1. Verify that

\[ 10^4 \sum_{n=0}^{\infty} \left(\frac{1}{10^4}\right)^n = \sum_{n=1}^{100} 2n. \]  

(10 marks)

2. Write down the first four terms of the Taylor series expansion of \( f(x) = e^{3x-1} \) about the point \( x = 0 \).

(10 marks)

3. Find and classify all the critical points of the function

\[ f(x, y) = x^3 + 2y^3 - 3x - 6y. \]

(10 marks)

4. Find:
   (i) \[ \int x \sin(x^2) \, dx \]
   (ii) \[ \int_{2}^{10} \frac{1}{x^2 - x} \, dx \]

(10 marks)
5 Let \( f(x,y) = x + y - 1 \) and \( D \subset \mathbb{R}^2 \) be the region bounded by the triangle with vertices \((0,0), (1,0), (0,1)\). Find

\[
\iint_D f(x,y) dA.
\]

(10 marks)

6 Use Gaussian elimination to solve the following system of equations:

\[
\begin{align*}
3x + 2y + 3z - 2w &= 1; \\
x + y + z &= 3; \\
x + 2y + z - w &= 2.
\end{align*}
\]

(10 marks)

7 Let

\[
A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad v = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.
\]

Show that \( \{v, Av, A^2v\} \) is a basis for \( \mathbb{R}^3 \). (10 marks)

8 Let \( a, b \) be real numbers. Find

\[
\begin{pmatrix} a & b \\ b & a \end{pmatrix}^{2014}
\]

**Hint:** It is easy to take powers of diagonal matrices. (10 marks)

End of Question Paper