, leading to a two-sided (or two-tailed) test.[1] The standard Normal table shown in table 1 is two-sided[†]. In this two-sided table the value tabulated is the probability, α, that a random variable, Normally distributed with mean zero and standard deviation one, will be either greater than *z* or less than $-z$ (as shown in the diagram at the top of the table). The total area under the curve represents the total probability space for the standard Normal distribution and sums to 1, and the shaded areas at either end are equal to $\alpha/2$. A one-tailed probability can be calculated by halving the tabulated probabilities in table 1. As the Normal distribution is symmetrical it is not necessary for tables to include the probabilities for both positive and negative *z* values.

WORKED EXAMPLES

From our first example above we want to know what the probability is that a student chosen at random will have a test score of 89, given a population mean of 72 and standard deviation of 8. The *z*-score calculated above is 2.13. In order to obtain the *P*-value that corresponds to this *z*-score we first look at the row in the table that corresponds to a *z*-score of 2.1. We then need to look down the column that is headed 0.03. The corresponding *P*-value is 0.0198. However, this is a two-sided probability and corresponds to the probability that a *z*-score is either –2.13 or 2.13 (see figure 1). To get the probability that a student chosen at random will have a test score of at least 89 we need to halve the tabulated *P*-value. This gives a *P*-value of 0.0099.

In a previous tutorial we used the Normal approximation to the binomial to examine whether there were significant differences in the proportion of patients with healed leg ulcers at 12 weeks, between standard treatment and treatment in a specialised leg ulcer clinic.[4] The null hypothesis was that there was no difference in healing rates between the two groups. From this test we obtained a *z* score of 0.673. Looking this up in table 1 we can see that it corresponds to a two-sided *P*-value of 0.503. Thus we cannot reject the null, and we conclude that there is no reliable evidence of a difference in ulcer healing rates at 12 weeks between the two groups.

STUDENT'S *t*-DISTRIBUTION

The *t*-test is used for continuous data to compare differences in means between two groups (either paired or unpaired).[2] It is based on Student's *t*-distribution (sometimes referred to as just the *t*-distribution). This distribution is particularly important when we wish to estimate the mean (or mean difference between groups) of a Normally distributed population but have only a small sample. This is because the *t*-test, based on the *t*-distribution, offers more precise estimates for small sample sizes than the tests associated with the Normal distribution. It is closely related to the Normal distribution and as the sample size tends towards infinity the probabilities of the *t*-distribution approach those of the standard Normal distribution.

The main difference between the *t*-distribution and the Normal distribution is that the *t* depends only on one parameter, v, the degrees of freedom (d.f.), not on the mean or standard deviation. The degrees of freedom are based on the sample size, *n*, and are equal to $n-1$. If the *t* statistic calculated in the test is greater than the critical value for the chosen level of statistical significance (usually $P = 0.05$) the null hypothesis for the particular test being carried out is rejected in favour of the alternative. The critical value that is compared to the *t* statistic is taken from the table of probabilities for the *t*-distribution, an extract of which is shown in table 2.

Unlike the table for the Normal distribution described above, the tabulated values relate to particular levels of statistical significance, rather than the actual *P*-values. Each of the columns represents the cut-off points for declaring statistical significance for a given level of (two-sided) significance. For example, the column headed 0.05 in table 2 gives the values which a calculated *t*-statistic must be above in order for a result to be statistically significant at the two-sided 5 per cent level. Each row represents the cut-offs for different degrees of freedom. Any test which results in a *t* statistic less than the tabulated value will not be statistically significant at that level and the *P*-value will be greater than the value indicated in the column heading. As the *t*-distribution is symmetrical about the mean, it is not necessary for tables to include the probabilities for both positive and negative *t* statistics.

Consider, for example, a *t*-test from which a *t* value of 2.66 on 30 d.f. was obtained. Looking at the row corresponding to 30 d.f. in table 2 this value falls between the tabulated values for 0.02 (=2.457) and 0.01 (=2.75). Thus, the *P*-value that corresponds with this particular *t* value will be less than 0.02, but greater than 0.01. In fact the actual (two-tailed) *P*-value is 0.012.

CHI-SQUARED DISTRIBUTION

The final statistical table being considered in this tutorial is that of the Chi-squared distribution. There are a wide range of statistical tests that lead to use of the Chi-squared distribution, the most common of which is the Chi-squared test described in a previous tutorial.[4] Like the *t*-distribution the Chi-squared distribution has only one parameter, the degrees of freedom, *k*. A section of the Chi-squared distribution is shown in table 3. Like the table for the *t* distribution described above the tabulated values are the Chi-squared values that relate to particular levels of statistical significance, rather than actual *P*-values. Each of the columns represents the cut-off points for declaring statistical significance for a given level of significance. For example, the column headed 0.05 in table 3 gives the values above which a calculated Chi-squared statistic must be in order for a result to be statistically significant at the two-sided 5 per cent level, for degrees of freedom ranging from 1 to 30. Any test which results in a Chi-squared statistic less than the tabulated value will not be statistically significant at that level and the *P*-value will be greater than the value at the top of the column. Consider, for example, a Chi-squared value of 4.2 on 1 d.f. Looking at the row corresponding to 1 d.f. in table 3 this value falls between the tabulated values for 0.05 (=3.841) and 0.02 (=5.412). Thus, the *P*-value that corresponds with this particular Chi-squared statistic will be less than 0.05, but greater than 0.02.

As a second example consider the results of a Chi-squared test that was used to assess whether leg ulcer healing rates differed between two different treatment groups (group 1: standard care; treatment 2: specialised leg ulcer clinic).[4] From this significance test a Chi-squared value of 0.243 with 1 d.f. was obtained. Looking at the 1 d.f. row in table 3 it can be seen that all the values are greater than this value, including the value that corresponds with a *P*-value of 0.5, 0.455. Thus we can conclude that the *P*-value corresponding to a Chi-squared value of 0.243 is greater than 0.5; in fact the exact value is 0.62.

SUMMARY

In this tutorial we have shown how to read statistical tables of *P*-values for the standard Normal, *t* and Chi-squared distributions, and given examples to show how the values from these tables are used to make decisions in a variety of basic statistical tests.

REFERENCES

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4. **Freeman JV, Julious SA.** The analysis of categorical data. *Scope* 2007; 16(1):18–21.
5. **Freeman JV, Julious SA.** The Normal distribution. *Scope* 2005; 14(4).
6. **Swinscow TDV, Campbell MJ.** *Statistics at square one.* 10th ed. London: BMJ Books, 2002.

TABLE 3

	PROBABILITY*					
d.f.	0.5	0.1	0.05	0.02	0.01	0.001
1	0.455	2.706	3.841	5.412	6.635	10.827
2	1.386	4.605	5.991	7.824	9.210	13.815
3	2.366	6.251	7.815	9.837	11.345	16.268
4	3.357	7.779	9.488	11.668	13.277	18.465
5	4.351	9.236	11.070	13.388	15.086	20.517
6	5.348	10.645	12.592	15.033	16.812	22.457
7	6.346	12.017	14.067	16.622	18.475	24.322
8	7.344	13.362	15.507	18.168	20.090	26.125
9	8.343	14.684	16.919	19.679	21.666	27.877
10	9.342	15.987	18.307	21.161	23.209	29.588
11	10.341	17.275	19.675	22.618	24.725	31.264
12	11.340	18.549	21.026	24.054	26.217	32.909
13	12.340	19.812	22.362	25.472	27.688	34.528
14	13.339	21.064	23.685	26.873	29.141	36.123
15	14.339	22.307	24.996	28.259	30.578	37.697
16	15.338	23.542	26.296	29.633	32.000	39.252
17	16.338	24.769	27.587	30.995	33.409	40.790
18	17.338	25.989	28.869	32.346	34.805	42.312
19	18.338	27.204	30.144	33.687	36.191	43.820
20	19.337	28.412	31.410	35.020	37.566	45.315
21	20.337	29.615	32.671	36.343	38.932	46.797
22	21.337	30.813	33.924	37.659	40.289	48.268
23	22.337	32.007	35.172	38.968	41.638	49.728
24	23.337	33.196	36.415	40.270	42.980	51.745
25	24.337	34.382	37.652	41.566	44.314	52.620
26	25.336	35.563	38.885	42.479	45.642	54.707
27	26.336	36.741	40.113	44.140	45.963	55.476
28	27.336	37.916	41.337	45.419	48.278	56.893
29	28.336	39.087	42.557	46.693	49.588	58.302
30	29.336	40.256	43.773	47.962	50.892	59.703

TABLE 3. Distribution of X^2 taken from Swinscow & Campbell.[6]
*These are two-sided *P*-values.

† A simple trick for seeing whether a particular table is one-tailed or two-tailed is to look at the value that corresponds to a cut-off of 1.96. If the tabulated *P*-value is 0.05 then the table is for two-tailed *P*-values.