The laws of indices

Introduction

When a number is to be multiplied by itself a power, or an index, can be used to write this compactly. The plural of index is indices. In this leaflet we remind you of how this is done, and state a number of rules, or laws, which can be used to simplify expressions involving indices.

Powers, or indices

We write the expression
\[ 3 \times 3 \times 3 \times 3 \text{ as } 3^4 \]
We read this as ‘three to the power four’.

Similarly
\[ z \times z \times z = z^3 \]
We read this as ‘z to the power three’ or ‘z cubed’.

In the expression \( b^c \), the index is \( c \) and the number \( b \) is called the base. Your calculator will probably have a button to evaluate powers of numbers. It may be marked \( x^y \). Check this, and then use your calculator to verify that
\[ 7^4 = 2401 \quad \text{and} \quad 25^5 = 9765625 \]

Exercises

1. Without using a calculator work out the value of
   a) \( 4^2 \), b) \( 5^3 \), c) \( 2^5 \), d) \( \left( \frac{1}{2} \right)^2 \), e) \( \left( \frac{1}{3} \right)^2 \), f) \( \left( \frac{2}{3} \right)^3 \).

2. Write the following expressions more concisely by using an index.
   a) \( a \times a \times a \times a \), b) \( (yz) \times (yz) \times (yz) \), c) \( \left( \frac{a}{b} \right) \times \left( \frac{a}{b} \right) \times \left( \frac{a}{b} \right) \).

Answers

1. a) 16, b) 125, c) 32, d) \( \frac{1}{4} \), e) \( \frac{1}{5} \), f) \( \frac{8}{125} \).

2. a) \( a^4 \), b) \( (yz)^3 \), c) \( \left( \frac{a}{b} \right)^3 \).

The laws of indices

To manipulate expressions involving indices we use rules known as the laws of indices. The laws should be used precisely as they are stated - do not be tempted to make up variations of your own! The three most important laws are given here:
First law

\[ a^m \times a^n = a^{m+n} \]

When expressions with the same base are multiplied, the indices are added.

Example

We can write

\[ 7^6 \times 7^4 = 7^{6+4} = 7^{10} \]

You could verify this by evaluating both sides separately.

Example

\[ z^4 \times z^3 = z^{4+3} = z^7 \]

Second Law

\[ \frac{a^m}{a^n} = a^{m-n} \]

When expressions with the same base are divided, the indices are subtracted.

Example

We can write

\[ \frac{8^5}{8^3} = 8^{5-3} = 8^2 \quad \text{and similarly} \quad \frac{z^7}{z^4} = z^{7-4} = z^3 \]

Third law

\[ (a^m)^n = a^{mn} \]

Note that \( m \) and \( n \) have been multiplied to yield the new index \( mn \).

Example

\[ (6^4)^2 = 6^{4\times2} = 6^8 \quad \text{and} \quad (e^x)^y = e^{xy} \]

It will also be useful to note the following important results:

\[ a^0 = 1, \quad a^1 = a \]

Exercises

1. In each case choose an appropriate law to simplify the expression:
   a) \( 5^3 \times 5^{13} \),  
   b) \( 8^{13} \div 8^5 \),  
   c) \( x^6 \times x^5 \),  
   d) \( (a^3)^4 \),  
   e) \( y^7 \div y^y \),  
   f) \( \frac{x^4}{x^7} \).

2. Use one of the laws to simplify, if possible, \( a^6 \times b^5 \).

Answers

1. a) \( 5^{16} \),  
   b) \( 8^8 \),  
   c) \( x^{11} \),  
   d) \( a^{12} \),  
   e) \( y^4 \),  
   f) \( x^1 = x \).

2. This cannot be simplified because the bases are not the same.