Gaussianity in radar – rain gauge merging using kriging with external drift

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Merging Radar – Rain Gauge

Trying to keep the advantages of both:

- Radar: areal estimates, wide coverage, high spatial resolution
- Rain Gauges: higher accuracy
Kriging with External Drift

KED is one of the best and most efficient merging methods.

1. The estimate is based on the kriging interpolation of rain gauges.
2. The mean of the process is modelled as a linear function of the radar (external drift).
3. It also estimates the uncertainty associated with the prediction (kriging variance).
4. The process is assumed to be Gaussian.
Gaussian assumption

• Kriging methods assume the process to be Gaussian
• KED assumes the rainfall residuals to be Gaussian

\[
\text{Residuals} = \text{True rainfall} - \text{process mean (or drift)}
\approx \text{Rain gauge rain} - \text{linear function of radar rain}
\]

• Rainfall is not Gaussian, neither are the residuals.
• Transforming rainfall to a Gaussian variable improves Gaussianity of the residuals too.
Comparing methods

Possible solutions:

• Analytical transformations (Box-Cox)
• Empirical transformations (Normal Scores)
• Singularity analysis
Box-Cox Transformations:

\[ y = \begin{cases} 
\log(x) & \text{if } \lambda = 0 \\
\frac{x^{\lambda} - 1}{\lambda} & \text{if } \lambda \neq 0 
\end{cases} \]

1. \( \lambda = 0.5 \rightarrow \text{Square root} 
2. \( \lambda = 0.25 \rightarrow \text{Square root} - \text{Square root} 
3. \( \lambda = 0.1 \rightarrow \text{Almost Logarithmic} 
4. \text{Optimal time-variant } \lambda [0.2, 1] 

According to Erdin et al. (2012)
Normal Score Transformation (NST)

- Empirical relationship between quantiles
- It requires continuous strictly increasing CDF
- Some adaptations for rainfall

Rain Gauge data → transformation

Radar data → transformation

KED

Back transformation

KED merged rainfall estimate

Radar data transformation

Normal Score Transformation (NST)
Singularity analysis

• Fractal theories, adapted to Bayesian rainfall merging by Wang et al. (2015).

• **Local Singularity:** structure in which the areal average follows a power function of the considered area

• Singularities are characteristic of non-Gaussian structures, removing them makes a field more Gaussian.

• Need aereal characteristics, cannot be applied to point measurements in KED
Case study

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<td>Event 6</td>
<td>2009-12-05 15:00</td>
<td>2009-12-06 13:00</td>
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</tbody>
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Legend
- ◊ Radars
- ▀ Validation Rain Gauges
- ● Modelling Rain Gauges
- Study Area

[Map showing various locations and rain gauge data]
Evaluation techniques

How effective are the methods in generating Gaussian residuals?

How effective is the back-transformation in reproducing the original PDF of rain?

What is the quality and the reliability of the final rainfall product?
Gaussianity Test

- No Transformation
- Singularity Analysis
- Box-Cox
- Normal Scores
Rainfall distribution reconstruction

We use a linear function of the radar for the original distribution.
Validation with Rain Gauges

- X Singularity Analysis
- X Box-Cox $\lambda = 0.1$
- ✓ Other Box-Cox
- ✓ Normal Scores
- ✓ No Transformation
Qualitative evaluation

- Singularity Analysis
- No Transformation
- Box-Cox
- Normal Scores
1. Box-Cox with low $\lambda$ introduces a high bias

2. Singularity analysis not suitable for KED

3. Merging improves the results

4. Transformations are helpful, but more important in specific applications

5. Box-Cox with $\lambda = 0.5$ and $\lambda = 0.25$ have analytical back-transformation
Conclusions

• Square root or square root – square root transformations are recommended because of:
  ✓ Good skills
  ✓ Analytical back-transformation
  ✓ Simplicity

• Normal Score Transformation performs well, but more complex
• Box-Cox with low $\lambda$ and Singularity Analysis are not suitable
• Transformations improve the estimations, but not significantly
• In specific applications transformations may be important
Thank you!!!

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References:


