Title: Probability 2

Target: On completion of this worksheet you should be able to find the probability of independent events and the probability of a combination of events using ‘and’ and ‘or’.

Suppose a coin is tossed and a dice is thrown. The events ‘get a head’ and ‘get a 4’ are independent events as one event does not affect the other. In this case

\[ P(\text{head and 4}) = P(\text{head}) \times P(4) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} \]

In general if A and B are two independent events then

\[ P(A \text{ and } B) = P(A) \times P(B) \]

Exercise

1. Find the probability of getting two tails when two coins are tossed.
2. A manufacturing process uses two machines. The probability that the first breaks down is 0.02 and the probability that the second breaks down is 0.07. Any breakdowns are independent of each other. What is the probability that both machines will breakdown?

(Answers: 0.25, 0.0014)

Examples

1. If a red dice and a blue dice are thrown what is the probability of getting a 2 on the red dice and a 3 on the blue dice?

These events are independent so

\[ P(2 \text{ on red and 3 on blue}) = P(2 \text{ on red}) \times P(3 \text{ on blue}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \]

2. A coin is tossed twice. Find the probability of getting 2 heads.

\[ P(2 \text{ H}) = P(H \text{ on 1}\text{st throw}) \times P(H \text{ on 2}\text{nd throw}) = 0.5 \times 0.5 = 0.25 \]

Examples

1. A manufacturer produces components and packs them in boxes of 20. In one particular box it is known that there are 3 defective items. If two components are picked from this box what is the probability that both are defective?

\[ P(1\text{st defective}) = \frac{3}{20} \]

Now, having picked one item, there are only 19 left in the box and 2 defectives. So

\[ P(2\text{nd defective}) = \frac{2}{19} \]

\[ P(\text{both defective}) = \frac{3}{20} \times \frac{2}{19} = \frac{3}{190} \]

2. A bag contains 5 red sweets and 7 blue sweets. Find the probability that if two sweets are picked they are not both red.

\[ P(2\text{red}) = \frac{5}{12} \times \frac{4}{11} = \frac{5}{33} \]

so

\[ P(\text{not both red}) = 1 - \frac{5}{33} = \frac{28}{33} \]
Examples cont.
3. A company manufactures CDs of which 3% are faulty. What is the probability of getting exactly one faulty CD in a box of 3?

\[ P(\text{faulty CD}) = 0.03 \quad P(\text{good CD}) = 0.97 \]

\[ P(\text{faulty, good, good}) = 0.03 \times 0.97 \times 0.97 = 0.028227 \]

But the order could be good, faulty, good or good, good, faulty. In each of these the probability is the same as above. These are mutually exclusive events so we can add the probabilities ie multiply by 3.

\[ P(1 \text{ faulty CD in box of 3}) = 3 \times 0.028227 = 0.0847 \text{ to 3 sf} \]

4. A bag contains 5 red balls and 7 green balls. Four balls are picked out of the bag. What is the probability that two are red and two are green?

5 red to choose from, 4 red left
7 green to choose from, 6 green left

\[ P(RRGG) = \frac{5}{12} \times \frac{4}{11} \times \frac{7}{10} \times \frac{6}{9} \]

The denominator decreases by 1 each time a ball is taken out. We must find the probability for all the different ways of getting 2 red and 2 green and then add these together.

\[ P(\text{RRGG}) = \frac{5 \times 4 \times 7 \times 6}{12 \times 11 \times 10 \times 9} \]

Notice that we have the same numerators but in a different order (the denominators are exactly the same). This will be the same for all cases so we must find how many ways there are of picking 2 red and 2 green balls: RRGG, RGRG, RGGR, GRRG, GRGR, GGRG. There are 6 different ways so

\[ P(2R \& 2G) = \frac{5 \times 4 \times 7 \times 6}{12 \times 11 \times 10 \times 9} \times \frac{6 \text{ green left}}{10} \]

\[ = \frac{33}{5} \]

Exercise
1. A company has 3 photocopying machines. The probability that a machine breaks down is 0.01. What is the probability that only one is working. (Assume the machine breakdowns are independent).

2. Four coins are tossed. Find the probability of exactly 3 heads.

3. A dice is thrown 3 times. What is the probability of getting at least 2 sixes.

4. A bag of sweets has 5 with soft centres and 10 with hard centres. Three sweets are picked at random and eaten. What is the probability that only 1 is soft centred.

(Answers: 0.000297, 0.25, 2/27, 45/91)

Notation: In the above we take account of what has already happened. We say ‘probability of a red given that a red has already been picked’ and write P(red I red already picked).

In general if A and B are two events then P(B | A) means probability of B occurring given that A has already occurred. Also \[ P(A \text{ and } B) = P(A) \times P(B | A) \]

Exercise
1. Find the probability of getting a multiple of 3 or an even number when a dice is thrown.

2. A card is drawn from a pack of 52 playing cards. What is the probability of drawing a picture card or a heart?

(Answers: \( \frac{3}{2}, \frac{11}{25} \))